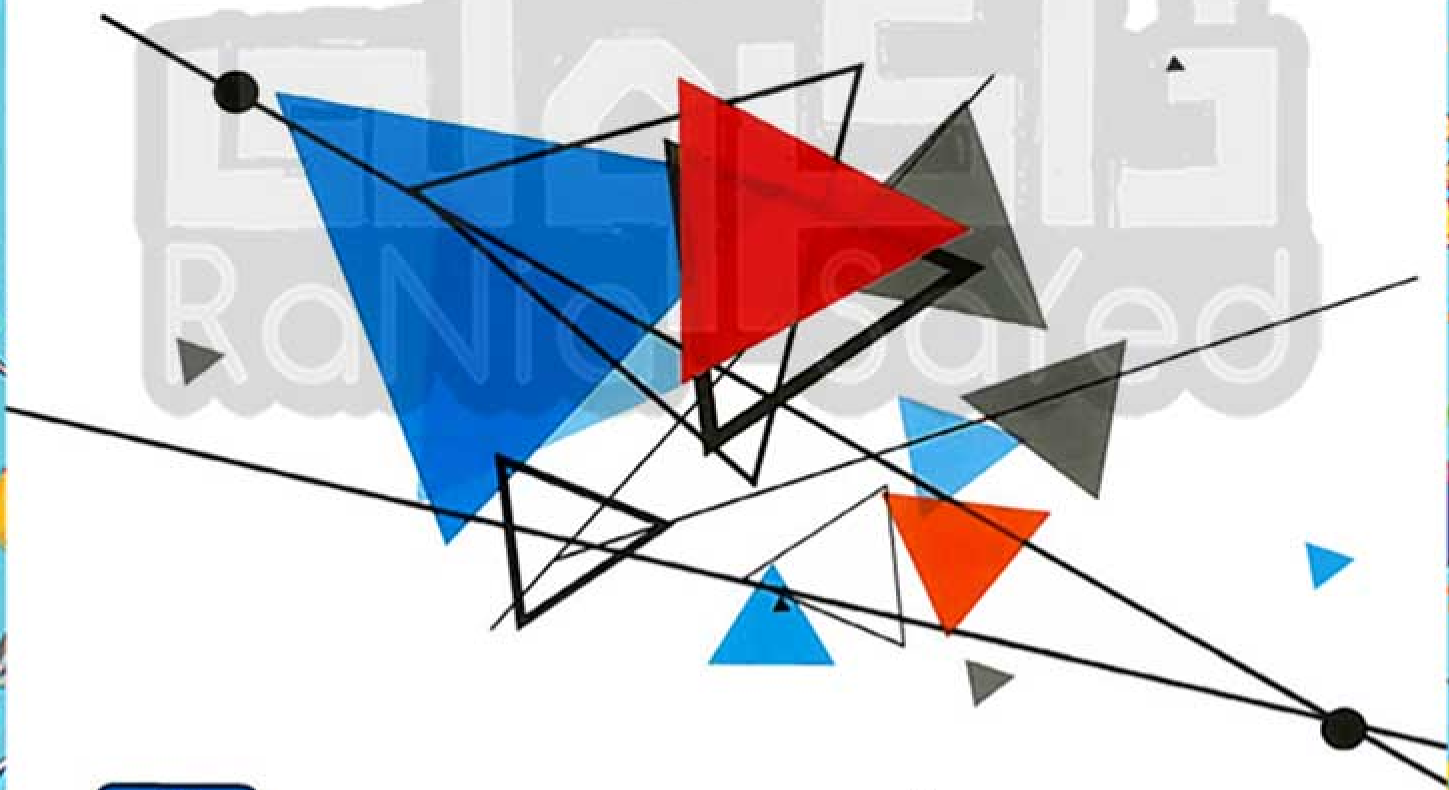
EL-MOASSER

In Mathematics

The Main Book

For 1st Prep.

Second Term



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Distribution of Maths Syllabus
First year preparatory - Second term

Month	Algebra and statistics	Geometry and measurement
	«One period weekly»	«One and half periods weekly»
February	**Numbers and algebra :** • Repeated multiplication in $\mathbb{Q}$ • Non-negative integer powers. • Negative integer powers.	• Deductive proof. • The polygon : - (convex - concave - regular) - The sum of measures of the interior angles of a polygon. - The sum of measures of the exterior angles of a polygon. - Parallelogram and its properties. - Parallelogram and its special cases.
March	• Scientific notation of the rational number. • Order of mathematical operations. • The square root of a perfect square rational number. • Solving equations in $\mathbb{Q}$ • Solving inequalities in $\mathbb{Q}$	• The triangle : - **Theorem (1) :** The sum of measures of the interior angles of a triangle is 180° - The exterior angles of a triangle. - **Theorem (2) :** The ray drawn from the midpoint of a side of a triangle parallel to ... - **Corollary :** The line segment joining the midpoints of two sides .. - **Theorem (3) :** The length of the line segment joining the midpoints of two sides ... • **Pythagoras' theorem**
April	**Statistics and probability :** • Samples • Probability	• Geometric transformations : - Reflection. - Translation. - Rotation.
May	General Exercises	

The used symbols in this book

$=$	is equal to
$\neq$	is not equal to
$\in$	belongs to
$\notin$	does not belong to
$\subset$	is a subset of
$\not\subset$	is not a subset of
$\cap$	intersection
$\cup$	union
$\varnothing$ or $\{\ \}$	empty set (null set)
$\mathbb{N}$	the set of natural numbers
$\mathbb{Z}$	the set of integers
$\mathbb{Q}$	the set of rational numbers
$<$	is less than
$>$	is more than
$\leq$	is less than or equal to
$\geq$	is more than or equal to
$\|a\|$	the absolute value of a
a^n	a to the power n (the n^{th} power of a)
$\sqrt{a}$	the positive square root of the rational number a
$\because$	since
$\therefore$	then , hence , therefore
$\overleftrightarrow{AB}$	the straight line AB
$\overrightarrow{AB}$	the ray AB
$\overline{AB}$	the line segment AB
AB	the length of the line segment AB
$\angle B$	the angle B
$m(\angle B)$	the measure of the angle B
$//$	is parallel to
$\perp$	is perpendicular to
$\llcorner$	a right angle
Δ	triangle
$\equiv$	is congruent to

CONTENTS

First Algebra and Statistics

UNIT 1

Numbers and Algebra

Lessons of the unit :

1. Repeated multiplication.
2. Non-negative integer powers.
3. Negative integer powers.
4. Scientific notation of the rational number.
5. Order of mathematical operations.
6. The square root of a perfect square rational number.
7. Solving equations in $\mathbb{Q}$.
8. Solving inequalities in $\mathbb{Q}$.

Unit Objectives :

By the end of this unit, student should be able to :

- remember what have been studied on repeated multiplication in $\mathbb{Z}$
- multiply repeated multiplication of the rational numbers.
- recognize the laws of powers in $\mathbb{Q}$
- recognize the negative power of the non-zero rational number.
- recognize the standard scientific notation of the rational number.
- write a rational number in the standard form.
- perform the mathematical operations according to the priority of their performances.
- recognize the square root of a perfect square rational number.
- find the square root of a perfect square rational number.
- solve an equation of the first degree in one unknown in $\mathbb{Q}$
- use the equation to solve word problems.
- solve an inequality of the first degree in one unknown in $\mathbb{Q}$

Ghiyath Al-Din Ibn Masoud Al-Kashi
(1380 A.D. - 1436 A.D.)

He was an Arab scientist who had many investigations in mathematics :

- He had invented the decimal fraction.
- He put a theory concerning the sum of the natural numbers that are raised to the fourth power.
- He reached a very accurate rate for the approximate ratio (π) that nearly equates the accuracy of the calculators.

Lesson 1

Repeated multiplication

We had known before in the set of integers that : $3^4 = 3 \times 3 \times 3 \times 3$ where we found that the number 3 has repeated 4 times in the multiplication operation and we read it as : «3 to the power 4»

Also , we can apply the previous on normal fractions :

For example: $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

From multiplying normal fractions , we find that :

$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} = \frac{2^4}{3^4}$ i.e. $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$

Generally

If $\frac{a}{b}$ is a rational number and n is a positive integer , then : $\left(\frac{a}{b}\right)^n = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}$ ... to n times

It is read as « $\frac{a}{b}$ to the power n » or «the n^{th} power of the number $\frac{a}{b}$ » i.e. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

For example: • $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125}$ • $(0.7)^2 = \left(\frac{7}{10}\right)^2 = \frac{7^2}{10^2} = \frac{49}{100}$

Remark

If $\frac{a}{b}$ is a rational number , then : $\left(\frac{a}{b}\right)^0 = 1$ where $a \neq 0$

For example: • $\left(\frac{1}{5}\right)^0 = 1$ • $\left(-\frac{3}{7}\right)^0 = 1$

Remark

If a is a rational number and m is a positive integer

then

$(-a)^m = (a)^m$ when m is an even number.	$(-a)^m = -(a)^m$ when m is an odd number.
For example: $\left(-\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$	For example: $\left(-\frac{1}{2}\right)^3 = -\left(\frac{1}{2}\right)^3 = -\frac{1}{8}$

Example 1 Find each of the following in the simplest form :

1. $\left(\frac{2}{3}\right)^2 \times \frac{9}{4}$
2. $\left(-\frac{5}{4}\right)^2 \times \left(\frac{2}{5}\right)^4$
3. $\left(3\frac{1}{2}\right)^2 \div \left(-10\frac{1}{2}\right)$
4. $\left(-\frac{2}{5}\right)^2 \times \left(-\frac{5}{2}\right)^3 \times \left(\frac{1}{5}\right)^0$

Solution

1. $\left(\frac{2}{3}\right)^2 \times \frac{9}{4} = \frac{2^2}{3^2} \times \frac{9}{4} = \frac{4}{9} \times \frac{9}{4} = 1$
2. $\left(-\frac{5}{4}\right)^2 \times \left(\frac{2}{5}\right)^4 = \frac{5^2}{4^2} \times \frac{2^4}{5^4} = \frac{25}{16} \times \frac{16}{625} = \frac{1}{25}$
3. $\left(3\frac{1}{2}\right)^2 \div \left(-10\frac{1}{2}\right) = \left(\frac{7}{2}\right)^2 \div \left(-\frac{21}{2}\right) = \frac{7^2}{2^2} \times \left(-\frac{2}{21}\right)$
   $= \frac{49}{4} \times \left(-\frac{2}{21}\right) = -\frac{7}{6}$
4. $\left(-\frac{2}{5}\right)^2 \times \left(-\frac{5}{2}\right)^3 \times \left(\frac{1}{5}\right)^0 = \frac{2^2}{5^2} \times \left(-\frac{5^3}{2^3}\right) \times 1 = \frac{4}{25} \times \left(-\frac{125}{8}\right) = -\frac{5}{2}$

TRY 1 by yourself

Find each of the following in its simplest form :

1. $\left(\frac{1}{5}\right)^2$
2. $\left(-\frac{2}{3}\right)^3$
3. $\left(-\frac{4}{5}\right)^4$
4. $\left(1\frac{1}{2}\right)^4$
5. $\left(-\frac{3}{9}\right)^2 \times \left(\frac{9}{4}\right)^2 \times \left(\frac{81}{16}\right)^0$

Final answers of try by yourself questions are at the end of each lesson to check your answer.

Unit 1

Example 2 If $x = -\frac{1}{2}$, $y = \frac{1}{4}$ and $z = 4$

Find the value of : $(x + y)^3 \times z^3$

Solution

$$(x + y)^3 \times z^3 = \left(-\frac{1}{2} + \frac{1}{4}\right)^3 \times 4^3 = \left(-\frac{2}{4} + \frac{1}{4}\right)^3 \times 4^3$$

$$= \left(-\frac{1}{4}\right)^3 \times 4^3 = -\frac{1^3}{4^3} \times 4^3 = -1$$

TRY 2 by yourself

If $x = -\frac{2}{3}$, $y = \frac{1}{2}$ and $z = -\frac{4}{3}$

, find the value of : $x^2 - y^2 z$

At the end of each lesson, you will find the final answers of try by yourself questions in the same form.

Answers of try by yourself

1 (1) $\frac{1}{25}$ (2) $-\frac{8}{27}$ (3) $\frac{256}{625}$

(4) $\frac{81}{16}$ (5) $\frac{9}{16}$

2 $x^2 - y^2 z = \frac{7}{9}$

Lesson 2

Non-negative integer powers

You have studied in the primary stage the laws of non-negative integer powers in $\mathbb{Z}$

In this lesson we will illustrate that these laws are also applicable for the rational numbers.

The first law

From the definition of repeated multiplication , you know that :

$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$, $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

i.e. $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^7$

Generally

If $\frac{a}{b}$ is a rational number , n and m are non-negative integers

, then : $\left(\frac{a}{b}\right)^n \times \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n+m}$

i.e. When multiplying the like bases, we add their powers (indices).

For example:

- $\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)^{3+2} = \left(\frac{2}{5}\right)^5$
- $\left(-\frac{1}{2}\right)^4 \times \left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right)^{4+3} = \left(-\frac{1}{2}\right)^7$

Unit 1

Example 1 Calculate each of the following , then put the result in its simplest form :

1. $\frac{2}{3} \times (\frac{2}{3})^2 \times (\frac{2}{3})^3$
2. $(-\frac{1}{3})^3 \times (\frac{1}{3})^2$
3. $\frac{3}{4} \times (-\frac{3}{4})^2$

Solution

1. $\frac{2}{3} \times (\frac{2}{3})^2 \times (\frac{2}{3})^3 = (\frac{2}{3})^{1+2+3} = (\frac{2}{3})^6 = \frac{2^6}{3^6} = \frac{64}{729}$

2. $(-\frac{1}{3})^3 \times (\frac{1}{3})^2 = -(\frac{1}{3})^3 \times (\frac{1}{3})^2$

$= -(\frac{1}{3})^5 = -\frac{1^5}{3^5} = -\frac{1}{243}$

Notice that : $(-\frac{1}{3})^3 = -(\frac{1}{3})^3$ because the index is an odd number.

3. $\frac{3}{4} \times (-\frac{3}{4})^2 = \frac{3}{4} \times (\frac{3}{4})^2$

$= (\frac{3}{4})^3 = \frac{3^3}{4^3} = \frac{27}{64}$

Notice that : $(-\frac{3}{4})^2 = (\frac{3}{4})^2$ because the index is an even number.

The second law

According to the first law , you know that : $a^6 = a^2 \times a^4$

, therefore : $a^6 \div a^2 = a^4$, $a^6 \div a^4 = a^2$

Generally

If $\frac{a}{b}$ is a rational number , where $\frac{a}{b} \neq 0$, n and m are non-negative integers , $n \geq m$

, then : $(\frac{a}{b})^n \div (\frac{a}{b})^m = (\frac{a}{b})^{n-m}$

i.e. When dividing like bases , we subtract their powers (indices).

For example:

- $(\frac{3}{8})^5 \div (\frac{3}{8})^2 = (\frac{3}{8})^{5-2} = (\frac{3}{8})^3$
- $(-\frac{2}{7})^4 \div (-\frac{2}{7})^2 = (-\frac{2}{7})^{4-2} = (-\frac{2}{7})^2$

Example 2 Calculate each of the following , then put the result in the simplest form :

1. $\left(\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5 \div \left(\frac{4}{5}\right)^7$
2. $\frac{2^5 \times 2^4}{2^6}$

Solution

1. $\left[\left(\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5\right] \div \left(\frac{4}{5}\right)^7 = \left(\frac{4}{5}\right)^{2+5} \div \left(\frac{4}{5}\right)^7$
$= \left(\frac{4}{5}\right)^7 \div \left(\frac{4}{5}\right)^7$
$= \left(\frac{4}{5}\right)^{7-7} = \left(\frac{4}{5}\right)^0 = 1$
2. $\frac{2^5 \times 2^4}{2^6} = \frac{2^{5+4}}{2^6} = \frac{2^9}{2^6} = 2^{9-6} = 2^3 = 8$

TRY 1 by yourself

Find each of the following in the simplest form :

1. $\left(\frac{1}{5}\right)^2 \times \left(\frac{1}{5}\right)^2$
2. $\left(\frac{3}{7}\right)^8 \div \left(\frac{3}{7}\right)^6$
3. $\left(-\frac{2}{3}\right)^5 \times \left(-\frac{2}{3}\right)^2 \div \left(-\frac{2}{3}\right)^6$
4. $\left(-\frac{1}{4}\right)^7 \div \left(\frac{1}{4}\right)^6 \times \frac{1}{4}$

Remark 1

- From the repeated multiplication , notice that :

$\left(\frac{3}{4} \times \frac{5}{7}\right)^3 = \left(\frac{3}{4} \times \frac{5}{7}\right) \times \left(\frac{3}{4} \times \frac{5}{7}\right) \times \left(\frac{3}{4} \times \frac{5}{7}\right)$
$= \left(\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right) \times \left(\frac{5}{7} \times \frac{5}{7} \times \frac{5}{7}\right)$
$= \left(\frac{3}{4}\right)^3 \times \left(\frac{5}{7}\right)^3$

Generally:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , n is a non-negative integer ,

then : $\boxed{\left(\frac{a}{b} \times \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \times \left(\frac{c}{d}\right)^n}$

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Remark 2

- From the repeated multiplication , notice that :

$$\left(\frac{2}{3} \div \frac{5}{11}\right)^4 = \left(\frac{\frac{2}{3}}{\frac{5}{11}}\right)^4 = \frac{\frac{2}{3}}{\frac{5}{11}} \times \frac{\frac{2}{3}}{\frac{5}{11}} \times \frac{\frac{2}{3}}{\frac{5}{11}} \times \frac{\frac{2}{3}}{\frac{5}{11}}$$

$$= \frac{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}}{\frac{5}{11} \times \frac{5}{11} \times \frac{5}{11} \times \frac{5}{11}}$$

$$= \left(\frac{2}{3}\right)^4 \div \left(\frac{5}{11}\right)^4$$

Generally:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , $\frac{c}{d} \neq 0$, n is a non-negative integer

, then : $\boxed{\left(\frac{a}{b} \div \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \div \left(\frac{c}{d}\right)^n}$ (where $\frac{c}{d} \neq 0$)

Example 3 Find the result of each of the following in its simplest form :

1 $\left(\frac{x\,y}{z}\right)^2$ 2 $\left(\frac{2x}{3y}\right)^4$

Solution

1 $\left(\frac{x\,y}{z}\right)^2 = \frac{(x\,y)^2}{z^2} = \frac{x^2 y^2}{z^2}$

2 $\left(\frac{2x}{3y}\right)^4 = \frac{2^4 x^4}{3^4 y^4} = \frac{16 x^4}{81 y^4}$

The third law

You know that : $(a^2)^3 = a^2 \times a^2 \times a^2$ and according to the first law : $a^2 \times a^2 \times a^2 = a^6$

i.e. $(a^2)^3 = a^6$

Generally

If $\frac{a}{b}$ is a rational number , n and m are non-negative integers

, then : $\left[\left(\frac{a}{b}\right)^n\right]^m = \left(\frac{a}{b}\right)^{n \times m}$

For example:

- $\left[\left(\frac{3}{5}\right)^3\right]^2 = \left(\frac{3}{5}\right)^{3 \times 2} = \left(\frac{3}{5}\right)^6$
- $\left[\left(-\frac{1}{2}\right)^4\right]^2 = \left(-\frac{1}{2}\right)^{4 \times 2} = \left(-\frac{1}{2}\right)^8$

Example 4 Find each of the following , in the simplest form :

1 $\left[\left(-2\frac{1}{2}\right)^2\right]^2$ 2 $\left(\frac{x^2}{y^3}\right)^3$ 3 $\frac{(-4x^3y^4)^2}{(-2xy^2)^4}$

Solution

1 $\left[\left(-2\frac{1}{2}\right)^2\right]^2 = \left(-2\frac{1}{2}\right)^{2\times 2} = \left(-2\frac{1}{2}\right)^4 = \left(2\frac{1}{2}\right)^4 = \left(\frac{5}{2}\right)^4 = \frac{5^4}{2^4} = \frac{625}{16}$

2 $\left(\frac{x^2}{y^3}\right)^3 = \frac{(x^2)^3}{(y^3)^3} = \frac{x^{2\times 3}}{y^{3\times 3}} = \frac{x^6}{y^9}$

3 $\frac{(-4x^3y^4)^2}{(-2xy^2)^4} = \frac{(-4)^2 \times x^{3\times 2} \times y^{4\times 2}}{(-2)^4 \times x^4 \times y^{2\times 4}} = \frac{\cancel{16}\, x^6 \cancel{y^8}}{\cancel{16}\, x^4 \cancel{y^8}} = x^{6-4} = x^2$

Example 5 If $x = \frac{1}{2}$, $y = -\frac{3}{4}$ and $z = \frac{3}{2}$, find the numerical value of each of the following in the simplest form :

1 $\left(\frac{x^2}{z}\right)^3$ 2 $\left(\frac{x^2 z}{y}\right)^2$

Solution

1 $\left(\frac{x^2}{z}\right)^3 = \left[\left(\frac{1}{2}\right)^2 \div \frac{3}{2}\right]^3 = \left(\frac{1^2}{2^2} \times \frac{2}{3}\right)^3$

$= \left(\frac{1}{4} \times \frac{2}{3}\right)^3 = \left(\frac{1}{6}\right)^3 = \frac{1^3}{6^3} = \frac{1}{216}$

Notice that :

$\frac{x^2}{z} = x^2 \div z$

2 $\left(\frac{x^2 z}{y}\right)^2 = \frac{x^{2\times 2} z^2}{y^2} = \frac{x^4 z^2}{y^2}$

$= \frac{\left(\frac{1}{2}\right)^4 \times \left(\frac{3}{2}\right)^2}{\left(-\frac{3}{4}\right)^2} = \frac{1^4}{2^4} \times \frac{\cancel{3^2}}{2^2} \times \frac{4^2}{\cancel{3^2}} = \frac{1}{16} \times \frac{1}{4} \times 16 = \frac{1}{4}$

TRY 2 by yourself

Calculate each of the following , then put the result in the simplest form :

1 $\left[\left(\frac{1}{2}\right)^2\right]^3$ 2 $\left(\frac{a^2 b^2}{c^3 d^4}\right)^2$ 3 $\left(\frac{5^2 \times 5^4}{5^5}\right)^2$

Answers *of try by yourself*

1 1 $\frac{1}{625}$ 2 $\frac{9}{49}$ 3 $-\frac{2}{3}$ 4 $-\frac{1}{16}$

2 1 $\frac{1}{64}$ 2 $\frac{a^4 b^4}{c^6 d^8}$ 3 25

Lesson 3 Negative integer powers

You know that , if a is a rational number , $a \neq 0$, then : $a^0 = 1$

therefore , $\frac{1}{a^n} = \frac{a^0}{a^n} = a^{0-n} = a^{-n}$ i.e. $a^{-n} = \frac{1}{a^n}$

Definition

If a is a rational number , $a \neq 0$ and n is a positive integer ,

then $a^{-n} = \frac{1}{a^n}$ and $a^n = \frac{1}{a^{-n}}$

For example:

- $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$
- $3 \times 5^{-1} = 3 \times \frac{1}{5} = \frac{3}{5}$
- $\frac{2}{5^{-2}} = 2 \times 5^2 = 2 \times 25 = 50$
- $0.1 = \frac{1}{10} = 10^{-1}$, $0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$, ... and so on.

Remarks

1. If a is a rational number , $a \neq 0$ and n is a positive integer ,

   then $a^n \times a^{-n} = a^n \times \frac{1}{a^n} = 1$ (the multiplicative neutral)

   i.e. each of a^n and a^{-n} is the multiplicative inverse of the other

2. If $\frac{a}{b}$ is a rational number not equal to zero and n is a positive integer

   , then : $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

   For example: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Example 1 Find the value of each of the following in the simplest form :

1. $2^4 \times 2^{-2}$
2. $\frac{5^{-2}}{5^{-3}}$
3. $(3^2)^{-2}$
4. $\frac{6^{-3} \times 6^5}{6^2}$
5. $\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2}$
6. $(7^3)^2 \times (7^{-2})^2$
7. $\left(\frac{3}{5}\right)^{-3} \div \left(\frac{4}{5}\right)^{-3}$

Solution

1. $2^4 \times 2^{-2} = 2^4 \times \frac{1}{2^2} = \frac{2^4}{2^2} = 2^{4-2} = 2^2 = 4$
2. $\frac{5^{-2}}{5^{-3}} = \frac{5^3}{5^2} = 5^{3-2} = 5$
3. $(3^2)^{-2} = \frac{1}{(3^2)^2} = \frac{1}{3^4} = \frac{1}{81}$
4. $\frac{6^{-3} \times 6^5}{6^2} = \frac{6^5}{6^3 \times 6^2} = \frac{6^5}{6^5} = 1$
5. $\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2} = \left(\frac{5^3 \times 5}{5^2 \times 5^4}\right)^{-2} = \left(\frac{5^4}{5^6}\right)^{-2}$
   $= \left(\frac{5^6}{5^4}\right)^2 = (5^{6-4})^2 = (5^2)^2 = 5^4 = 625$
6. $(7^3)^2 \times (7^{-2})^2 = (7^3)^2 \times \left(\frac{1}{7^2}\right)^2 = 7^6 \times \frac{1}{7^4} = 7^{6-4} = 7^2 = 49$
7. $\left(\frac{3}{5}\right)^{-3} \div \left(\frac{4}{5}\right)^{-3} = \left(\frac{5}{3}\right)^3 \div \left(\frac{5}{4}\right)^3 = \left(\frac{5}{3} \div \frac{5}{4}\right)^3$
   $= \left(\frac{\cancel{5}}{3} \times \frac{4}{\cancel{5}}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{4^3}{3^3} = \frac{64}{27}$

Remark

All laws of powers that we have studied in the previous lesson are correct in the case of the negative powers. So , the previous example can be solved by using laws of powers as follows :

1. $2^4 \times 2^{-2} = 2^{4+(-2)} = 2^2 = 4$
2. $\frac{5^{-2}}{5^{-3}} = 5^{-2-(-3)} = 5^{-2+3} = 5$
3. $(3^2)^{-2} = 3^{2 \times (-2)} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$
4. $\frac{6^{-3} \times 6^5}{6^2} = 6^{-3+5-2} = 6^0 = 1$

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5) $\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2} = \left(5^{3+(-2)-(-1)-4}\right)^{-2} = \left(5^{3-2+1-4}\right)^{-2}$

$= \left(5^{-2}\right)^{-2} = 5^{(-2)\times(-2)} = 5^4 = 625$

6) $(7^3)^2 \times (7^{-2})^2 = \left(7^3 \times 7^{-2}\right)^2 = \left(7^{3+(-2)}\right)^2 = 7^2 = 49$

7) $\left(\frac{3}{5}\right)^{-3} \div \left(\frac{4}{5}\right)^{-3} = \left(\frac{3}{5} \div \frac{4}{5}\right)^{-3} = \left(\frac{3}{\cancel{5}} \times \frac{\cancel{5}}{4}\right)^{-3} = \left(\frac{3}{4}\right)^{-3} = \left(\frac{4}{3}\right)^3 = \frac{4^3}{3^3} = \frac{64}{27}$

TRY 1 by yourself

Find the value of each of the following in the simplest form :

1) 5^{-3}

2) $\left(\frac{3}{7}\right)^{-2}$

3) $(2^{-3})^2$

4) $\left(\frac{2^{-2} \times 2^6}{2^3}\right)^{-3}$

Example 2 Simplify each of the following to the simplest form where $x \neq 0$:

1) $x^5 \times x^{-2} \times x^{-3}$

2) $(x^2)^{-3} \div (x^{-1})^2$

3) $\left(\frac{x^4 \times x^{-3}}{x^{-4} \times x}\right)^{-2}$

Solution

1) $x^5 \times x^{-2} \times x^{-3} = x^{5+(-2)+(-3)} = x^{5-2-3} = x^0 = 1$

2) $(x^2)^{-3} \div (x^{-1})^2 = x^{-6} \div x^{-2} = x^{-6-(-2)} = x^{-6+2} = x^{-4} = \frac{1}{x^4}$

3) $\left(\frac{x^4 \times x^{-3}}{x^{-4} \times x}\right)^{-2} = \left(x^{4+(-3)-(-4)-1}\right)^{-2} = \left(x^{4-3+4-1}\right)^{-2}$

$= (x^4)^{-2} = x^{-8} = \frac{1}{x^8}$

TRY 2 by yourself

Simplify each of the following to the simplest form putting the result in positive integer power where the denominator doesn't equal zero :

1) $(x^{-2})^{-5}$

2) $\left(\frac{a^4}{a^{-3}}\right)^{-2}$

3) $(y^5 \times y^{-2})^3$

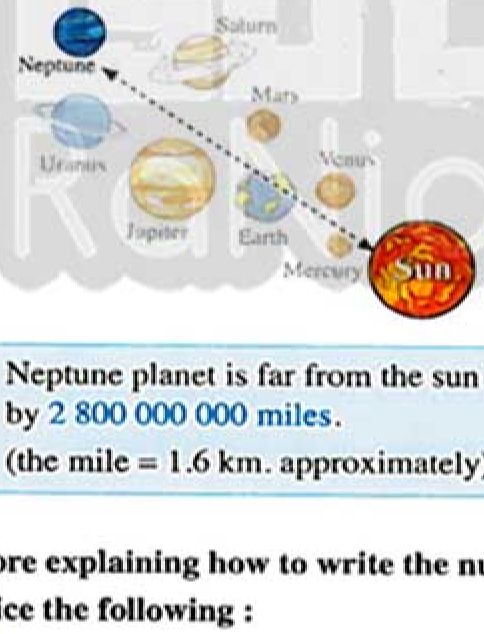
Answers of try by yourself

1 1) $\frac{1}{125}$ 2) $\frac{49}{9}$ 3) $\frac{1}{64}$ 4) $\frac{1}{8}$

2 1) x^{10} 2) $\frac{1}{a^{14}}$ 3) y^9

Lesson 4 Scientific notation of the rational number

- The scientific notation of the number is a useful method to deal with the very large numbers or the very small numbers like the numbers in the following examples.



Neptune planet is far from the sun by 2 800 000 000 miles.
(the mile = 1.6 km. approximately).

The diameter length of a virus ≈ 0.0000000025 cm.

- **Before explaining how to write the numbers in their scientific notation , we should notice the following :**

1 $10 = 10^1$, $100 = 10 \times 10 = 10^2$, $1000 = 10 \times 10 \times 10 = 10^3$ and so on

Hence we find that :

$2000 = 2 \times 1000 = 2 \times 10^3$, $50\,000 = 5 \times 10\,000 = 5 \times 10^4$

2 $0.1 = \frac{1}{10} = 10^{-1}$, $0.01 = \frac{1}{100} = \frac{1}{10 \times 10} = 10^{-2}$,

$0.001 = \frac{1}{1000} = \frac{1}{10 \times 10 \times 10} = 10^{-3}$ and so on

Hence we find that :

- $0.03 = \frac{3}{100} = \frac{3}{10 \times 10} = 3 \times 10^{-2}$
- $0.0007 = \frac{7}{10\,000} = \frac{7}{10 \times 10 \times 10 \times 10} = 7 \times 10^{-4}$

The standard scientific notation of a number

The number is written in the standard form as : $a \times 10^n$ where $1 \le |a| < 10$ and $n \in \mathbb{Z}$

There are examples for some numbers written in its standard form :

- 4.6×10^8
- 5.236×10^{-6}
- -9.6×10^{10}
- -1.001×10^{-5}
- -3×10^{12}
- 1×10^{-7}

Each of the previous numbers is the product of two numbers :

- The first number could be positive or negative and its absolute value must be greater than or equal 1 and less than 10
- The second number expresses the powers of the number 10 (These powers could be positive or negative)

Examples for some numbers not in the standard form :

- 8 200 000 000
- 0.000 000 135
- 45×10^8 (because $45 > 10$)
- 706.4×10^5 (because $706.4 > 10$)
- 0.248×10^{-7} (because $0.248 < 1$)
- -0.0015×10^{-9} (because $|-0.0015| < 1$)

and the next example will show how to write these numbers in the standard form.

Writing the number in the standard form

Example 1 Write each of the following numbers in the standard form :

1 8 200 000 000
2 0.000 000 135
3 45×10^8
4 706.4×10^5
5 0.248×10^{-7}
6 $-0.001\,5 \times 10^{-9}$

Solution

1 $8\,200\,000\,000.0 = 8.2 \times 10^9$

Moving the decimal point 9 places towards left

Using the power 9 of the number 10

2 $0.000\,000\,135 = 1.35 \times 10^{-7}$

Moving the decimal point 7 places towards right

Using the power -7 of the number 10

3 To put 45.0×10^8 in the standard form we move the decimal point one place to the left , then we multiply by 10

$\therefore 45.0 \times 10^8 = 4.5 \times 10^8 \times 10 = 4.5 \times 10^9$

4 To put 706.4×10^5 in the standard form we move the decimal point two places to the left , then we multiply by 10^2

$\therefore 706.4 \times 10^5 = 7.064 \times 10^5 \times 10^2 = 7.064 \times 10^7$

5 To put 0.248×10^{-7} in the standard form we move the decimal point one place to the right , then we multiply by 10^{-1}

$\therefore 0.248 \times 10^{-7} = 2.48 \times 10^{-7} \times 10^{-1} = 2.48 \times 10^{-8}$

6 To put -0.0015×10^{-9} in the standard form we move the decimal point three places to the right , then we multiply by 10^{-3}

$\therefore -0.0015 \times 10^{-9} = -1.5 \times 10^{-9} \times 10^{-3} = -1.5 \times 10^{-12}$

Remark

The standard form for 1 is 1×10^{zero}

, So is the standard form for 2 is 2×10^{zero} , etc ...

Unit 1

TRY 1 by yourself

In the following , determine the numbers that are not in the standard form , then write them in the standard form :

1. 8.5×10^{-4}
2. 17×10^{8}
3. 0.5×10^{-7}
4. 530.5×10^{9}
5. -0.999×10^{-5}
6. 6×10^{6}
7. 650 000 000
8. 0.000 001 02
9. -2.5×10^{8}

Operations on the numbers in the standard form

Example 2 Write the result of each of the following in the standard form :

1. $(1.2 \times 10^{5}) \times (4 \times 10^{3})$
2. $(6.5 \times 10^{4}) \times (8 \times 10^{2})$
3. $(2.4 \times 10^{11}) \div (1.2 \times 10^{-4})$
4. $(6.6 \times 10^{7}) \times (3 \times 10)^{4}$
5. $(2.3 \times 10^{6}) + (3.7 \times 10^{5})$

Solution

1. $(1.2 \times 10^{5}) \times (4 \times 10^{3}) = (1.2 \times 4) \times (10^{5} \times 10^{3}) = 4.8 \times 10^{8}$
2. $(6.5 \times 10^{4}) \times (8 \times 10^{2}) = (6.5 \times 8) \times (10^{4} \times 10^{2})$
   $= 52 \times 10^{6}$
   $= 5.2 \times 10^{7}$

Notice that : 52×10^{6} is not in the standard form , then we should put it in the standard form.

3. $(2.4 \times 10^{11}) \div (1.2 \times 10^{-4}) = \frac{2.4}{1.2} \times \frac{10^{11}}{10^{-4}} = 2 \times 10^{15}$
4. $(6.6 \times 10^{7}) \times (3 \times 10)^{4} = (6.6 \times 10^{7}) \times (3^{4} \times 10^{4})$
   $= (6.6 \times 3^{4}) \times (10^{7} \times 10^{4})$
   $= 534.6 \times 10^{11} = 5.346 \times 10^{13}$
5. $(2.3 \times 10^{6}) + (3.7 \times 10^{5}) = 10^{5} (2.3 \times 10 + 3.7)$
   $= 10^{5} (23 + 3.7) = 10^{5} \times 26.7 = 2.67 \times 10^{6}$

Example 3 Write the result of each of the following in the standard form :

1. $30\,000 \times 400\,000$
2. $140\,000 \times 0.005$
3. $0.000\,015 \div 30$
4. $(50\,000)^3$
5. $(0.000\,3)^5$
6. $(-0.001)^6$

Solution

1. $30\,000 \times 400\,000 = (3 \times 10^4) \times (4 \times 10^5) = (3 \times 4) \times (10^4 \times 10^5)$
$= 12 \times 10^9 = 1.2 \times 10^{10}$

2. $140\,000 \times 0.005 = (1.4 \times 10^5) \times (5 \times 10^{-3})$
$= (1.4 \times 5) \times (10^5 \times 10^{-3}) = 7 \times 10^2$

3. $0.000\,015 \div 30 = (1.5 \times 10^{-5}) \div 3 \times 10$
$= \frac{1.5}{3} \times \frac{10^{-5}}{10} = 0.5 \times 10^{-6} = 5 \times 10^{-7}$

4. $(50\,000)^3 = (5 \times 10^4)^3 = 5^3 \times 10^{12} = 125 \times 10^{12} = 1.25 \times 10^{14}$

5. $(0.000\,3)^5 = (3 \times 10^{-4})^5 = 3^5 \times 10^{-20} = 243 \times 10^{-20} = 2.43 \times 10^{-18}$

6. $(-0.001)^6 = (0.001)^6 = (1 \times 10^{-3})^6 = 1^6 \times 10^{-18} = 10^{-18}$

TRY 2 by yourself

Write the result of each of the following in the standard form :

1. $(5.3 \times 10^7) \times (3 \times 10^5)$
2. $0.000\,6 \div 20$
3. $(400\,000)^2$
4. $(3.2 \times 10^9) - (0.2 \times 10^8)$

Answers of try by yourself

1 [2] 1.7×10^9 [3] 5×10^{-8} [4] 5.305×10^{11} [5] -9.99×10^{-6}

[7] 6.5×10^8 [8] 1.02×10^{-6}

2 [1] 1.59×10^{13} [2] 3×10^{-5} [3] 1.6×10^{11} [4] 3.18×10^9

Lesson 5 Order of mathematical operations

We know that : addition , subtraction , multiplication and division are the basic mathematical operations which are performed on the numbers.

Sometimes one problem contains the four operations or some of them , so it is necessary to be agreed on rules which determine the priority of performance of these operations.

The following situation shows the importance of that :

The following problem was given to each of Heba and Ahmed. Calculate : $3 + 4 \times 2$

Their answers were as follows :

Heba multiplied at first , then she added.

She got : 11

Ahmed added at first , then he multiplied.

He got : 14

For the results are different, then it is necessary to be agreed on some rules that determine the order of performing the mathematical operations , that are :

First Order of performing the mathematical operations in an expression has no parentheses

1. Evaluate the powers.
2. Perform multiplications and divisions in order from left to right.
3. Perform additions and subtractions in order from left to right.

Second Order of performing the mathematical operations in an expression has parentheses

1. Perform the operations within parentheses (interior parentheses then exterior ones).
2. Evaluate the powers.
3. Perform multiplications and divisions in order from left to right.
4. Perform additions and subtractions in order from left to right.

- According to these rules , we can determine that Heba held the correct answer because she performed the multiplication operation first , then the addition operation.

Notice that :

The scientific calculators and computers follow the same rules of ordering the operations.

Example 1 Calculate the value of each of the following :

1. $3 + 6 \times (5 + 4) \div 3 - 7$
2. $9 - 5 \div (8 - 3) \times 2 + 6$

Solution

1. $3 + 6 \times (5 + 4) \div 3 - 7 = 3 + 6 \times 9 \div 3 - 7$ (parentheses)
   $= 3 + 54 \div 3 - 7$ (multiplication)
   $= 3 + 18 - 7$ (division)
   $= 21 - 7$ (addition)
   $= 14$ (subtraction)

2 $9 - 5 \div (8 - 3) \times 2 + 6 = 9 - 5 \div 5 \times 2 + 6$ (parentheses)
$= 9 - 1 \times 2 + 6$ (division)
$= 9 - 2 + 6$ (multiplication)
$= 7 + 6$ (subtraction)
$= 13$ (addition)

Example 2 **Calculate the value of each of the following :**

1 $4 - 3\left[4 - 2(6 - 3)\right] \div 2$ 2 $16 \div \left[8 - 3(4 - 2)\right] + 1$

Solution

1 $4 - 3\left[4 - 2(6 - 3)\right] \div 2 = 4 - 3\left[4 - 2 \times 3\right] \div 2$ (the interior parentheses)
$= 4 - 3\left[4 - 6\right] \div 2$ (multiplication inside parentheses)
$= 4 - 3\left[-2\right] \div 2$ (subtraction inside parentheses)
$= 4 + 6 \div 2$ (multiplication by parentheses)
$= 4 + 3$ (division)
$= 7$ (addition)

2 $16 \div \left[8 - 3(4 - 2)\right] + 1 = 16 \div \left[8 - 3 \times 2\right] + 1$ (the interior parentheses)
$= 16 \div \left[8 - 6\right] + 1$ (multiplication inside parentheses)
$= 16 \div 2 + 1$ (subtraction inside the parentheses)
$= 8 + 1$ (division)
$= 9$ (addition)

Example 3 **Calculate the value of each of the following :**

1 $8 \times 2^2 - 7 \times (4 + 1)$ 2 $2 + 3\left[5 + (4 - 1)^2\right]$
3 $3\left[(3^2 + 1) - (2^3 - 2)\right]$

Solution

1 $8 \times 2^2 - 7 \times (4 + 1) = 8 \times 2^2 - 7 \times 5$ (addition inside parentheses)
$= 8 \times 4 - 7 \times 5$ (powers)
$= 32 - 35$ (multiplication)
$= -3$ (subtraction)

2 $2 + 3\left[5 + (4 - 1)^2\right] = 2 + 3\left[5 + 3^2\right]$ (subtraction inside interior parentheses)

$= 2 + 3\left[5 + 9\right]$ (powers inside parentheses)

$= 2 + 3 \times 14$ (addition inside parentheses)

$= 2 + 42$ (multiplication)

$= 44$ (addition)

3 $3\left[(3^2 + 1) - (2^3 - 2)\right] = 3\left[(9 + 1) - (8 - 2)\right]$ (powers)

$= 3\left[10 - 6\right]$ (the interior parentheses)

$= 3 \times 4$ (subtraction inside parentheses)

$= 12$ (multiplication)

Remark

In the problems containing fractions , we should perform the operations in the numerator and denominator before division.

Example 4 Calculate the value of each of the following :

1 $\frac{36 - 6}{3 + 12}$

2 $\frac{11 - (5 - 4)}{5^2 - 10 \times 2}$

3 $7 + 8 \div \frac{4 + 12 - 2}{3^2 - 2} - (2^3 + 2)$

Solution

1 $\frac{36 - 6}{3 + 12} = \frac{30}{15} = 2$

2 $\frac{11 - (5 - 4)}{5^2 - 10 \times 2} = \frac{11 - 1}{25 - 20} = \frac{10}{5} = 2$

3 $7 + 8 \div \frac{4 + 12 - 2}{3^2 - 2} - (2^3 + 2) = 7 + 8 \div \frac{14}{7} - (2^3 + 2)$

$= 7 + 8 \div 2 - 10 = 7 + 4 - 10 = 1$

TRY by yourself

Calculate the value of each of the following :

1 $20 \div (12 - 2) \times 3^2 - 2$

2 $\frac{6 \times 3 + 10 \div 5}{2 - (10 - 2^2)}$

Answers of try by yourself

1 16

2 −5

Lesson 6 The square root of a perfect square rational number

Definition :

The square root of the perfect square rational number «a» is the number whose square equals «a»

For example:

- The number $\boxed{6}$ is a square root of the number $\boxed{36}$ because : $6^2 = 36$
- Also , the number $\boxed{-6}$ is a square root of the number $\boxed{36}$ because : $(-6)^2 = 36$

i.e. Finding the square root is the inverse operation of finding the square of a number. It means that , to find the square root of a number , we search a number which , when multiplied by itself , equals this number.

Generally	Examples :
• The **positive** square root of the number a is symbolized by $\sqrt{a}$	The positive square root of 25 is $\sqrt{25} = 5$
• The **negative** square root of the number a is symbolized by $-\sqrt{a}$	The negative square root of 16 is $-\sqrt{16} = -4$
• The two square roots of the number a is symbolized by $\pm\sqrt{a}$ which means $\sqrt{a}$, $-\sqrt{a}$, and each of them is the additive inverse of the other.	The two square roots of 49 are $\pm\sqrt{49} = \pm 7$

Remarks

1. $\sqrt{0} = 0$
2. In the set of rational numbers it is meaningless to find $\sqrt{a}$ if a is a negative rational number because there is no rational number if it is multiplied by itself , the result will be negative.
3. $\sqrt{a^2} = |a|$

   For example: • $\sqrt{(-3)^2} = |-3| = 3$ • $\sqrt{\left(-\frac{4}{5}\right)^2} = \left|-\frac{4}{5}\right| = \frac{4}{5}$
4. $\sqrt{a^2 b^2} = \sqrt{(ab)^2} = |ab|$

   For example: $\sqrt{a^4 b^6} = \sqrt{(a^2 b^3)^2} = |a^2 b^3|$
5. If $x^2 = a$ where $a \geq 0$, then $x = \pm\sqrt{a}$

Example 1 Find each of the following in the simplest form :

1. $\sqrt{36}$
2. $-\sqrt{\frac{16}{25}}$
3. $\pm\sqrt{2\frac{1}{4}}$
4. $\sqrt{\left(-\frac{2}{7}\right)^2}$
5. $-\sqrt{0.25}$
6. $\pm\sqrt{\frac{3.6}{10}}$
7. $\sqrt{16+9}$
8. $\sqrt{100-36}$
9. $\sqrt{\frac{36\,a^8}{49\,d^4}}$

Solution

1. $\sqrt{36} = 6$ because $6^2 = 36$
2. $-\sqrt{\frac{16}{25}} = -\frac{4}{5}$ because $\left(\frac{4}{5}\right)^2 = \frac{16}{25}$
3. $\pm\sqrt{2\frac{1}{4}} = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$
4. $\sqrt{\left(-\frac{2}{7}\right)^2} = \left|-\frac{2}{7}\right| = \frac{2}{7}$
5. $-\sqrt{0.25} = -\sqrt{\frac{25}{100}} = -\frac{5}{10} = -\frac{1}{2}$
6. $\pm\sqrt{\frac{3.6}{10}} = \pm\sqrt{\frac{36}{100}} = \pm\frac{6}{10} = \pm\frac{3}{5}$
7. $\sqrt{16+9} = \sqrt{25} = 5$
8. $\sqrt{100-36} = \sqrt{64} = 8$
9. $\sqrt{\frac{36\,a^8}{49\,d^4}} = \frac{6\,a^4}{7\,d^2}$

Notice that :
When there is an addition or a subtraction operation under the square root , it must be performed first before finding the square root.

Unit 1

TRY 1 by yourself

Complete the following :

1) $\sqrt{64} =$ ....
2) $-\sqrt{900} =$ ....
3) $-\sqrt{\frac{36}{25}} =$ ....
4) $\pm\sqrt{6\frac{1}{4}} =$ ....
5) $\sqrt{0.64} =$ ....
6) $\sqrt{100-64} =$ ....

Remark

In some cases , it is more easy to use factorization in finding the square root of a number , to do that we factorize the given number into its prime factors , then we take one factor from each two equal factors , then the product of these taken factors is the square root of this number.

Example 2 Find : $\sqrt{441}$

Solution

$\because 441 = 3 \times 3 \times 7 \times 7$

$\therefore \sqrt{441} = 3 \times 7$

$= 21$

3	441
3	147
7	49
7	7
	1

Example 3 Simplify each of the following to the simplest form :

1) $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times (\frac{2}{7})^2$
2) $(-\frac{3}{2})^2 \times \sqrt{\frac{64}{9}} \times (\frac{5}{2})^0$
3) $(2\,\frac{7}{9})^2 \div \sqrt{\frac{25}{9}}$

Solution

1) $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times (\frac{2}{7})^2 = -\frac{2}{7} \times \frac{7}{2} \times \frac{4}{49} = -\frac{4}{49}$

2) $(-\frac{3}{2})^2 \times \sqrt{\frac{64}{9}} \times (\frac{5}{2})^0 = \frac{9}{4} \times \frac{8}{3} \times 1 = 6$

3) $(2\,\frac{7}{9})^2 \div \sqrt{\frac{25}{9}} = (\frac{25}{9})^2 \div \frac{5}{3} = ((\frac{5}{3})^2)^2 \div \frac{5}{3}$

$= (\frac{5}{3})^4 \div \frac{5}{3} = (\frac{5}{3})^{4-1} = (\frac{5}{3})^3 = \frac{125}{27}$

TRY 2 by yourself

Simplify to the simplest form :

1. $\left(\frac{2}{3}\right)^2 \times \sqrt{\frac{81}{16}} \times \left(\frac{7}{9}\right)^0$

2. $\frac{5}{7} \times \sqrt{\frac{49}{36}} \div \left(-\frac{5}{3}\right)^2$

Example 4

The base length of a triangle is 16 cm. and its corresponding height is 8 cm. Find the side length of a square having the same area of that triangle.

Solution

$\because$ The area of the triangle = $\frac{1}{2}$ of the base length × its corresponding height.

$\therefore$ The area of the triangle = $\frac{1}{2} \times 16 \times 8 = 64$ cm^2.

$\therefore$ The area of the square = 64 cm^2.

$\therefore$ The side length of the square = $\sqrt{64} = 8$ cm.

TRY 3 by yourself

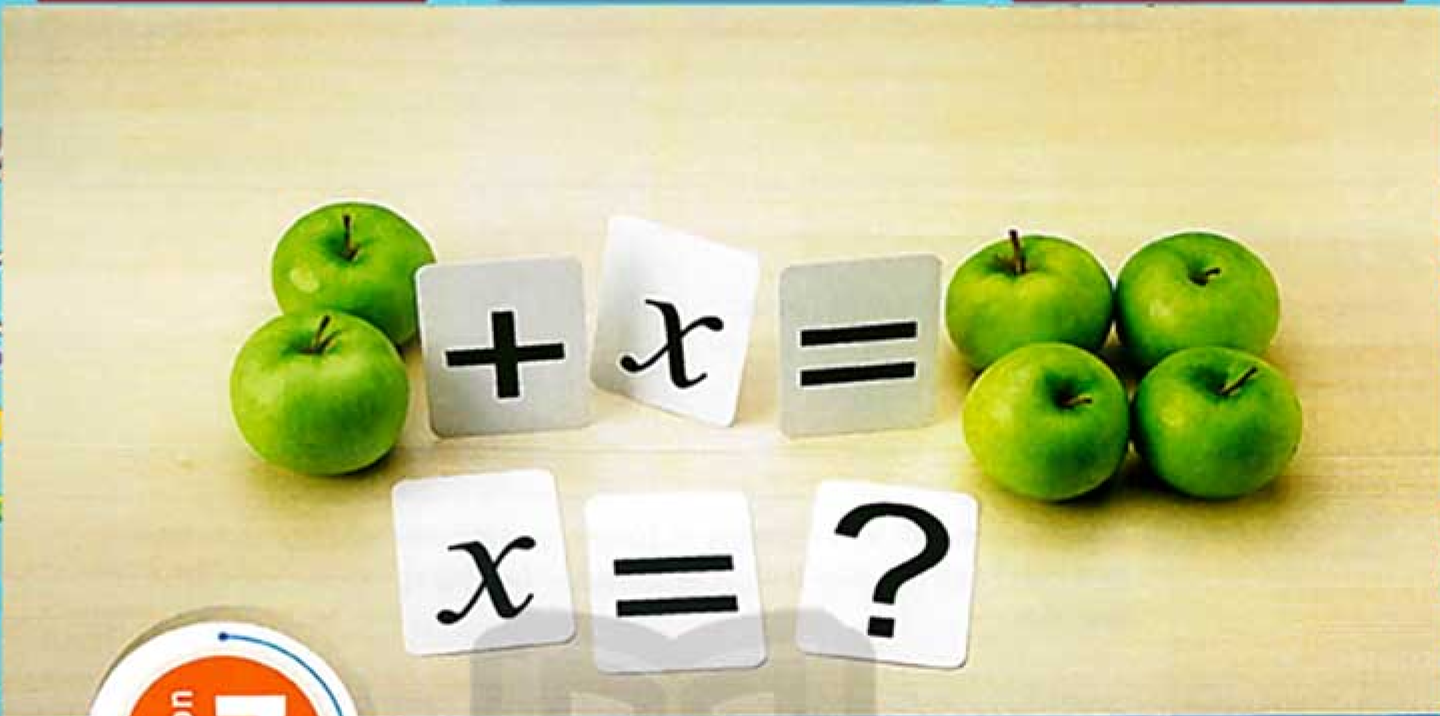
The area of a square is 1.44 cm^2. Find its perimeter.

Answers *of try by yourself*

1 1. 8 2. −30 3. $-\frac{6}{5}$
4. $\pm\frac{5}{2}$ 5. 0.8 6. 6

2 1. 1 2. 0.3

3 The perimeter of square = 4.8 cm.



Lesson 7

Solving equations in $\mathbb{Q}$

The equation

is a mathematical statement which contains one variable as x (or more as x and y) and contains equality relation « = »

as : $2x = 6$, $x + 3 = 5$, $2x - y = 3$ and $x^2 = 25$

The degree of the equation

is determined by the heighest degree of the terms forming the equation.

For example:

- $5x + 2 = 7$ is an equation of **the first** degree in one unknown x
- $x^2 + x - 3 = 0$ is an equation of **the second** degree in one unknown x
- $2x + 3y = 5$ is an equation of **the first** degree in two unknowns x and y

The substitution set

is the set that contains the probable values of the unknown.

The solution set (The S.S.)

is the set whose elements satisfy the equality of the equation and it is a subset of the substitution set.

For example:

If $x + 3 = 5$ and the substitution set is $\{2, 3\}$

- Putting $x = 2$, **we get** that the left side = $2 + 3 = 5$ = the right side.

  i.e. $x = 2$ is a solution of the equation.

- Putting $x = 3$, **we get** that the left side = $3 + 3 = 6$ $\neq$ the right side.

  i.e. $x = 3$ is not a solution of the equation.

$\therefore$ The S.S. = $\{2\}$ and it is a subset of the substitution set $\{2, 3\}$

- The previous method for solving the equation is called substitution method and we notice that it is a long method and it may be impossible if the number of elements of the substitution set is infinite as we see in $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$

  Therefore, we will use another easier method that will need studying the properties of the equality relation to enable us to put the unknown x in one side of the equation alone.

The properties of the equality relation

• We can add any rational number to both sides of the equation. For example: If $x - 1 = 5$, then $x - 1 + 1 = 5 + 1$ i.e. $\boxed{x = 6}$	• We can subtract any rational number from both sides of the equation. For example: If $x + 3 = 2$, then $x + 3 - 3 = 2 - 3$ i.e. $\boxed{x = -1}$
• We can multiply both sides of the equation by the same rational number. For example: If $\frac{1}{5} x = 2$, then $\frac{1}{5} x \times 5 = 2 \times 5$ i.e. $\boxed{x = 10}$	• We can divide both sides of the equation by the same rational number not equal to zero. For example: If $7 x = 14$, then $\frac{7 x}{7} = \frac{14}{7}$ i.e. $\boxed{x = 2}$

Then by applying any of the previous properties on any equation, then we will get an equivalent equation to the origin equation that has the same solution.

Generally : If a , b and c are three rational numbers , then these numbers have the following properties :

1 If $a = b$	then $a + c = b + c$
2 If $a + c = b + c$	then $a = b$
3 If $a = b$	then $a \times c = b \times c$
4 If $a \times c = b \times c$, $c \neq 0$	then $a = b$

Unit 1

The following examples show how to use the equality properties to solve an equation of the first degree in one unknown.

Example 1 Find the solution set of the equation $x + 5 = 4$ if the substitution set is :

1. $\mathbb{Z}$
2. $\mathbb{N}$

Solution

1 If the substitution set is $\mathbb{Z}$

$\because x + 5 = 4$

Adding -5 to both sides

(-5 is the additive inverse of 5)

$\therefore x + 5 + (-5) = 4 + (-5)$

i.e. $x + 0 = -1$ **i.e.** $x = -1$

You can check the truth of your solution by substituting by $x = -1$ in the origin equation , you will get the left side $= -1 + 5 = 4 =$ the right side.

$\therefore$ The S.S. $= \{-1\}$

> **Another method :**
> You can imagine that 5 is moved from the left side to the right side and became -5
> $x + 5 = 4 \Rightarrow x = 4 - 5$

2 If the substitution set is $\mathbb{N}$

$\because x + 5 = 4$ Subtracting 5 from both sides

$\therefore x + 5 - 5 = 4 - 5$ $\therefore x = 4 - 5$

$\because$ The subtraction $(4 - 5)$ is impossible in $\mathbb{N}$

$\therefore$ The S.S. in $\mathbb{N}$ is $\varnothing$

Example 2 Find the solution set of each of the following equations in $\mathbb{Q}$:

1. $2x - 5 = 13$
2. $2\frac{1}{2} - \frac{3}{2}x = 5$

Solution

1 $\because 2x - 5 = 13$

Adding 5 to both sides

(5 is the additive inverse of (-5))

$\therefore 2x - 5 + 5 = 13 + 5$

i.e. $2x = 18$

Dividing both sides by 2

$\therefore \frac{2x}{2} = \frac{18}{2}$

$\therefore x = 9$ $\therefore$ The S.S. $= \{9\}$

"Check the truth of the solution"

> **Another method :**
> You can imagine that 2 is moved from the left side to the right side and it became divisor
> $2x = 18 \Rightarrow x = \frac{18}{2}$

2 $\because 2\frac{1}{2} - \frac{3}{2}x = 5$ Subtracting $2\frac{1}{2}$ from both sides

$\therefore 2\frac{1}{2} - \frac{3}{2}x - 2\frac{1}{2} = 5 - 2\frac{1}{2}$

$\therefore -\frac{3}{2}x = 2\frac{1}{2}$

$\therefore -\frac{3}{2}x = \frac{5}{2}$

Multiplying both sides by $(-\frac{2}{3})$ ($-\frac{2}{3}$ is the multiplicative inverse of $-\frac{3}{2}$)

$\therefore -\frac{3}{2}x \times (-\frac{2}{3}) = \frac{5}{2} \times (-\frac{2}{3})$

$\therefore x = -\frac{5}{3}$

$\therefore$ The S.S. $= \{-\frac{5}{3}\}$ "Check the truth of the solution"

Example 3 **Find the S.S. of each of the following equations :**

1 $2(x+3) = 4$, where $x \in \mathbb{Z}$

2 $5(x+2) - 1 = 19$, where $x \in \mathbb{Q}$

Solution

1 $\because 2(x+3) = 4$ Dividing both sides by 2

$\therefore \frac{2(x+3)}{2} = \frac{4}{2}$ $\therefore x + 3 = 2$

Adding (-3) to both sides $\therefore x + 3 - 3 = 2 - 3$

$\therefore x = -1$ $\therefore$ The S.S. $= \{-1\}$

2 $\because 5(x+2) - 1 = 19$

Using the distribution property

$\therefore 5x + 10 - 1 = 19$

$\therefore 5x + 9 = 19$

> **Notice that :**
> $5(x+2) - 1 = 19$, $5x + 9 = 19$ and $5x = 10$ are equivalent equations.

Adding (-9) to both sides $\therefore 5x + 9 - 9 = 19 - 9$

$\therefore 5x = 10$ Dividing both sides by 5

$\therefore \frac{5x}{5} = \frac{10}{5}$ $\therefore x = 2$

$\therefore$ The S.S. $= \{2\}$

Unit 1

Example 4 Find in $\mathbb{Q}$ the solution set of each of the following equations :

1 $3x + 4 = 2(x + 1)$ 2 $2(x + 3) - (x - 2) = 4(x - 1) + 3$

Solution Notice that the variable (x) exists in the two sides , then we try to collect it in one side (say the left side)

1 $\because 3x + 4 = 2(x + 1)$

Using the distribution property

$\therefore 3x + 4 = 2x + 2$

Subtracting $2x$ from both sides

$\therefore 3x - 2x + 4 = 2x - 2x + 2$

$\therefore x + 4 = 2$

Subtracting 4 from both sides

$\therefore x + 4 - 4 = 2 - 4$ $\therefore x = -2$

$\therefore$ The S.S. $= \{-2\}$

Another method :

(-4)

$3x + 4 = 2x + 2$

$(-2x)$

$\therefore 3x - 2x = 2 - 4$

i.e. $x = -2$

2 $\because 2(x + 3) - (x - 2) = 4(x - 1) + 3$

Using the distribution property : $\therefore 2x + 6 - x + 2 = 4x - 4 + 3$

$\therefore x + 8 = 4x - 1$

Subtracting x from both sides : $\therefore x - x + 8 = 4x - x - 1$

$\therefore 8 = 3x - 1$

Adding 1 to both sides :

$\therefore 8 + 1 = 3x - 1 + 1$

$\therefore 9 = 3x$

Dividing both sides by 3

$\therefore \frac{9}{3} = \frac{3x}{3}$

$\therefore 3 = x$

$\therefore$ The S.S. $= \{3\}$

Another method :

$(+1)$

$x + 8 = 4x - 1$

$(-x)$

$\therefore 8 + 1 = 4x - x$

i.e. $9 = 3x$

TRY 1 by yourself

Find the solution set of each of the following equations :

1 $x - 5 = 2$, where $x \in \mathbb{N}$

2 $2x + 11 = 3$, where $x \in \mathbb{Z}$

3 $2x - 3 = 5x + 6$, where $x \in \mathbb{Q}$

Using equations in solving word problems

To solve the word problems in algebra , we translate the verbal statements into algebraic symbols and expressions , and the following table shows some examples for that.

Verbal statement	Algebraic expression
• Two numbers , their sum is 9	$x , 9 - x$
• Two numbers , the difference between them is 4	$x , x - 4$ (or $x , x + 4$)
• Two numbers , their product is 10	$x , \frac{10}{x}$
• Two numbers , one of them is twice the other.	$x , 2x$ (or $x , \frac{1}{2}x$)
• Two numbers , one of them is third of the other.	$x , \frac{1}{3}x$ (or $x , 3x$)
• Eight subtracted from three times of a number.	$3x - 8$
• Two numbers , one of them increases than twice of the other by 5	$x , 2x + 5$
• Three consecutive integers.	$x , x + 1 , x + 2$
• Three consecutive even numbers.	$x , x + 2 , x + 4$
• Three consecutive odd numbers.	$x , x + 2 , x + 4$

Example 5 **Two natural numbers , one of them is thrice of the other. If the sum of them is 16, find the two numbers.**

Solution

• We give one of the two number the symbol x

• Using the information given , we form a first degree equation in one unknown.

$\because$ The other number is thrice of the number x

$\therefore$ The other number = $3x$

$\because$ The sum of the two numbers = 16

$\therefore$ The equation is $x + 3x = 16$

• We solve the equation we get to find the value of the unknown.

$\because x + 3x = 16$ $\therefore 4x = 16$ Dividing by 4 : $\therefore x = 4$

i.e. one of the two numbers = 4 , the other number = $3 \times 4 = 12$

• We make sure that the solution is right by using the problem itself , not by using the equation.

$\because$ 12 is the thrice of 4 , $12 + 4 = 16$ $\therefore$ The solution is true.

Unit 1

Example 6 **Three natural consecutive odd numbers whose sum is 27, find these numbers.**

Solution Let the smallest odd number = x

$\because$ Each odd number exceeds the odd number just before it by 2

$\therefore$ The next odd number = $x + 2$ and the third odd number = $x + 4$

$\because$ The sum of the numbers = 27

$\therefore x + (x + 2) + (x + 4) = 27$

$\therefore 3x + 6 = 27$ $\qquad \therefore 3x = 27 - 6$

$\therefore 3x = 21$ $\qquad \therefore x = \frac{21}{3}$

$\therefore x = 7$

i.e. The numbers are 7 , 9 and 11

To check the solution :

The numbers 7 , 9 and 11 are natural consecutive odd numbers

, $7 + 9 + 11 = 27$

$\therefore$ The solution is true.

Remember that

- The perimeter of a rectangle = 2 (length + width)
- The perimeter of a square = side length × 4
- The perimeter of the triangle = the sum of its side lengths
- The area of the triangle = $\frac{1}{2}$ the base length × the height
- The sum of measures of the interior angles of the triangle = 180°

Example 7 **A rectangle with length equals twice its width and its perimeter = 18 cm. Find the dimensions of the rectangle.**

Solution Let the width of the rectangle = x cm.

$\therefore$ Its length = $2x$ cm.

$\because$ The perimeter of the rectangle = 2 (length + width)

$\therefore 18 = 2(2x + x)$ $\qquad \therefore 18 = 2 \times 3x$

$\therefore 18 = 6x$ $\qquad \therefore x = 3$

i.e. The width of the rectangle = 3 cm. and its length = 6 cm.

To check the solution :

$\because$ The length of the rectangle = 6 cm. equals twice its width 3 cm.

, the perimeter of the rectangle = $2(6 + 3) = 2 \times 9 = 18$ cm.

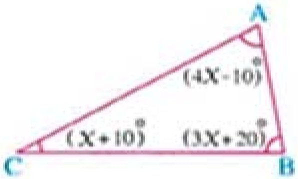
$\therefore$ The solution is true.

Example 8 **In the opposite figure :**

ABC is a triangle in which m (∠ A) = $(4X - 10)^\circ$

, m (∠ B) = $(3X + 20)^\circ$, m (∠ C) = $(X + 10)^\circ$

Find the measures of the angles of the triangle.



Solution

∵ The sum of measures of the interior angles of the triangle = 180°

∴ $(4X - 10) + (3X + 20) + (X + 10) = 180$

∴ $8X + 20 = 180$

Subtracting 20 from both sides

∴ $8X = 180 - 20$ ∴ $8X = 160$

∴ $X = \frac{160}{8}$ ∴ $X = 20$

∴ m (∠ A) = $(4 \times 20) - 10 = 80 - 10 = 70^\circ$

, m (∠ B) = $(3 \times 20) + 20 = 60 + 20 = 80^\circ$

, m (∠ C) = $20 + 10 = 30^\circ$

To check the solution :

∵ m (∠ A) + m (∠ B) + m (∠ C) = $70^\circ + 80^\circ + 30^\circ = 180^\circ$

∴ The solution is true.

TRY 2 by yourself

The difference between two integers is 4 and their sum is 14 , find the two numbers.

Answers of try by yourself

1 [1] {7} [2] {-4} [3] {-3}

2 The two numbers are : 9 , 5

Lesson 8

Solving inequalities in $\mathbb{Q}$

- We had studied before some concepts as the substitution set and the solution set in equations which they are also the same concepts for inequalities.
- The solution set of the inequality is the set whose elements satisfy the inequality and it is a subset of the substitution set.

Illustrated example

Find the solution set of the inequality $x + 3 < 5$ if the substitution set is $\{-1, 0, 1, 2, 3\}$, then represent the S.S. on the number line.

Solution

Substitute for the value of x by each element of the substitution set.

- At $x = -1$ $\therefore$ The left side $= -1 + 3 = 2$
  $\because 2 < 5$ $\therefore -1$ is a solution to the inequality.
- At $x = 0$ $\therefore$ The left side $= 0 + 3 = 3$
  $\because 3 < 5$ $\therefore 0$ is a solution to the inequality.
- At $x = 1$ $\therefore$ The left side $= 1 + 3 = 4$
  $\because 4 < 5$ $\therefore 1$ is a solution to the inequality.
- At $x = 2$ $\therefore$ The left side $= 2 + 3 = 5$
  $\because 5 = 5$ $\therefore 2$ is not a solution to the inequality.
- At $x = 3$ $\therefore$ The left side $= 3 + 3 = 6$
  $\because 6 > 5$ $\therefore 3$ is not a solution to the inequality.

From the previous :

$\therefore$ The solution set $= \{-1, 0, 1\}$

Remark

"The substitution method which is followed in the previous example sometimes be long and not easy and it may be impossible if the substitution set is infinite. Therefore we use another easier method for the solution and that requires studying the properties of inequalities."

Properties of inequalities

We know that $6 > -9$ is a true inequality.

But do the following operations lead to true inequalities ?

1. **Add 2 to the two sides of the inequality**

   $\therefore 6 + 2 > -9 + 2 \longrightarrow 8 > -7$ (true inequality)

   Generally: We can add a constant number to both sides of the inequality without change in the inequality relation.

2. **Subtract 7 from the two sides of the inequality**

   $\therefore 6 - 7 > -9 - 7 \longrightarrow -1 > -16$ (true inequality)

   Generally: We can subtract a constant number from the two sides of the inequality without change in the inequality relation.

3. **Multiply the two sides of the inequality by 5** (positive number)

   $\therefore 6 \times 5 > -9 \times 5 \longrightarrow 30 > -45$ (true inequality)

   Generally: Multiplying the two sides of the inequality by a positive number does not change the inequality relation.

4. **Divide the two sides of the inequality by 3** (positive number)

   $\therefore \frac{6}{3} > -\frac{9}{3} \longrightarrow 2 > -3$ (true inequality)

   Generally: Dividing the two sides of the inequality by a positive number does not change the inequality relation.

5. **Multiply the two sides of the inequality by – 1** (negative number)

   $\therefore 6 \times (-1) > -9 \times (-1) \longrightarrow -6 > 9$ (false inequality) because $-6 < 9$

   Generally: If we multiply the two sides of the inequality by a negative number , then we change the sign of the inequality to the opposite sign.

6. **Divide the two sides of the inequality by – 3** (negative number)

   $\therefore \frac{6}{-3} > \frac{-9}{-3} \longrightarrow -2 > 3$ (false inequality) because $-2 < 3$

   Generally: If we divide the two sides of the inequality by a negative number , then we change the inequality sign to the opposite sign.

Unit 1

We can summarize the properties of inequality that noticed before as follows :

Assuming that a , b , c are three rational numbers , then :

1 If $a < b$	, then $a + c < b + c$
2 If $a < b$	, then $a - c < b - c$
3 If $a < b$, c is a positive number	, then $ac < bc$
4 If $a < b$, c is a positive number	, then $\frac{a}{c} < \frac{b}{c}$
5 If $a < b$, c is a negative number	, then $ac > bc$
6 If $a < b$, c is a negative number	, then $\frac{a}{c} > \frac{b}{c}$

Remark

" If a and b are two non-zero rational numbers have the same sign and $a > b$, then : $\frac{1}{a} < \frac{1}{b}$ "

Example 1 **Find the solution set of the inequality : $x + 2 < 5$, in each of the two following cases :**

1 If $x \in \mathbb{Z}$ 2 If $x \in \mathbb{N}$

, then represent the solution set on the number line in each case.

Solution

$\because x + 2 < 5$

Subtracting 2 from the two sides

$\therefore x + 2 - 2 < 5 - 2$ i.e. $x < 3$

1 When $x \in \mathbb{Z}$

The solution set is all the integers which are less than 3

i.e. The S.S. $= \{2 , 1 , 0 , -1 , ...\}$

2 When $x \in \mathbb{N}$

The solution set is all the natural numbers which are less than 3

i.e. The S.S. $= \{2 , 1 , 0\}$

We notice from the previous example that :

The solution set of the inequality depends on the substitution set , we find that :

The solution set in $\mathbb{N}$ differs from the solution set in $\mathbb{Z}$

Example 2 **Find the solution set of the inequality : $2x - 5 > 5$, in each of the two following cases :**

1 If $x \in \mathbb{Q}$ 2 If $x \in \mathbb{Z}$

Solution

$\because 2x - 5 > 5$

Adding 5 to both sides

$\therefore 2x - 5 + 5 > 5 + 5$ $\therefore 2x > 10$

Multiplying both sides by $\frac{1}{2}$

$\therefore \frac{1}{2} \times 2x > \frac{1}{2} \times 10$ **i.e.** $x > 5$

1 When $x \in \mathbb{Q}$

The S.S. is all the rational numbers which are greater than 5 , then we write it by characterized property method because it is difficult to list all its members.

i.e. The S.S. $= \{x : x \in \mathbb{Q} , x > 5\}$

2 When $x \in \mathbb{Z}$

The solution set is all the integers which are greater than 5

i.e. The S.S. $= \{6 , 7 , 8 , \cdots\}$

Example 3 Find in $\mathbb{Q}$ the solution set of each of the following inequalities :

1 $4 - 2x \leq 2$

2 $7(x - 1) > 9x - 6$

Solution

1 $\because 4 - 2x \leq 2$

Adding -4 to both sides

$\therefore -4 + 4 - 2x \leq -4 + 2$

$\therefore -2x \leq -2$

Dividing both sides by (-2)

$\therefore \frac{-2x}{-2} \geq \frac{-2}{-2}$ (**Notice** the change of inequality sign)

$\therefore x \geq 1$ **i.e.** The S.S. $= \{x : x \in \mathbb{Q}, x \geq 1\}$

2 $\because 7(x - 1) > 9x - 6$

$\therefore 7x - 7 > 9x - 6$

Subtracting $(9x)$ from both sides

$\therefore 7x - 9x - 7 > 9x - 9x - 6$

$\therefore -2x - 7 > -6$

Adding 7 to both sides

$\therefore -2x - 7 + 7 > -6 + 7$

$\therefore -2x > 1$

Dividing both sides by (-2)

$\therefore \frac{-2x}{-2} < \frac{1}{-2}$ (**Notice** the change of inequality sign)

$\therefore x < -\frac{1}{2}$ **i.e.** The S.S. $= \{x : x \in \mathbb{Q}, x < -\frac{1}{2}\}$

Example 4 **Find in $\mathbb{Z}$ the solution set of the inequality $-11 \le 3x - 5 < 4$, then represent it on the number line.**

Solution

$\because -11 \le 3x - 5 < 4$

Adding 5 to the three sides

$\therefore -11 + 5 \le 3x - 5 + 5 < 4 + 5$ $\therefore -6 \le 3x < 9$

Dividing all sides by 3

$\therefore \frac{-6}{3} \le \frac{3x}{3} < \frac{9}{3}$ $\therefore -2 \le x < 3$

i.e. The S.S. $= \{-2, -1, 0, 1, 2\}$

TRY by yourself

Find the solution set of each of the following inequalities :

1. $2x - 3 \ge 5$, where $x \in \mathbb{Q}$
2. $5x - 10 < 2x - 1$, where $x \in \mathbb{N}$

Answers of try by yourself

1. $\{x : x \in \mathbb{Q}, x \ge 4\}$
2. $\{0, 1, 2\}$

UNIT 2

Statistics and Probability

Lessons of the unit :

1. **Samples :** - Systematic sample.
 - Random sample.
2. **Probability :** - Experimental probability.
 - Theoretical probability.

Pierre Simon Marquis de Laplace
(1749 - 1827)

He was a French mathematician and astronomer.
His first work was published in 1771 starting with differential equations however he had already started to think about the mathematical and philosophical concepts of probability and statistics.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the sample and how to select it.
- classify the samples according to the selecting method of its elements.
- select a random sample from a society distributed randomly.
- use a calculator to select a random sample.
- perform a random experiment and write the sample space.
- recognize the concept of the event.
- calculate the probability of an event.
- recognize the impossible event.
- recognize the certain event.

Lesson 1 Samples

Prelude

When we check the production of a factory to know if its products are manufactured according to the certain specifications , usually we don't check the whole production of this factory , but it is sufficient to check a part of this production under certain conditions , in order to represent the whole production , then we generalize the results for all the production.

This part is called "a sample".

Definition

A sample is a small part from a large society that looks like this society and represents it well and is selected randomly.

- And notice that the selected sample should wholly represent the society (the object of study) and it shouldn't be based on a certain group and neglect the other , so that the results of the study can be near reality and we can make decisions according to these results , so we can generalise these results on the society as a whole.

Types of samples

Samples are classified according to the way used in selecting its items , and in this lesson , we introduce two types of samples :

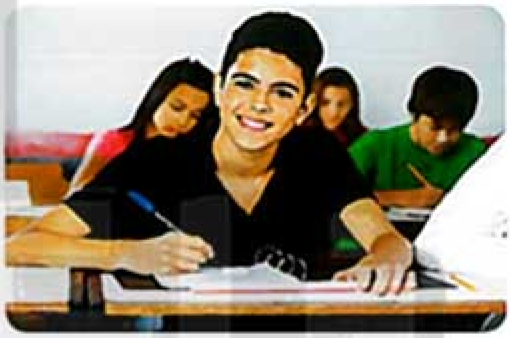
1. Systematic sample.
2. Random sample.

1 Systematic sample

Systematic sample is the sample whose elements are selected from the elements of a society distributed randomly by following a certain system or method in selection.

For example:

To select a systematic sample representing 10% of the marks of students in a preparatory school in the mid-year exam in maths , to study the standards of students , we will do as follows :



1. Students must be distributed randomly in a numbered list i.e. selection shouldn't be from certain classes as excellent students' classes or selecting certain classes and neglecting others.
2. We select in a regular way the tenth student in each 10 student from the students list i.e. we select the mark of the tenth , twentieth , thirtieth , ... students is the list.

Remark

If the society (the object of study) is allready divided into classes or groups as the school is divided into classes for boys and others for girls , then we select a part of each group to be represented in the sample so that the sample represents the society as a whole.

2 Random sample

Random sample is the sample whose elements are selected from the elements of a society distributed randomly by following a random and irregular method of selecting.

In this sample , each individual must get the same chance of selecting.

So , we can select its elements by two methods :

- Manual method.
- Using the scientific calculator.

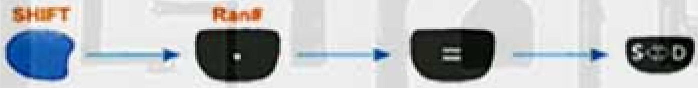
The first method (manual method) :

It is done as follows :

1. Every member of the society is given a number , then this number is written on a piece of paper such that all pieces of paper are of the same colour and size.
2. Each piece of paper is folded perfectly such that the written number does not appear and is put in a bag or a box and mixed together very well.
3. The selection is carried out by drawing a piece after piece without looking inside the paper till the operation of drawing is finished when the required number of selected members is done.

The second method (using the scientific calculator) :

This method depends on using the random number function on the scientific calculator shown in the opposite figure by pressing the following keys successively from the left :



Then a random number in the range 0.000 to 0.999 will appear every time. Take the apparent numbers without the decimal point. The numbers which are greater than the whole number of the society under study should be ignored and also if a number is repeated it must be ignored and taken once only.

In any survey , a 10% sample is considered adequate to provide reliable information about the whole society.

Example **A factory has 300 workers. The people in charge of the monthly magazine of this factory want to develop this magazine by doing a survey of a sample representing 10% of the total number of the workers in this factory. Show how the selection of this sample can be carried out using the calculator.**

Solution

$\because$ The number of workers in the factory = 300 workers.

$\therefore$ The number of the random sample = $\frac{10}{100} \times 300 = 30$ workers.

Then we want to select 30 workers to do this survey. The selection operation can be carried out as follows :

1. Each worker in the factory is given a number from 1 to 300
2. Use the calculator to select 30 numbers randomly by the method mentioned before such that these numbers are included between 0 and 301 and the number that is more than 300 should be ignored.

For example:

By pressing the keys SHIFT → Ran# → = → S⇔D successively.

- If we get the decimal 0.56 , then the number of the selected person is 56
- If we get the decimal 0.049 , then the number of the selected person is 49
- If we get the decimal 0.132 , then the number of the selected person is 132
- If we get the decimal 0.453 , it must be ignored because 453 is more than 300 and so on till we get 30 numbers.

Assuming that the calculator gave us the shown numbers in the opposite table , then the workers who carry these numbers are the selected sample to carry out this survey.

56	49	132	141	249	272
254	256	4	213	74	198
131	2	156	47	172	13
8	3	85	82	9	38
41	14	34	279	118	103

Lesson 2

Probability

Prelude

In our daily life , many times we ask ourselves about some affairs that may happen in the future and that we cannot give a certain result for them.

For example:

- If the Egyptian football team get to the finals of African nations championship , what is its chance for winning the cup ?
- If an Egyptian citizen puts himself up for parliamentary elections in one of elections zones , what is his chance for winning in the elections ?

All these questions' answers are expectations to what may happen (occur) in the future referring to previous experience , studies or observations.

When we answer these questions , we use some words as «may be , chance or probable».

In mathematics , we call this probability. In this lesson , we will study :

1. Experimental probability.
2. Theoretical probability.

1 Experimental probability

• If one of the Olympic swimmers wants to achieve a new record in the next Olympic Games , what is the probability that this swimmer achieves this record ?

The answer to this question cannot be got by expecting , hoping or by doing a survey of the opinions of the trainers or by asking the swimmer himself , but by trying.

i.e. The swimmer covers the needed distance in the race several times , then we record the number of times in which he could achieve the requested number and divide it by the total number of times , so the quotient is the probability of achieving the new record in the next Olympics.

The experimental probability depends on performing an experiment , then we record the results and use them to calculate the value of probability of an event occurrence using the rule :

$$\text{Experimental probability} = \frac{\text{Number of trials in which the outcome occurs}}{\text{Total number of trials}}$$

It is noticed that the more we carry out the experiment , the more we obtain an accurate value for the probability.

Example 1 **If we tossed a piece of coin with double face 200 times and the results of appearance of a head or a tail in each toss were recorded in a table as shown :**

	Heads (H)	Tails (T)	Total
Statistics Tallies	卌 /	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 ////	
Frequency	106	94	200

Calculate :

1 The probability of appearance of a head.

2 The probability of appearance of a tail.

Solution

1 The probability of appearance of a head $= \frac{\text{Number of getting heads}}{\text{Total number of tosses}}$

$= \frac{106}{200} = 0.53$

2 The probability of appearance of a tail $= \frac{\text{Number of getting tails}}{\text{Total number of tosses}}$

$= \frac{94}{200} = 0.47$

TRY by yourself 1

Roll a fair die 25 times and record the results of appearance of a number on the upper face in a table, then calculate :

1 The probability of appearance of the number 4

2 The probability of appearance of the number 3

2 Theoretical probability

In the previous , we carried out the experiment of tossing a piece of coin and we found that :

- The probability of appearance of a head = 0.53
- The probability of appearance of a tail = 0.47

But when we study this experiment theoretically ,
we find that :

If we tossed the coin piece once , then we obtain either a head or a tail.

i.e. the number of possible outcomes = 2
and there is one chance to obtain a head and also one chance to obtain a tail.

(i.e. all outcomes of the experiment have the same chance to happen).

i.e. the probability of appearance of a head $= \frac{1}{2} = 0.50$
and the probability of appearance of a tail $= \frac{1}{2} = 0.50$

Notice that :
We can express the probability by percentage.
i.e. we write the probability of appearance of a head = 50%

Remark

Notice the difference between the experimental probability of appearance of a head (0.53) and the theoretical probability of appearance of a head (0.50)

i.e. When the number of times of carrying out the experiment increases , the value of experimental probability approaches the value of theoretical probability.

Definition of random experiment

Random experiment is an experiment in which we can specify all its possible outcomes before carrying it out but we cannot determine certainly which of them will occur.

Sample space

Sample space is the set of all possible outcomes of a random experiment and it is denoted by S

For example:

- When we toss a piece of coin once , then the sample space is $S = \{H , T\}$

- When we roll a fair die once observing the apparent number on the upper face , then the sample space is $S = \{1 , 2 , 3 , 4 , 5 , 6\}$

Event

Event is a subset of the sample space.

For example:

If A is the event of appearance of an odd number when rolling a fair die once and observing the apparent number on the upper face , then $A = \{1 , 3 , 5\}$, $A \subset S$

Generally

The probability of any event occurrence $A \subset S$ is denoted by P (A) and it is given by using the relation :

$$P(A) = \frac{\text{The number of elements of the event « A »}}{\text{The number of elements of sample space « S »}} = \frac{n « A »}{n « S »}$$

Unit 2

Example 2 **If a fair die is rolled once and we observe the apparent number on the upper face , find the probability of each of the following events :**

1. A is the event of appearance of a number more than 4 (Approximating the result to the nearest hundredth)
2. B is the event of appearance of an even number.
3. C is the event of appearance of a number equal to 5 (Approximating the result to the nearest tenth)
4. D is the event of appearance of a number equal to 7
5. E is the event of appearance of a number less than 7

Solution $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

1. $A = \{5, 6\}$, $n(A) = 2$

   $\therefore P(A) = \frac{2}{6} = \frac{1}{3} \simeq 0.33$ (to the nearest hundredth)

2. $B = \{2, 4, 6\}$, $n(B) = 3$ $\therefore P(B) = \frac{3}{6} = 0.5$

3. $C = \{5\}$, $n(C) = 1$

   $\therefore P(C) = \frac{1}{6} \simeq 0.2$ (to the nearest tenth)

4. $D = \{\ \}$ or $\varnothing$, $n(D) = \text{zero}$

   $\therefore P(D) = \frac{0}{6} = \text{zero}$ (the impossible event)

5. $E = \{1, 2, 3, 4, 5, 6\}$, $n(E) = 6$

   $\therefore P(E) = \frac{6}{6} = 1$ (the certain event)

Remarks

1. **The impossible event :** is the event that has no chance for occurring.
   i.e. the probability of the impossible event = Zero
2. **The certain event :** is the event that has all the possible outcomes.
   i.e. the probability of the certain event = 1
3. The value of probability of any event is not less than zero and not more than one
   i.e. $0 \le$ the probability of an event occurrence ≤ 1

Example 3 **From the set of digits $\{3, 4, 5\}$, form a two-digit number, then find the probability of each of the following events :**

1. A « the event that the units digit is odd »
2. B « the event that the tens digit is even »
3. C « the event that the two digits are odd »
4. D « the event that the sum of the two digits = 8 »
5. E « the event that the product of the two digits = 20 »

Solution $S = \{33, 43, 53, 34, 44, 54, 35, 45, 55\}$, $n(S) = 9$

1. $A = \{33, 43, 53, 35, 45, 55\}$, $n(A) = 6$ $\therefore P(A) = \frac{6}{9} = \frac{2}{3}$
2. $B = \{43, 44, 45\}$, $n(B) = 3$ $\therefore P(B) = \frac{3}{9} = \frac{1}{3}$
3. $C = \{33, 53, 35, 55\}$, $n(C) = 4$ $\therefore P(C) = \frac{4}{9}$
4. $D = \{53, 44, 35\}$, $n(D) = 3$ $\therefore P(D) = \frac{3}{9} = \frac{1}{3}$
5. $E = \{54, 45\}$, $n(E) = 2$ $\therefore P(E) = \frac{2}{9}$

Example 4 **A bag has an amount of marbles of the same size and touch. If 2 marbles are red, 3 are blue and 5 are white and a marble is drawn randomly, calculate :**

1. The probability of that (the drawn marble is red)
2. The probability of that (the drawn marble is blue)
3. The probability of that (the drawn marble is white)
4. The probability of that (the drawn marble is not blue)

Solution The probability of occurrence of a certain outcome

$= \dfrac{\text{The number of possible chances to get this outcome}}{\text{The total number of chances}}$

$\because$ The total number of marbles $= 2 + 3 + 5 = 10$

1 The probability of that (the drawn marble is red)

$= \dfrac{\text{The number of red marbles}}{\text{The total number of marbles}} = \dfrac{2}{10} = \dfrac{1}{5}$

2 The probability of that (the drawn marble is blue)

$= \dfrac{\text{The number of blue marbles}}{\text{The total number of marbles}} = \dfrac{3}{10}$

3 The probability of that (the drawn marble is white)

$= \dfrac{\text{The number of white marbles}}{\text{The total number of marbles}} = \dfrac{5}{10} = \dfrac{1}{2}$

4 The probability of that (the drawn marble is not blue)

$= \dfrac{\text{The number of marbles which aren't blue}}{\text{The total number of marbles}} = \dfrac{10 - 3}{10} = \dfrac{7}{10}$

Remark

In the previous example , notice that :

P (red marble) $= \dfrac{2}{10}$, P (blue marble) $= \dfrac{3}{10}$,

P (white marble) $= \dfrac{5}{10}$ $\because$ $\dfrac{2}{10} + \dfrac{3}{10} + \dfrac{5}{10} = 1$

$\therefore$ The sum of probabilities of all outcomes of a random experiment = 1

So , if the probability of occurrence of an event is a , then the probability that it doesn't occur = 1 – a

So , we can find the probability that the drawn marble is not blue as follows :

The probability that the drawn marble is not blue

= 1 – the probability that it is blue $= 1 - \dfrac{3}{10} = \dfrac{7}{10}$

Example 5 **A Class has some students who wear glasses and other students who don't wear glasses. If a student is chosen randomly from this class and the probability that this student wears glasses is 0.1**

1. Find the probability that this student does not wear glasses.
2. If the number of students in this class is 30 students , find the expected number of students who wear glasses.

Solution

1. The probability that this student does not wear glasses
$= 1 -$ the probability that the student wears glasses $= 1 - 0.1 = 0.9$

2. $\because$ The expected number of outcomes of an event = the probability of occurrence of this event × the total number of all possible outcomes.
$\therefore$ The expected number of students who wear glasses $= 0.1 \times 30$
$= 3$ students.

Example 6 **A spinner game was divided into some equal sectors. 2 of them are green , 4 are blue and the rest are red. If the probability that the pointer stops pointing at a green sector is $\frac{1}{6}$, then find the number of red sectors.**

Solution

$\because$ The probability that the pointer stops pointing at a green sector

$= \frac{\text{The number of green sectors}}{\text{The number of all sectors}}$

$\therefore \frac{1}{6} = \frac{2}{\text{The number of all sectors}}$

$\therefore$ The number of all sectors $= 2 \times 6 = 12$ sectors.

$\therefore$ The number of red sectors $= 12 - (2 + 4) = 6$ sectors.

Unit 2

TRY 2 by yourself

1. A box contains cards numbered from 1 to 15. If a card is drawn randomly , what is the probability that the written number on the card is divisible by 5 ?

2. An experiment has 3 outcomes. If the probability of occurrence of the first outcome is 0.3 and the probability of the second is 0.45 , calculate the probability of the third outcome.

3. A farm has 2000 cows. If the probability of cow madness infection in this farm is 0.17 , what is the expected number of infected cows ?

Answers of try by yourself

1 Answer by yourself after doing experiment.

2
1. The probability that the written number on the card is divisible by 5 $= \frac{3}{15} = \frac{1}{5}$
2. The probability of the third outcome $= 1 - (0.3 + 0.45) = 0.25$
3. The expected number of infected cows = 340 cows.

Second

Geometry and Measurement

UNIT 3

Geometry and Measurement

Lessons of the unit :

Unit Objectives :

By the end of this unit, student should be able to :

- use the deductive proof to prove the verity of the theorems.
- recognize the polygon and the difference between the convex polygon and the concave polygon.
- find the sum of the measures of the interior angles and the exterior angles of any polygon.
- recognize the regular polygon and find the measure of its interior angle.
- recognize the parallelogram and its properties.
- deduce when the quadrilateral be a parallelogram.
- recognize the special cases of the parallelogram (Rectangle - Rhombus - Square).
- deduce that the sum of measures of the interior angles of a triangle is 180°
- recognize the exterior angle of a triangle and its measure.
- deduce the relation between the length of the line segment joining the midpoints of two sides of a triangle and the length of the third side.
- recognize the Pythagoras' theorem.
- recognize the properties of the reflection in a straight line, the reflection in a point, the translation and the rotation.
- find an image of a geometric figure by using the reflection, the translation and the rotation.

Euclid

Euclid

(325 B.C. - 265 B.C.)

He was a Greek mathematician.
He lived in Alexandria.
Euclid introduced the system of axioms.
Since this time, geometry of Euclid was considered a model of logical proof.

Euclid's Axioms (notations) :

- Things which are equal to one thing are equal to each other.
- If equals are added to equals, then the sums are equal.
- Things which coincide with one another are equal to each other.
- The whole is greater than the part.

Lesson 1 Deductive proof

- Deductive proof is a theoretical method to prove theorems and reaching to results. In a deductive proof , we do not need to use geometric instruments in measuring , but we use definitions , properties , facts and previous theorems to get results , by writing mathematical statements , which means that when we write any statement , we have to mention the reason for which this statement is true.

For example:

If you know that
ABCD is a rectangle ,
you can write the following

Mathematical statement	Reason
• ABCD is a rectangle	Given
• AB = CD	opposite sides in the rectangle are equal in length
• m (∠ B) = 90°	Angles of the rectangle are right
• $\overline{AD}$ // $\overline{BC}$	opposite sides in the rectangle are parallel

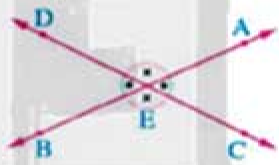
How to write the proof in geometry ?

1. Read carefuly the problem , in order to determine what is **"given"** , which are all information given in the problem , and **"required"** , which is the question we need to answer in the problem.

2. Use the given information in the problem to draw a clear geometric figure (if the figure is not already given) and put on the figure the given information in the problem as : side lengths , angles measures and so on.
3. Write the given in form of points.
4. Write the required to be proved.
5. Think about a plan for the proof which is the main steps needed in order to arrive to the required.
6. Write the proof by writing a mathematical statements mention for each statement a reason that makes this statement is true.
7. Be sure that you answered the question of the problem.

In the following, some examples showing how to write the deductive proof :

1 **If two straight lines intersect , then the measures of each two vertically opposite angles are equal.**



Given $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two straight lines intersecting at E

Required to prove (R.T.P.) m (∠ AED) = m (∠ BEC)

Proof

∵ ∠ AED and ∠ AEC are two adjacent angles where $\overrightarrow{EC} \cup \overrightarrow{ED} = \overleftrightarrow{CD}$

∴ m (∠ AED) + m (∠ AEC) = 180°

, ∵ ∠ AEC and ∠ BEC are two adjacent angles where $\overrightarrow{EA} \cup \overrightarrow{EB} = \overleftrightarrow{AB}$

∴ m (∠ AEC) + m (∠ BEC) = 180°

∴ m (∠ AED) + m (∠ AEC) = m (∠ AEC) + m (∠ BEC)

∴ m (∠ AED) = m (∠ BEC) (Q.E.D.)*

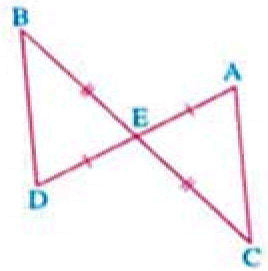
Similarly , you can prove that m (∠ AEC) = m (∠ BED)

* Q.E.D. is an abbreviation for quod erat demonstrandum.
It is a Latin abbreviation which means to be demonstrated

Example 1 In the opposite figure :

$\overline{AD} \cap \overline{BC} = \{E\}$ where AE = DE and BE = CE

Prove that : Δ AEC $\equiv \Delta$ DEB



Solution

Given $\overline{AD} \cap \overline{BC} = \{E\}$ where AE = DE , BE = CE

R.T.P. Δ AEC $\equiv \Delta$ DEB

Proof $\because \overline{AD} \cap \overline{BC} = \{E\}$

$\therefore$ m ($\angle$ AEC) = m ($\angle$ DEB) (V.O.A)

$\therefore$ In $\Delta\Delta$ AEC and DEB :

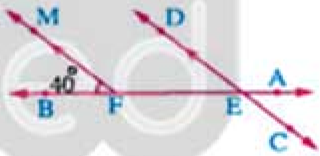
$\begin{cases} AE = DE \text{ (given)} \\ CE = BE \text{ (given)} \\ m(\angle AEC) = m(\angle DEB) \text{ (by proof)} \end{cases}$

$\therefore \Delta$ AEC $\equiv \Delta$ DEB (Q.E.D.)

TRY 1 by yourself

In the opposite figure :

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$, $\overleftrightarrow{CD} \parallel \overrightarrow{FM}$, $F \in \overrightarrow{AB}$ and m ($\angle$ MFB) = 40°



Complete the following proof to find : m ($\angle$ AEC)

Given ....

Required to find (R.T.F.) ....

Proof $\because \overleftrightarrow{CD}$ // .... (given) and $\overleftrightarrow{AB}$ is a transversal.

$\therefore$ m ($\angle$ DEB) = m ($\angle$ ....) = 40° (corresponding angles)

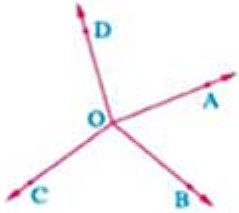
$\because \overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$

$\therefore$ m ($\angle$ AEC) = m ($\angle$ ....) (V.O.A)

$\therefore$ m ($\angle$ AEC) = ...° (The req.)

2 The sum of the measures of the accumulative angles at a point is equal to 360°

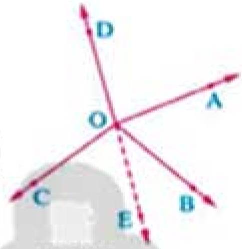
Given $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ are rays that start at O



R.T.P. The sum of the measures of the accumulative angles at O is 360°

Construction Draw $\overleftrightarrow{DO}$ and $E \in \overleftrightarrow{DO}$

Proof



$\because$ m ($\angle$ EOB) + m ($\angle$ BOA) + m ($\angle$ AOD) = 180°

, m ($\angle$ EOC) + m ($\angle$ COD) = 180°

$\therefore$ m ($\angle$ EOB) + m ($\angle$ BOA) + m ($\angle$ AOD) + m ($\angle$ EOC) + m ($\angle$ COD) = 180° + 180° = 360°

$\therefore$ m ($\angle$ AOB) + m ($\angle$ BOC) + m ($\angle$ COD) + m ($\angle$ DOA) = 360°

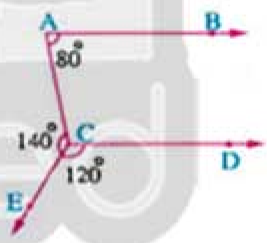
(Q.E.D.)

Example 2 In the opposite figure :

m ($\angle$ BAC) = 80° , m ($\angle$ DCE) = 120°

and m ($\angle$ ACE) = 140°

Prove that : $\overrightarrow{AB}$ // $\overrightarrow{CD}$



Solution

Given m ($\angle$ BAC) = 80° , m ($\angle$ DCE) = 120° , m ($\angle$ ACE) = 140°

R.T.P. $\overrightarrow{AB}$ // $\overrightarrow{CD}$

Proof $\because$ m ($\angle$ DCA) + m ($\angle$ DCE) + m ($\angle$ ACE) = 360°

(accumulative angles at C)

$\therefore$ m ($\angle$ DCA) = 360° – (120° + 140°) = 100°

$\therefore$ m ($\angle$ BAC) + m ($\angle$ DCA) = 80° + 100° = 180°

And they are interior angles in the same side of the transversal $\overleftrightarrow{AC}$

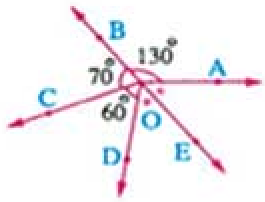
$\therefore$ $\overrightarrow{AB}$ // $\overrightarrow{CD}$

(Q.E.D.)

Unit 3

TRY 2 by yourself

In the opposite figure :



m (∠ AOB) = 130° , m (∠ BOC) = 70° ,

m (∠ COD) = 60° and $\overrightarrow{OE}$ bisects ∠ AOD

Complete the following proof to prove that :

$\overrightarrow{OE}$ and $\overrightarrow{OB}$ are on one straight line.

Given ....

R.T.P. ....

Proof ∵ m (∠ AOB) + m (∠ BOC) + m (∠ COD) + m (∠ AOD) = ... °

(Accumulative angles at O)

∴ m (∠ AOD) = ...° − .... ° = ... °

∵ $\overrightarrow{OE}$ bisects ∠ .... (given)

∴ m (∠ AOE) = $\frac{1}{2}$ m (∠ ....)

∴ m (∠ AOE) = $\frac{1}{2}$ × ....° = ...°

∴ m (∠ AOE) + m (∠ AOB) = ...° + ...° = ...°

∴ $\overrightarrow{OE}$ and $\overrightarrow{OB}$ are on one straight line. (Q.E.D.)

Lesson 2

The polygon

The polygon

It is a simple closed line that consists of three line segments , or more. The polygon is named according to the number of its sides.

Notice that :

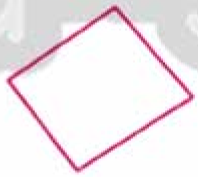
The simple line is the line that does not cut itself

Examples for some polygons :

3 sides

Triangle

4 sides

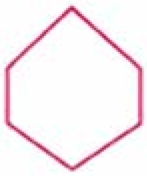


Quadrilateral

5 sides

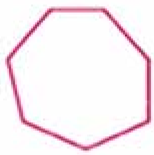
Pentagon

6 sides



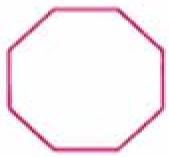
Hexagon

7 sides



Heptagon

8 sides

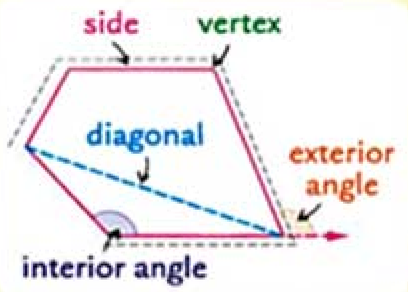


Octagon

Unit 3

Remarks

1. Each line segment of the line segments forming the polygon is called a **side**.
2. Each point resulted from intersecting of two adjacent sides of the polygon is called a **vertex**.
3. The sum of the side lengths of the polygon is called the **perimeter of the polygon**.
4. Each line segment joining two non-adjacent vertices of the polygon is called a **diagonal**.
5. The included angle between two adjacent sides of the polygon is called an **interior angle**.



6. The included angle between a side of the polygon and the extension of its adjacent side is called an **exterior angle**.
7. The number of sides of any polygon = the number of its vertices = the number of its interior angles.

Convex polygon and concave polygon

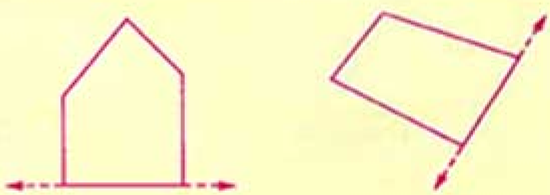
- The polygon is **convex** if the measure of any of its interior angles is less than 180°

- The polygon is **concave** if the measure of one of its interior angles at least is greater than 180° "reflex angle"

Remarks

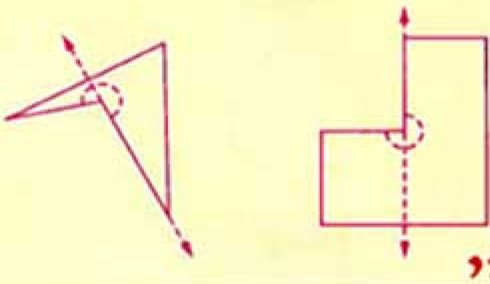
- **In the convex polygon :**

If a straight line is drawn to pass through any two consecutive vertices, then the remaining vertices lie on one side of this straight line.



- **In the concave polygon :**

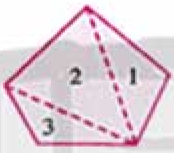
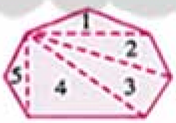
There are straight lines passing through two consecutive vertices and the remaining vertices lie on two different sides of the straight lines.



The sum of measures of the interior angles of the polygon :

We knew before that : The sum of measures of the interior angles of the triangle equals 180°
We can use that to deduce a general rule to find the sum of measures of the interior angles of any polygon whose number of sides is n

If we draw from any vertex of the polygon all diagonals that pass through this vertex , then the surface of this polygon will be divided into a number of triangles as shown in the following table :

The polygon	The number of its sides	The number of the resulted triangles	The sum of measures of the interior angles of the polygon
 quadrilateral	4	2	$2 \times 180° = 360°$
 pentagon	5	3	$3 \times 180° = 540°$
 hexagon	6	4	$4 \times 180° = 720°$
 heptagon	7	5	$5 \times 180° = 900°$

From the previous, you notice that :

The number of resulted triangles = the number of the sides of the polygon – 2

Generally : If we draw all the possible diagonals that out of a vertex of a polygon having (n) sides, then the surface of this polygon will be divided into $(n-2)$ triangles.

$\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore$ The sum of measures of the interior angles of a polygon of n sides equals $(n-2) \times 180°$

Unit 3

For example:

- The sum of measures of the interior angles of the octagon = $(8-2) \times 180° = 1080°$
- The sum of measures of the interior angles of the enneagon (nonagon) $= (9-2) \times 180° = 1260°$

Example 1 **Complete the following table :**

Number of sides of the polygon	10	3	12	15
Sum of measures of its interior angles	....	....	....	....

Solution

Number of sides of the polygon	10	3	12	15
Sum of measures of its interior angles	$8 \times 180°$ $= 1440°$	$1 \times 180°$ $= 180°$	$10 \times 180°$ $= 1800°$	$13 \times 180°$ $= 2340°$

Example 2 **The sum of measures of the interior angles of a polygon is 2160° Find the number of its sides.**

Solution

$\because$ The sum of measures of the interior angles of a polygon of n sides equals $(n-2) \times 180°$

$\therefore 2160° = (n-2) \times 180°$ $\therefore n - 2 = \frac{2160}{180} = 12$ $\therefore n = 14$

$\therefore$ The number of sides of this polygon is 14 sides.

TRY by yourself 1

Complete the following table :

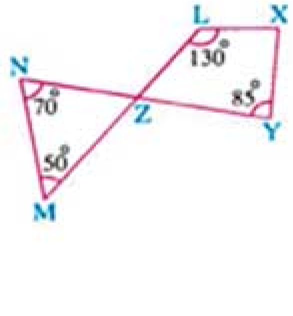
Number of sides of the polygon	11	....	16	....
Sum of measures of its interior angles	....	900°	....	540°

Final answers of try by yourself questions are at the end of each lesson to check your answer.

Example 3 **In the opposite figure :**

$\overline{LM} \cap \overline{YN} = \{Z\}$, m (∠ M) = 50°, m (∠ N) = 70° , m (∠ Y) = 85° and m (∠ L) = 130°

Find : m (∠ X)



Solution

Given $m(\angle M) = 50^\circ$, $m(\angle N) = 70^\circ$, $m(\angle Y) = 85^\circ$, $m(\angle L) = 130^\circ$

R.T.F. $m(\angle X)$

Proof In Δ ZMN :

$\because m(\angle M) = 50^\circ$, $m(\angle N) = 70^\circ$

$\therefore m(\angle NZM) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ$

$\because m(\angle LZY) = m(\angle NZM)$ (V.O.A.)

$\therefore m(\angle LZY) = 60^\circ$

$\because$ The figure XYZL is a quadrilateral.

$\therefore$ The sum of measures of its interior angles $= (4 - 2) \times 180^\circ$

$= 2 \times 180^\circ = 360^\circ$

$\therefore m(\angle X) = 360^\circ - (130^\circ + 85^\circ + 60^\circ) = 85^\circ$ (The req.)

Example 4 **If the ratio among the measures of the interior angles of a quadrilateral is 2 : 3 : 3 : 4 , find the smallest measure of these angles of that quadrilateral.**

Solution

$\because$ The ratio among the measures of the interior angles of a quadrilateral is 2 : 3 : 3 : 4

$\therefore$ The measures of the interior angles of this figure are $2x$, $3x$, $3x$ and $4x$

$\because$ The sum of measures of the interior angles of the quadrilateral $= (4 - 2) \times 180^\circ = 2 \times 180^\circ = 360^\circ$

$\therefore 2x + 3x + 3x + 4x = 360^\circ$ $\therefore 12x = 360^\circ$ $\therefore x = \frac{360^\circ}{12} = 30^\circ$

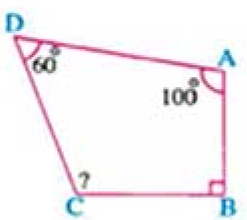
$\because$ The smallest measure $= 2x$

$\therefore$ The smallest measure of the angles $= 2 \times 30^\circ = 60^\circ$

TRY 2 by yourself

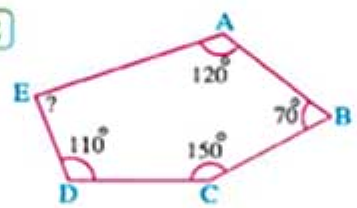
By using the given data for each figure find the measure of the unknown angle:

1



$m(\angle C) = \dots\dots\dots^\circ$

2

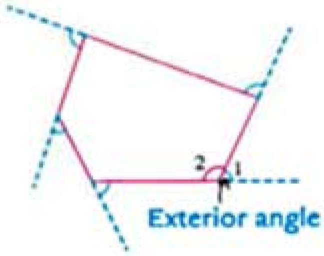


$m(\angle E) = \dots\dots\dots^\circ$

Unit 3

The sum of measures of the exterior angles of the convex polygon which has n sides

- We mentioned that the exterior angle of the polygon is the angle included between one side and the extension of its adjacent side, and although we can draw two exterior angles equal in measure at each vertex of the polygon, but the rule of the sum of measures of the exterior angles use only one exterior angle at each vertex.



- At any vertex of the polygon, we find that the sum of measures of the interior angle and the exterior angle equals 180°

In the previous figure : $m(\angle 1) + m(\angle 2) = 180°$ as an example.

In the previous pentagon, we find that the sum of measures of the five exterior and five interior angles of the pentagon equals $5 \times 180°$

Since the sum of measures of the interior angles equals $3 \times 180°$

$\therefore$ The sum of measures of the five exterior angles of the pentagon $= 2 \times 180° = 360°$

We can deduce that for any convex polygon of n sides as follows :

The sum of measures of the exterior angles + the sum of measures of the interior angles $= n \times 180°$

$\therefore$ The sum of measures of the exterior angles $+ (n - 2) \times 180° = n \times 180°$

$\therefore$ The sum of measures of the exterior angles $= n \times 180° - (n - 2) \times 180°$

$= 180° n - 180° n + 360° = 360°$

So we get :

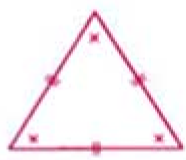
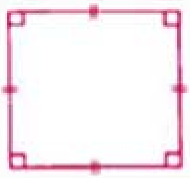
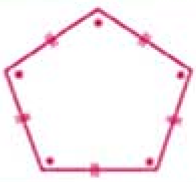
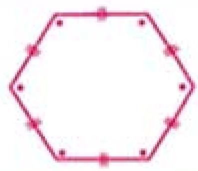
The sum of measures of the exterior angles of a convex polygon of n sides = 360°
(taking into account one exterior angle at each vertex)

The regular polygon

The polygon is regular if :

1. All its sides are equal in length.
2. All its angles are equal in measure.

As examples for the regular polygons :

Equilateral triangle	Square	Regular pentagon	Regular hexagon
			

The measure of the interior angle of a regular polygon

We knew that the sum of measures of the interior angles of a polygon of n sides = $(n-2) \times 180°$

Then :

If the polygon is regular , then its interior angles (whose number is n) are equal in measure.

$\therefore$ The measure of each interior angle of the regular polygon of n sides $= \frac{(n-2) \times 180°}{n}$

For example:

- The measure of each interior angle of the equilateral triangle $= \frac{(3-2) \times 180°}{3} = 60°$
- The measure of each interior angle of the square $= \frac{(4-2) \times 180°}{4} = 90°$

Example 5 Complete the following table :

Number of sides of the regular polygon	5	8	12	6
Measure of one of its interior angles	....	....	....	....

Solution

Number of sides of the regular polygon	5	8	12	6
Measure of one of its interior angles	$\frac{3 \times 180°}{5} = 108°$	$\frac{6 \times 180°}{8} = 135°$	$\frac{10 \times 180°}{12} = 150°$	$\frac{4 \times 180°}{6} = 120°$

Example 6 The measure of one of the interior angles of a regular polygon is 144°
Find the number of its sides.

Solution

$\because$ The measure of each interior angle of the regular polygon of n sides $= \frac{(n-2) \times 180°}{n}$

$\therefore \frac{(n-2) \times 180°}{n} = 144°$ $\quad\quad$ $\therefore (n-2) \times 180° = 144°\, n$

$\therefore 180°\, n - 360° = 144°\, n$ $\quad\quad$ $\therefore 180°\, n - 144°\, n = 360°$

$\therefore 36°\, n = 360°$ $\quad\quad$ $\therefore n = 10$

$\therefore$ The number of sides = 10 sides.

Another solution

∵ The measure of the exterior angle of the polygon

= 180° − the measure of the interior angle = 180° − 144° = 36°

∵ The sum of the measures of the exterior angles = 360°

∴ The number of the exterior angles = $\frac{360°}{36°}$ = 10 angles.

∴ The number of sides = 10 sides.

Notice that :

The number of the polygon sides = The number of its vertices = The number of its interior angles = The number of its exterior angles

Remark

" The number of sides of the regular polygon in which the measure of one of its interior angles is $x°$ = $\frac{360°}{180° - x}$ "

For example:

The number of sides of the regular polygon which the measure of one of its interior angles is 144° = $\frac{360°}{180° - 144°}$ = 10 sides.

TRY 3 by yourself

Complete the following table :

The number of sides of the regular polygon	3	10	....	....
The measure of one of its interior angles	..........°	..........°	135°	160°

At the end of each lesson , you will find the final answers of try by yourself questions in the same form.

Answers of try by yourself

1
- The number of sides of the polygon : 7 , 5
- The sum of measures of the interior angles : 1620° , 2520°

2 (1) 110° (2) 90°

3
- The number of sides of the regular polygon : 8 , 18
- The measure of one of its interior angles : 60° , 144°

Lesson 3 The parallelogram and its properties

* In primary stage you have studied the parallelogram and its properties and in this lesson you will remember first what you studied before , then you will study when a quadrilateral will be a parallelogram.

Definition

Parallelogram is a quadrilateral , in which each two opposite sides are parallel.

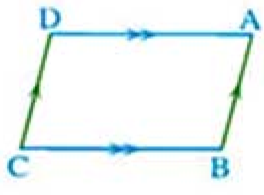
For example:

In the opposite figure :

If ABCD is a quadrilateral in which

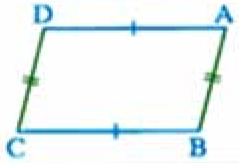
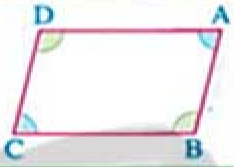
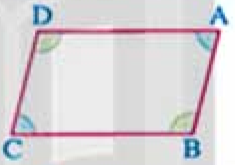
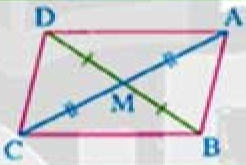
$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$

, then ABCD is a parallelogram.



Unit 3

Properties of parallelogram

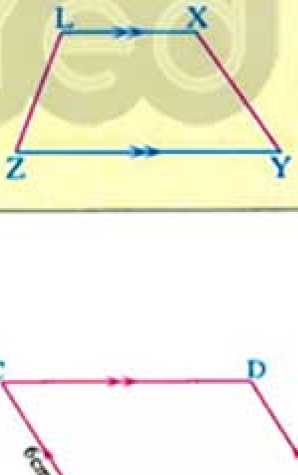
1 Each two opposite sides are equal in length.		• AB = DC • AD = BC
2 Each two opposite angles are equal in measure.		• m (∠ A) = m (∠ C) • m (∠ B) = m (∠ D)
3 The sum of measures of each two consecutive angles is 180°		• m (∠ A) + m (∠ B) = 180° • m (∠ B) + m (∠ C) = 180° • m (∠ C) + m (∠ D) = 180° • m (∠ D) + m (∠ A) = 180°
4 The two diagonals bisect each other.		• AM = CM • BM = DM

The perimeter of the parallelogram = The sum of two consecutive sides × 2

Remark

A quadrilateral in which only two sides are parallel is called a trapezium , as shown in the opposite figure in which :

$\overline{XL} \parallel \overline{YZ}$



Example 1 **In the opposite figure :**

ABCD is a parallelogram in which :

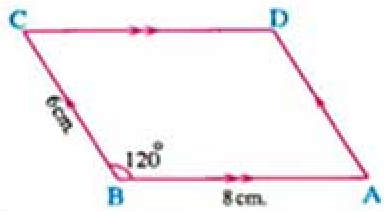
AB = 8 cm. , BC = 6 cm. and m (∠ B) = 120°

Find :

1 The length of each of $\overline{CD}$ and $\overline{DA}$

2 The measure of each of ∠ D , ∠ A and ∠ C

3 The perimeter of ABCD



Solution

Given ABCD is a parallelogram , AB = 8 cm. , BC = 6 cm. and $m(\angle B) = 120°$

R.T.F. 1 CD and DA

2 $m(\angle D)$, $m(\angle A)$ and $m(\angle C)$

3 The perimeter of ABCD

Proof $\because$ ABCD is a parallelogram

$\therefore$ CD = AB = 8 cm. (Properties of a parallelogram)

and DA = CB = 6 cm. (Properties of a parallelogram) (First req.)

, $m(\angle D) = m(\angle B) = 120°$ (Properties of a parallelogram)

, $\because m(\angle A) + m(\angle B) = 180°$ (Properties of a parallelogram)

, $m(\angle B) = 120°$

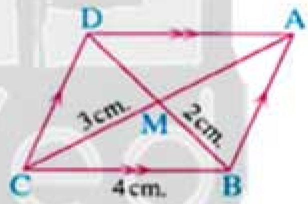
$\therefore m(\angle A) = 180° - 120° = 60°$ and $m(\angle C) = m(\angle A) = 60°$ (Second req.)

The perimeter of ABCD $= (AB + BC) \times 2$

$= (8 + 6) \times 2 = 14 \times 2 = 28$ cm. (Third req.)

Example 2 **In the opposite figure :**

ABCD is a parallelogram whose diagonals intersect at M. If BC = 4 cm. , BM = 2 cm. and MC = 3 cm. , then find the perimeter of Δ AMD



Solution

Given ABCD is a parallelogram whose diagonals intersect at M , BC = 4 cm. , BM = 2 cm. and MC = 3 cm.

R.T.F. The perimeter of Δ AMD

Proof $\because$ ABCD is a parallelogram

$\therefore$ AD = BC = 4 cm. (Two opposite sides in a parallelogram)

$\because$ The two diagonals bisect each other.

$\therefore$ MD = MB = 2 cm. and AM = MC = 3 cm.

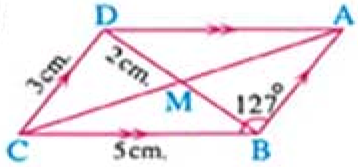
$\therefore$ The perimeter of Δ AMD = AD + MD + AM = 4 + 2 + 3 = 9 cm.

(The req.)

Unit 3

TRY 1 by yourself

In the opposite figure :



ABCD is a parallelogram whose diagonals intersect at M

If BC = 5 cm. , DC = 3 cm. , DM = 2 cm.

and m (∠ ABC) = 127° ,

Complete the following :

1. AB = ……… cm. and AD = ……… cm.
2. BD = ……… cm.
3. m (∠ ADC) = ………° , m (∠ BAD) = ………° and m (∠ BCD) = ………°
4. The perimeter of ▱ ABCD = ……… cm.

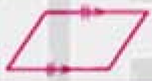
When does a quadrilateral represent a parallelogram ?

A quadrilateral represents a parallelogram if one of the following conditions satisfies

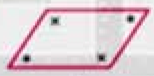
- Each two opposite sides are parallel.

- Each two opposite sides are equal in length.

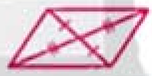
- Two opposite sides are parallel and equal in length.



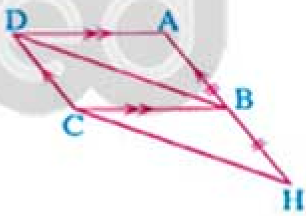
- Each two opposite angles are equal in measure.



- The two diagonals bisect each other.



Example 3 **In the opposite figure :**



ABCD is a parallelogram , H ∈ $\overrightarrow{AB}$

where : AB = BH

Prove that : BHCD is a parallelogram.

Solution

Given ABCD is a parallelogram and AB = BH

R.T.F. BHCD is a parallelogram.

Proof

∵ ABCD is a parallelogram ∴ AB = CD

∵ AB = BH (Given) ∴ DC = BH (1)

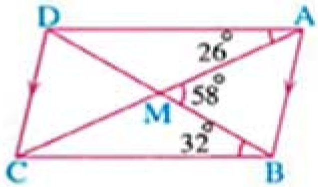
∵ $\overline{AB}$ // $\overline{DC}$, H ∈ $\overrightarrow{AB}$ ∴ $\overline{BH}$ // $\overline{DC}$ (2)

From (1) and (2) : ∴ DC = BH and $\overline{DC}$ // $\overline{BH}$

∴ BHCD is a parallelogram. (Q.E.D.)

TRY 2 by yourself

In the opposite figure :



ABCD is a quadrilateral , its diagonals intersect at M , $\overline{AB} \parallel \overline{CD}$, m (∠ AMB) = 58° , m (∠ MBC) = 32° and m (∠ MAD) = 26°

Complete the following proof to prove that : ABCD is a parallelogram.

Given ....

R.T.P. ....

Proof

∵ M ∈ $\overline{AC}$

∴ m (∠ BMC) = 180° − ....° = ...°

∴ In Δ BMC : m (∠ MCB) = 180° − (..........° + ...°) = ...°

∴ m (∠ MCB) = m (∠ ....) and they are .... angles

∴ $\overline{\cdots} \parallel \overline{\cdots}$ ∵ $\overline{\cdots} \parallel \overline{\cdots}$ (Given)

∴ ABCD is a parallelogram. (Q.E.D.)

Lesson 4 The special cases of the parallelogram

We studied in the previous lesson that the parallelogram is a quadrilateral in which each two opposite sides are parallel, we notice that this condition is verified also in each of rectangle, rhombus and square.

So, we said that each of rectangle, rhombus and square is a special case of the parallelogram with the same properties of it which stated in the previous lesson, as well as some another properties for each figure. In this lesson, we will represent each figure of them and its properties.

1 Rectangle

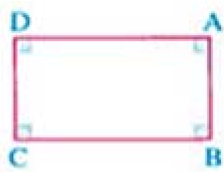
The rectangle is a parallelogram with a right angle.

Properties of the rectangle

The rectangle has the same properties of the parallelogram and some additional properties as the following :

1. The four angles of the rectangle are all equal in measure and the measure of each is 90°

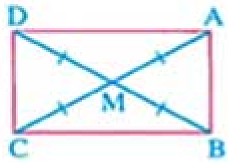
$m(\angle A) = m(\angle B) = m(\angle C) = m(\angle D) = 90^\circ$



❷ The two diagonals of the rectangle are equal in length.

AC = BD and as the two diagonals bisect each other

, then AM = BM = CM = DM



The perimeter of the rectangle = (length + width) × 2

2 Rhombus

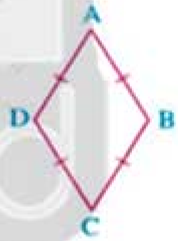
The rhombus is a parallelogram in which two adjacent sides are equal in length.

Properties of the rhombus

The rhombus has the same properties of the parallelogram and some additional properties as the following :

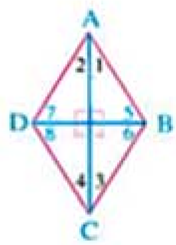
❶ The four sides of the rhombus are all equal in length.

AB = BC = CD = DA



❷ The two diagonals of the rhombus are perpendicular and bisect each of its interior angles.

- $\overline{AC} \perp \overline{BD}$
- m (∠ 1) = m (∠ 2) = m (∠ 3) = m (∠ 4)
- m (∠ 5) = m (∠ 6) = m (∠ 7) = m (∠ 8)

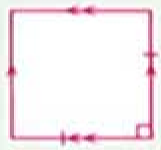


The perimeter of the rhombus = the length of one side × 4

Unit 3

3 Square

The square is a parallelogram with a right angle and two adjacent sides are equal in length.

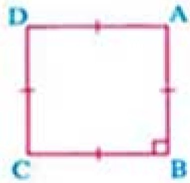


Properties of the square

The square has the same properties of the parallelogram and some additional properties as the following :

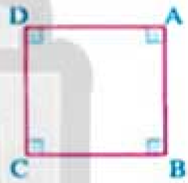
1. The four sides of the square are all equal in length.

   AB = BC = CD = DA



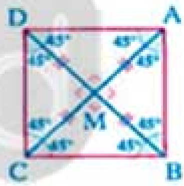
2. The four angles of the square are all equal in measure and each of them is of measure 90°

   m (∠ A) = m (∠ B) = m (∠ C) = m (∠ D) = 90°



3. The two diagonals of the square are equal in length , perpendicular and each diagonal bisects the two vertices angles which this diagonal joins into two angles each one of measure 45°.

   - AC = BD and hence AM = BM = CM = DM
   - $\overline{AC} \perp \overline{BD}$



The perimeter of the square = the length of one side × 4

Notice that :

We can also define the square as follows :

1. A square is a rectangle with two adjacent sides equal in length.
2. A square is a rectangle with two perpendicular diagonals.
3. A square is a rhombus with a right angle.
4. A square is a rhombus with two diagonals equal in length.

Notice that :

To prove that the quadrilateral is a rectangle , a rhombus or a square , we must first prove that it is a parallelogram , as we see in the previous lesson , then :

The parallelogram is

a rectangle

if :

One of its angles is a right angle

or

Its two diagonals are equal in length.

a rhombus

if :

Two adjacent sides are equal in length.

or

Its two diagonals are perpendicular.

a square

if :

One of its angles is right and two adjacent sides are equal in length.

or

One of its angles is right and its two diagonals are perpendicular.

or

Its two diagonals are perpendicular and equal in length.

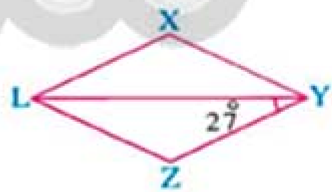
or

Two adjacent sides are equal in length and its two diagonals are equal in length.

Example 1 **In the opposite figure :**

XYZL is a rhombus in which m (∠ LYZ) = 27°

Calculate the measures of the angles of the rhombus XYZL



Solution

Given XYZL is a rhombus in which m (∠ LYZ) = 27°

R.T.F. m (∠ XYZ) , m (∠ XLZ) , m (∠ X) and m (∠ Z)

Proof ∵ $\overline{YL}$ is a diagonal in the rhombus XYZL

∴ $\overrightarrow{YL}$ bisects ∠ XYZ ∴ m (∠ XYZ) = 2 × 27° = 54°

, ∵ Each two opposite angles in the rhombus are equal in measure.

∴ m (∠ XLZ) = 54°

$\because$ The rhombus is a special case of the parallelogram

$\therefore$ Each two consecutive angles are supplemantry

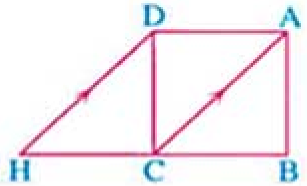
$\therefore m(\angle X) + m(\angle XYZ) = 180°$

$\therefore m(\angle X) + 54° = 180°$ $\therefore m(\angle X) = 126°$

$\therefore m(\angle Z) = 126°$ (The req.)

Try to solve this example by another method using the properties of the rhombus

Example 2 **In the opposite figure :**



ABCD is a square , $H \in \overrightarrow{BC}$ such that $\overline{DH} \parallel \overline{AC}$

1 **Prove that :** CH = BC

2 **Find :** $m(\angle ADH)$

Solution

Given ABCD is a square and $\overline{DH} \parallel \overline{AC}$

R.T.P. CH = BC

R.T.F. $m(\angle ADH)$

Proof $\because \overline{AD} \parallel \overline{BC}$ (two opposite sides in the square) and $H \in \overrightarrow{BC}$

$\therefore \overline{AD} \parallel \overline{CH}$

$\because \overline{DH} \parallel \overline{AC}$ (given)

$\therefore$ ACHD is a parallelogram. $\therefore$ CH = AD

But AD = BC (two opposite sides in the square)

$\therefore$ CH = BC (First req.)

$\because \overline{AC}$ is a diagonal in the square. $\therefore \overrightarrow{CA}$ bisects $\angle BCD$

$\because m(\angle BCD) = 90°$ $\therefore m(\angle ACD) = 45°$

$\because \overleftrightarrow{DH} \parallel \overleftrightarrow{AC}$ and $\overleftrightarrow{CD}$ is their transversal.

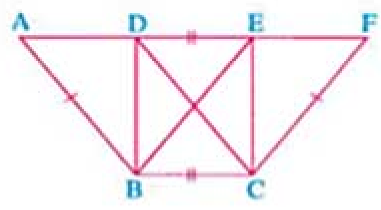
$\therefore m(\angle CDH) = m(\angle ACD) = 45°$ (two alternate angles)

, $\because m(\angle ADC) = 90°$ (property of the square)

$\therefore m(\angle ADH) = m(\angle ADC) + m(\angle CDH)$

$= 90° + 45° = 135°$ (Second req.)

Example 3 **In the opposite figure :**



ABCD , EBCF are two parallelograms ,

D and E belong to $\overleftrightarrow{AF}$, AB = FC , BC = DE

Prove that : The figure DBCE is a rectangle.

Solution

Given ABCD and EBCF are two parallelograms , AB = FC , BC = DE

R.T.P. The figure DBCE is a rectangle.

Proof

$\because$ ABCD is a parallelogram. $\therefore \overline{AD} \parallel \overline{BC}$

, $\because$ D and E belong to $\overleftrightarrow{AF}$ $\therefore \overline{DE} \parallel \overline{BC}$

, $\because$ DE = BC $\therefore$ DBCE is a parallelogram.

, $\because$ ABCD is a parallelogram. $\therefore$ AB = DC

, $\because$ EBCF is a parallelogram. $\therefore$ FC = EB but : AB = FC

$\therefore$ DC = EB

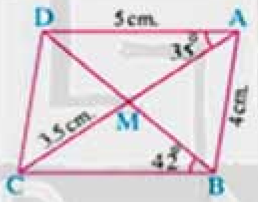
$\therefore$ DBCE is a parallelogram and its diagonals are equal in length.

$\therefore$ DBCE is a rectangle. (Q.E.D.)

TRY by yourself

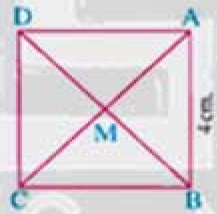
Using the given in each figure , complete where M is the intersection point of the diagonals :

1



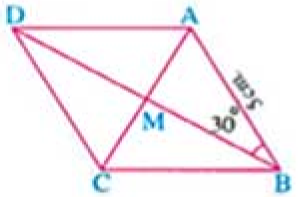
ABCD is a parallelogram :
- The perimeter of Δ ABC = ... cm.
- m (∠ AMB) = ...°

2



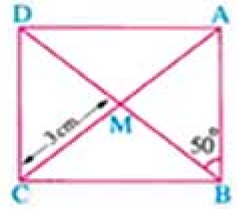
ABCD is a square :
- The perimeter of the square = ... cm.
- m (∠ BAC) = ...°

3



ABCD is a rhombus :
- AD = ... cm.
- m (∠ BAM) = ...°

4



ABCD is a rectangle :
- BD = ... cm.
- m (∠ MCD) = ...°

Answers of try by yourself

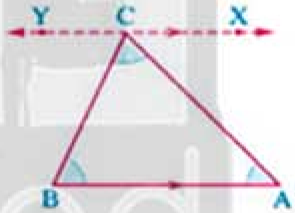
1 16 , 77   2 16 , 45   3 5 , 60   4 6 , 50

Lesson 5

The triangle

Theorem 1

The sum of the measures of the interior angles of a triangle is 180°



Given: ABC is a triangle

R.T.P.: $m(\angle A) + m(\angle B) + m(\angle ACB) = 180°$

Construction: Draw $\overrightarrow{CX} \parallel \overline{AB}$

Proof

$\because \angle XCY$ is a straight angle

$\therefore m(\angle XCA) + m(\angle ACB) + m(\angle YCB) = 180°$

$\because \overleftrightarrow{XY} \parallel \overline{AB}$

$\therefore m(\angle XCA) = m(\angle A)$ (alternate angles)

$, m(\angle YCB) = m(\angle B)$ (alternate angles)

$\therefore m(\angle A) + m(\angle ACB) + m(\angle B) = 180°$ (Q.E.D.)

Example 1 **In the opposite figure :**

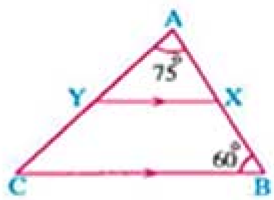
ABC is a triangle in which $m(\angle A) = 75°$,

$m(\angle B) = 60°$,

$X \in \overline{AB}$ and $Y \in \overline{AC}$

Such that : $\overline{XY} \parallel \overline{BC}$

Find : $m(\angle AYX)$



Solution

Given $\overline{XY} \parallel \overline{BC}$, m (∠ A) = 75° and m (∠ B) = 60°

R.T.F. m (∠ AYX)

Proof ∵ m (∠ A) = 75° and m (∠ B) = 60° (given)

, the sum of measures of the interior angles of Δ ABC = 180°

∴ m (∠ C) = 180° – (75° + 60°)

= 180° – 135° = 45°

∵ $\overline{XY} \parallel \overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal.

∴ m (∠ AYX) = m (∠ C) = 45° (corresponding angles) (The req.)

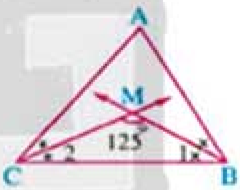
Try to solve the example by another method.

Example 2 **In the opposite figure :**

$\overrightarrow{BM}$ bisects ∠ ABC , $\overrightarrow{CM}$ bisects ∠ ACB

and m (∠ BMC) = 125°

Find : m (∠ A)



Solution

Given $\overrightarrow{BM}$ bisects ∠ ABC , $\overrightarrow{CM}$ bisects ∠ ACB and m (∠ BMC) = 125°

R.T.F. m (∠ A)

Proof ∵ The sum of measures of the interior angles of Δ MBC = 180°

and m (∠ BMC) = 125°

∴ m (∠ 1) + m (∠ 2) = 180° – 125° = 55°

But m (∠ ABC) = 2 m (∠ 1) and m (∠ ACB) = 2 m (∠ 2)

∴ m (∠ ABC) + m (∠ ACB) = 2 × 55° = 110°

∵ The sum of measures of the interior angles of Δ ABC = 180°

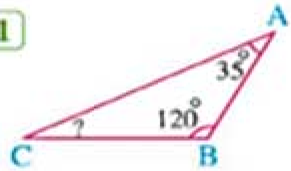
∴ m (∠ A) = 180° – 110° = 70° (The req.)

Unit 3

TRY by yourself 1

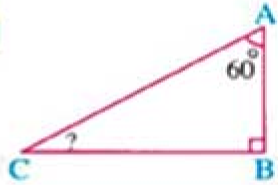
In each of the following figures , find the measure of the angle marked by (?) :

1



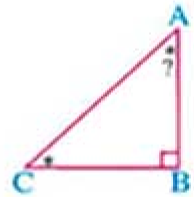
m (∠ C) = ...°

2



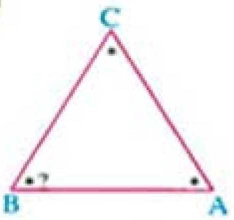
m (∠ C) = ...°

3



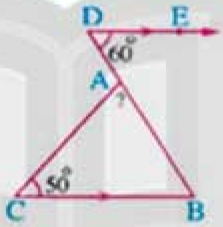
m (∠ A) = ...°

4



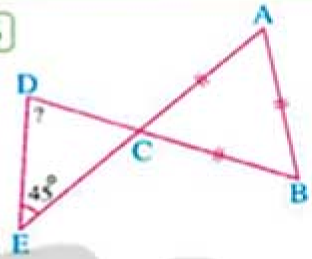
m (∠ B) = ...°

5



m (∠ BAC) = ...°

6

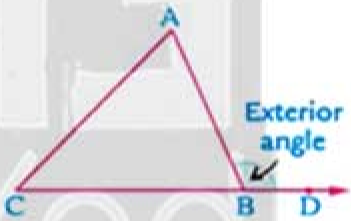


m (∠ D) = ...°

The exterior angle of the triangle

In the opposite figure :

If ABC is a triangle , $D \in \overrightarrow{CB}$ and $D \notin \overline{CB}$, then ∠ ABD is called an exterior angle of Δ ABC

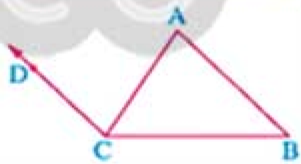


Notice that :

In the opposite figure :

∠ ACD is not an exterior angle of Δ ABC

because $D \notin \overrightarrow{BC}$



The measure of an exterior angle of a triangle

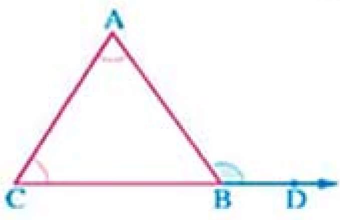
The measure of an exterior angle of a triangle is equal to the sum of the measures of its non adjacent interior angles.

In the opposite figure :

If ABC is a triangle , $D \in \overrightarrow{CB}$ and $D \notin \overline{CB}$,

then m (∠ ABD) = m (∠ A) + m (∠ C)

We can prove that as follows :



$\because$ m ($\angle$ A) + m ($\angle$ C) + m ($\angle$ ABC) = 180°

, m ($\angle$ ABD) + m ($\angle$ ABC) = 180°

$\therefore$ m ($\angle$ ABD) + m ($\angle$ ABC) = m ($\angle$ A) + m ($\angle$ C) + m ($\angle$ ABC)

$\therefore$ m ($\angle$ ABD) = m ($\angle$ A) + m ($\angle$ C) (Q.E.D.)

Notice that :

The measure of the exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

i.e. In the previous figure : m ($\angle$ ABD) > m ($\angle$ A) and m ($\angle$ ABD) > m ($\angle$ C)

Example 3 **In the opposite figure :**

ABC is a triangle , D $\in \overrightarrow{BC}$ and E $\in \overline{AC}$

where $\overrightarrow{BE}$ bisects $\angle$ ABC ,

m ($\angle$ A) = 80° and

m ($\angle$ ACD) = 150°

Find :

1 m ($\angle$ ABC) 2 m ($\angle$ BEC)

Solution

Given $\overrightarrow{BE}$ bisects $\angle$ ABC , m ($\angle$ A) = 80° and m ($\angle$ ACD) = 150°

R.T.F. 1 m ($\angle$ ABC) 2 m ($\angle$ BEC)

Proof $\because$ $\angle$ ACD is an exterior angle of Δ ABC

$\therefore$ m ($\angle$ ACD) = m ($\angle$ A) + m ($\angle$ ABC)

$\therefore$ 150° = 80° + m ($\angle$ ABC)

$\therefore$ m ($\angle$ ABC) = 150° − 80° = 70° (First req.)

$\because$ $\overrightarrow{BE}$ bisects $\angle$ ABC (given)

$\therefore$ m ($\angle$ ABE) = $\frac{1}{2}$ m ($\angle$ ABC) = $\frac{70°}{2}$ = 35°

$\because$ $\angle$ BEC is an exterior angle of Δ ABE

$\therefore$ m ($\angle$ BEC) = m ($\angle$ A) + m ($\angle$ ABE) = 80° + 35° = 115° (Second req.)

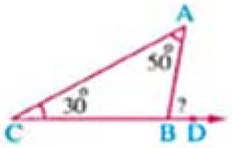
Try to solve the example by another method.

Unit 3

TRY 2 by yourself

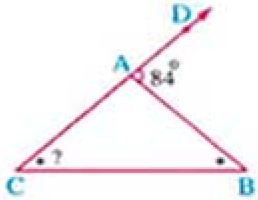
In each of the following figures , find the measure of the angle marked by (?) :

1



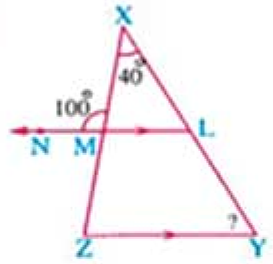
m (∠ ABD) = ………°

2



m (∠ C) = ………°

3



m (∠ Y) = ………°

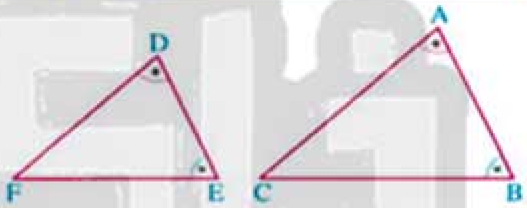
Remark 1

"If two angles of one triangle equal two angles of another triangle in measure , then the third angle of the first triangle is equal in measure to the third angle of the other triangle."

In ΔΔ ABC and DEF ,

if m (∠ A) = m (∠ D) and m (∠ B) = m (∠ E) ,

then m (∠ C) = m (∠ F)

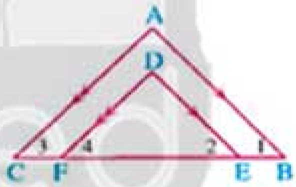


Example 4 **In the opposite figure :**

ABC and DEF are two triangles , E ∈ $\overline{BC}$,

F ∈ $\overline{BC}$, $\overline{DE}$ // $\overline{AB}$ and $\overline{DF}$ // $\overline{AC}$

Prove that : m (∠ A) = m (∠ D)



Solution

Given $\overline{DE}$ // $\overline{AB}$ and $\overline{DF}$ // $\overline{AC}$

R.T.P. m (∠ A) = m (∠ D)

Proof ∵ $\overline{DE}$ // $\overline{AB}$ and $\overleftrightarrow{BC}$ is a transversal to them.

∴ m (∠ 1) = m (∠ 2) (corresponding angles)

∵ $\overline{DF}$ // $\overline{AC}$ and $\overleftrightarrow{BC}$ is a transversal to them.

∴ m (∠ 3) = m (∠ 4) (corresponding angles)

In Δ Δ ABC and DEF :

∵ m (∠ 1) = m (∠ 2) and m (∠ 3) = m (∠ 4)

∴ m (∠ A) = m (∠ D) (Q.E.D.)

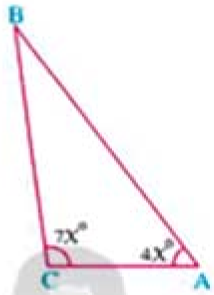
Remark 2

- If the sum of measures of two angles in a triangle **equals 90°** , then the third angle is **right**.
- If the sum of measures of two angles in a triangle is **less than 90°** , then the third angle is **obtuse**.
- If the sum of measures of two angles in a triangle is **more than 90°** , then the third angle is **acute**.

Example 5 **In the opposite figure :**

ABC is a triangle in which $m(\angle A) = 2\,m(\angle B) = 4\,x^\circ$ and $m(\angle C) = 7\,x^\circ$

Prove that : $\angle C$ is an obtuse angle.



Solution

Given $m(\angle A) = 2\,m(\angle B) = 4\,x^\circ$ and $m(\angle C) = 7\,x^\circ$

R.T.P. $\angle C$ is an obtuse angle.

Proof

$\because 2\,m(\angle B) = 4\,x^\circ$

$\therefore m(\angle B) = 2\,x^\circ$

$\therefore m(\angle A) + m(\angle B) = 4\,x^\circ + 2\,x^\circ = 6\,x^\circ$

$\because m(\angle C) = 7\,x^\circ$

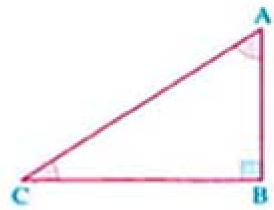
$\therefore m(\angle A) + m(\angle B) < m(\angle C)$

$\therefore \angle C$ is an obtuse angle. (Q.E.D.)

Remark 3

If the measure of an angle in a triangle equals the sum of measures of the other two angles , then the triangle is right-angled.

In the opposite figure :



If ABC is a triangle in which : $m(\angle A) + m(\angle C) = m(\angle B)$

, then $m(\angle B) = \frac{180^\circ}{2} = 90^\circ$

i.e. Δ ABC is right-angled at B

Example 6 ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 2 : 3 : 5
Prove that the triangle is right-angled , then mention the right angle.

Solution

Given ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 2 : 3 : 5

R.T.P. Δ ABC is right-angled and mention the right angle.

Proof

∵ m (∠ A) + m (∠ B) is equivalent to 5 parts and m (∠ C) is equivalent to 5 parts

∴ m (∠ A) + m (∠ B) = m (∠ C)

∵ The sum of measures of the interior angles of a triangle = 180°

∴ m (∠ A) + m (∠ B) + m (∠ C) = 180°

∴ m (∠ A) + m (∠ B) = m (∠ C) = 90°

∴ Δ ABC is right-angled at C (Q.E.D.)

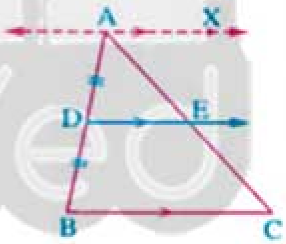
Answers of try by yourself

1. (1) 25° (2) 30° (3) 45°
2. (1) 80° (2) 42° (3) 60° (4) 60° (5) 70° (6) 75°

Lesson 6 Follow : The triangle

Theorem 2

The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.



Given D is the midpoint of $\overline{AB}$, $\overrightarrow{DE} \parallel \overline{BC}$

R.T.P. E is the midpoint of $\overline{AC}$

Construction Draw $\overleftrightarrow{AX} \parallel \overline{BC}$

Proof $\because \overleftrightarrow{AX} \parallel \overrightarrow{DE} \parallel \overline{BC}$

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two transversals

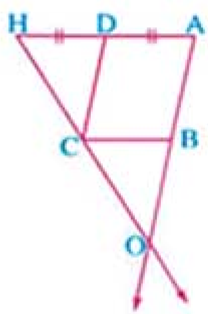
to them at D and E respectively.

, $\because AD = DB$ $\qquad \therefore AE = EC$

$\therefore$ E is the midpoint of $\overline{AC}$ (Q.E.D.)

Example 1

In the opposite figure :



ABCD is a parallelogram ,

$H \in \overrightarrow{AD}$ such that : $AD = DH$, $\overrightarrow{HC} \cap \overrightarrow{AB} = \{O\}$

Prove that :

1 $HC = CO$ $\qquad$ 2 $AB = BO$

Solution

Given ABCD is a parallelogram , AD = DH and $\overrightarrow{HC} \cap \overrightarrow{AB} = \{O\}$

R.T.P. 1 HC = CO   2 AB = BO

Proof In Δ HAO :

$\because$ D is the midpoint of $\overline{HA}$ (given) ,

$\overline{DC} \parallel \overline{AO}$ (definition of the parallelogram)

$\therefore$ C is the midpoint of $\overline{HO}$

i.e. HC = CO (theorem) (Q.E.D. 1)

$\because$ C is the midpoint of $\overline{OH}$ (proved)

, $\overline{CB} \parallel \overline{HA}$ (definition of the parallelogram)

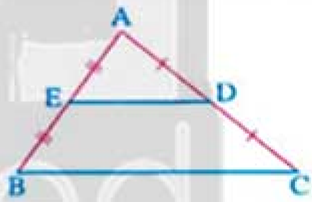
$\therefore$ B is the midpoint of $\overline{AO}$

i.e. AB = BO (theorem) (Q.E.D. 2)

Corollary

The line segment joining the midpoints of two sides of a triangle is parallel to the third side.

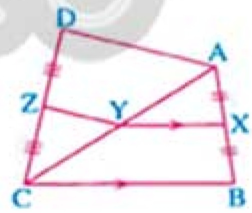
In the opposite figure :



If ABC is a triangle in which D is the midpoint of $\overline{AC}$,

E is the midpoint of $\overline{AB}$, then : $\overline{ED} \parallel \overline{BC}$

Example 2 **In the opposite figure :**



X is the midpoint of $\overline{AB}$,

$\overline{XY} \parallel \overline{BC}$ and Z is the midpoint of $\overline{DC}$

Prove that : $\overline{YZ} \parallel \overline{AD}$

Solution

Given X is the midpoint of $\overline{AB}$, Z is the midpoint of $\overline{CD}$ and $\overline{XY} \parallel \overline{BC}$

R.T.P. $\overline{YZ} \parallel \overline{AD}$

Proof In Δ ABC :

$\because$ AX = XB , $\overline{XY} \parallel \overline{BC}$ $\therefore$ AY = YC (theorem)

In Δ ACD : $\because$ AY = YC (proved)   , DZ = ZC (given)

$\therefore$ $\overline{YZ} \parallel \overline{AD}$ (corollary) (Q.E.D.)

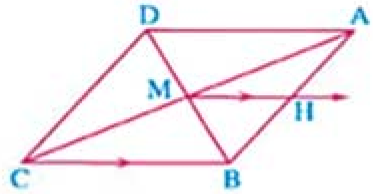
TRY 1 by yourself

In the opposite figure :

ABCD is a parallelogram and M is the point of intersection of its two diagonals.

Draw $\overrightarrow{MH} \parallel \overline{BC}$ to cut $\overline{AB}$ at H

Complete the solution to prove that : AH = HB



Given ....

R.T.P. ....

Proof

$\because$ ABCD is a parallelogram

$\therefore$ M is the midpoint of ....

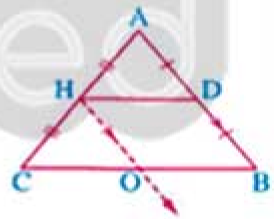
In Δ ABC :

$\because$ M is the midpoint of .... and .... // ....

$\therefore$ .... (theorem) (Q.E.D.)

Theorem 3

The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side.



Given ABC is a triangle , D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$

R.T.P. $DH = \frac{1}{2}$ BC

Construction Draw $\overrightarrow{HO} \parallel \overline{AB}$ to cut $\overline{BC}$ at O

Proof

$\because$ D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$

$\therefore \overline{DH} \parallel \overline{BC}$ (corollary)

, $\because \overline{HO} \parallel \overline{AB}$ (construction) , H is the midpoint of $\overline{AC}$

$\therefore$ O is the midpoint of $\overline{BC}$ $\therefore BO = \frac{1}{2}$ BC

, $\because$ The figure DHOB is a parallelogram.

$\therefore DH = BO = \frac{1}{2}$ BC (Q.E.D.)

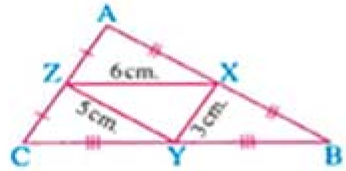
Unit 3

Example 3 **In the opposite figure :**

ABC is a triangle in which X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

If XY = 3 cm. , YZ = 5 cm. and ZX = 6 cm. ,

then find the perimeter of Δ ABC



Solution

Given ABC is a triangle in which X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively , XY = 3 cm. , YZ = 5 cm. and ZX = 6 cm.

R.T.F. The perimeter of Δ ABC

Proof In Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$ and Z is the midpoint of $\overline{AC}$

$\therefore$ XZ = $\frac{1}{2}$ BC (theorem)

$\therefore$ BC = 6 × 2 = 12 cm.

Similarly : $\because$ X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{BC}$

$\therefore$ XY = $\frac{1}{2}$ AC

$\therefore$ AC = 3 × 2 = 6 cm.

Similarly : $\because$ Y is the midpoint of $\overline{BC}$ and Z is the midpoint of $\overline{AC}$

$\therefore$ YZ = $\frac{1}{2}$ AB

$\therefore$ AB = 5 × 2 = 10 cm.

$\therefore$ The perimeter of Δ ABC = AB + BC + CA = 10 + 12 + 6 = 28 cm.

(The req.)

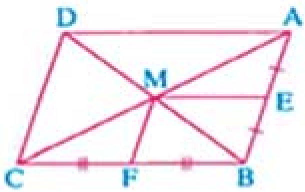
Example 4 **In the opposite figure :**

ABCD is a parallelogram in which :

$\overline{AC} \cap \overline{BD} = \{M\}$, E is the midpoint of $\overline{AB}$

, F is the midpoint of $\overline{BC}$

Prove that : The figure EBFM is a parallelogram.



Solution

Given ABCD is a parallelogram , E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$

R.T.P. The figure EBFM is a parallelogram.

Proof

$\because$ ABCD is a parallelogram whose diagonals intersect at M

$\therefore$ M is the midpoint of each of $\overline{AC}$ and $\overline{BD}$

$\therefore$ In Δ ABC :

$\because$ E is the midpoint of $\overline{AB}$, M is the midpoint of $\overline{AC}$

$\therefore$ $\overline{EM}$ // $\overline{BC}$

$\therefore$ $\overline{EM}$ // $\overline{BF}$

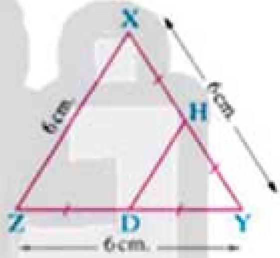
, EM = $\frac{1}{2}$ BC (theorem)

$\therefore$ EM = BF

$\therefore$ The figure EBFM is a parallelogram. (Q.E.D.)

TRY 2 by yourself

In the opposite figure :



XYZ is an equilateral triangle whose side is of length 6 cm. ,

D is the midpoint of $\overline{YZ}$ and H is the midpoint of $\overline{XY}$

Complete the following solution to prove that :

Δ HYD is an equilateral triangle and find its perimeter.

Given ....

R.T.P. ....

Proof

$\because$ D is the midpoint of .... $\therefore$ YD = ... cm. (1)

$\because$ H is the midpoint of .... $\therefore$ YH = ... cm. (2)

In Δ XYZ :

$\because$ D is the midpoint of .... and H is the midpoint of ....

$\therefore$ DH = $\frac{1}{2}$ .... = ... cm. (3)

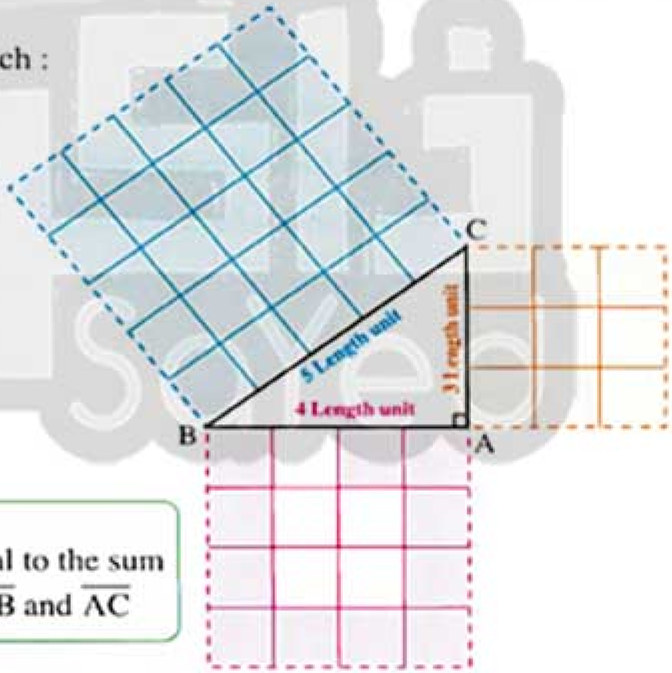
From (1) , (2) and (3) :

$\therefore$ Δ HYD is .... and its perimeter = ... cm. (The req.)

Lesson 7 Pythagoras' theorem

In the opposite figure :

- If ABC is a right-angled triangle at A in which :
  AB = 4 length units , AC = 3 length units
  , BC = 5 length units , then :
- The area of the square drawn on $\overline{AB}$
  $= (AB)^2 = 16$ square unit.
- The area of the square drawn on $\overline{AC}$
  $= (AC)^2 = 9$ square unit.
- The area of the square drawn on $\overline{BC}$
  $= (BC)^2 = 25$ square unit.



i.e.

The area of the square drawn on $\overline{BC}$ is equal to the sum of the areas of the two squares drawn on $\overline{AB}$ and $\overline{AC}$

In other words

$(BC)^2 = (AB)^2 + (AC)^2$

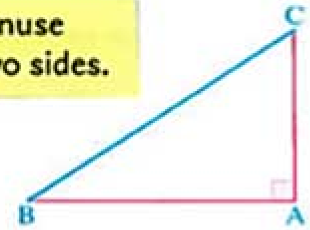
The verbal formula of this relation is defined by Pythagoras' theorem.

Pythagoras' theorem

The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.

We can also write the previous theorem as follows :

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



i.e. If ABC is a right-angled triangle at A, then :

$$(BC)^2 = (AB)^2 + (AC)^2$$

• **From the previous relation, we can deduce the following two relations :**

$$(AB)^2 = (BC)^2 - (AC)^2 \qquad (AC)^2 = (BC)^2 - (AB)^2$$

Example 1 **In each of the following figures :**

Find the side length which is denoted by sign (?) :

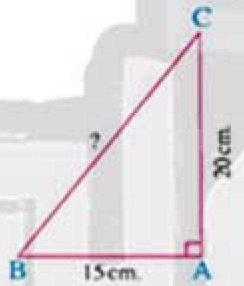


Fig. (1)

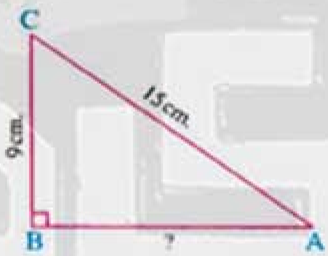


Fig. (2)

Fig. (3)

Solution

In fig. (1) :

$\because$ Δ ABC is right-angled at A

$\therefore (BC)^2 = (AB)^2 + (AC)^2 = (15)^2 + (20)^2 = 225 + 400 = 625$

$\therefore BC = \sqrt{625} = 25$ cm.

In fig. (2) :

$\because$ Δ ABC is right-angled at B

$\because (AB)^2 = (AC)^2 - (BC)^2 = (15)^2 - (9)^2 = 225 - 81 = 144$

$\therefore AB = \sqrt{144} = 12$ cm.

In fig. (3) :

$\because$ Δ ABC is right-angled at C

$\therefore (AC)^2 = (AB)^2 - (BC)^2 = (8.5)^2 - (7.5)^2 = 72.25 - 56.25 = 16$

$\therefore AC = \sqrt{16} = 4$ cm.

Unit 3

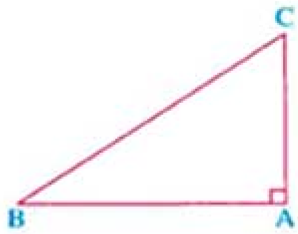
TRY 1 by yourself

In the opposite figure :

Δ ABC is right-angled at A

Complete the following table :

AB	8 cm.	12 cm.	12 cm.	20 cm.	....	....
AC	6 cm.	9 cm.	....	....	12 cm.	4.5 cm.
BC	....	....	13 cm.	25 cm.	20 cm.	7.5 cm.



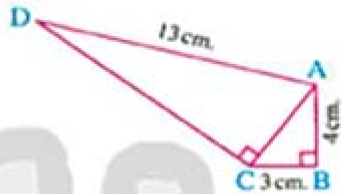
Example 2 **In the opposite figure :**

ABCD is a quadrilateral in which :

m (∠ B) = m (∠ ACD) = 90°

, AB = 4 cm. , BC = 3 cm. and AD = 13 cm.

Find : The length of each of $\overline{AC}$ and $\overline{CD}$



Solution

Given m (∠ B) = m (∠ ACD) = 90° , AB = 4 cm. , BC = 3 cm. and AD = 13 cm.

R.T.F. The length of each of $\overline{AC}$ and $\overline{CD}$

Proof $\because$ Δ ABC is a right-angled triangle at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras' theorem)

$\therefore (AC)^2 = (4)^2 + (3)^2 = 16 + 9 = 25$

$\therefore AC = \sqrt{25} = 5$ cm. (First req.)

$\because$ Δ ACD is a right-angled triangle at C

$\therefore (CD)^2 = (AD)^2 - (AC)^2$ (Pythagoras' theorem)

$\therefore (CD)^2 = (13)^2 - (5)^2 = 169 - 25 = 144$

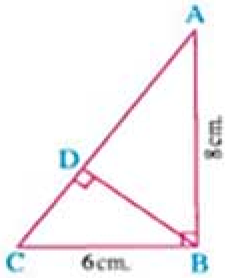
$\therefore CD = \sqrt{144} = 12$ cm. (Second req.)

Example 3 **In the opposite figure :**

ABC is a right-angled triangle at B , $D \in \overline{AC}$

such that $\overline{BD} \perp \overline{AC}$, AB = 8 cm. and CB = 6 cm.

Find : The length of $\overline{BD}$



Solution

Given Δ ABC is right-angled at B , $\overline{BD} \perp \overline{AC}$, AB = 8 cm. and CB = 6 cm.

R.T.F. The length of $\overline{BD}$

Proof

$\because$ Δ ABC is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras' theorem)

$\therefore (AC)^2 = 64 + 36 = 100$ $\qquad \therefore AC = \sqrt{100} = 10$ cm.

, $\because$ The area of $\Delta ABC = \frac{1}{2} BC \times AB = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$

, $\because$ The area of $\Delta ABC = \frac{1}{2} AC \times BD$

$\therefore 24 = \frac{1}{2} \times 10 \times BD$ $\qquad \therefore 24 = 5\, BD$

$\therefore BD = \frac{24}{5} = 4.8$ cm. (The req.)

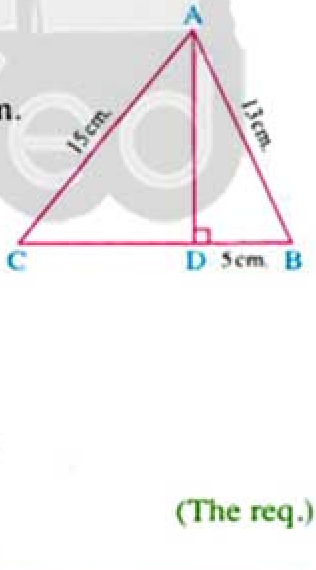
TRY 2 by yourself

In the opposite figure :

ABC is a triangle in which AB = 13 cm.

, AC = 15 cm. , $D \in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$ and BD = 5 cm.

Complete the following proof to find : The length of $\overline{DC}$



$\because$ Δ ABD is right-angled at D

$\therefore (AD)^2 = (AB)^2 - (\cdots\cdots)^2 = (\cdots\cdots)^2 - (\cdots\cdots)^2 = \cdots\cdots$

$\therefore AD = \sqrt{\cdots\cdots} = \cdots\cdots$ cm.

, $\because$ Δ ADC is right-angled at D

$\therefore (DC)^2 = (\cdots\cdots)^2 - (AD)^2 = (\cdots\cdots)^2 - (\cdots\cdots)^2 = \cdots\cdots$

$\therefore DC = \sqrt{\cdots\cdots} = \cdots\cdots$ cm. (The req.)

Answers *of try by yourself*

1 • AB = 16 cm. , AB = 6 cm. • AC = 5 cm. , AC = 15 cm.

• BC = 10 cm. , BC = 15 cm.

Unit 3

Enrichment information (for reading only)

You can get three numbers representing the lengths of sides of a right-angled triangle as follows :

1 If M is an even number bigger than 2, then the numbers $\boxed{M , \left(\frac{M}{2}\right)^2 - 1 , \left(\frac{M}{2}\right)^2 + 1}$ represent three lengths of sides of a right-angled triangle as shown in the following table :

M	$\left(\frac{M}{2}\right)^2 - 1$	$\left(\frac{M}{2}\right)^2 + 1$	Side lengths of the right-angled triangle
4	$\frac{16}{4} - 1 = 3$	$\frac{16}{4} + 1 = 5$	4 , 3 , 5
6	$\frac{36}{4} - 1 = 8$	$\frac{36}{4} + 1 = 10$	6 , 8 , 10
8	$\frac{64}{4} - 1 = 15$	$\frac{64}{4} + 1 = 17$	8 , 15 , 17
10	$\frac{100}{4} - 1 = 24$	$\frac{100}{4} + 1 = 26$	10 , 24 , 26

2 If M is an odd number bigger than 2, then the numbers $\boxed{M , \frac{M^2 - 1}{2} , \frac{M^2 + 1}{2}}$ represent three lengths of sides of a right-angled triangle as shown in the following table :

M	$\frac{M^2 - 1}{2}$	$\frac{M^2 + 1}{2}$	Side length of the right angled triangle
3	$\frac{9 - 1}{2} = 4$	$\frac{9 + 1}{2} = 5$	3 , 4 , 5
5	$\frac{25 - 1}{2} = 12$	$\frac{25 + 1}{2} = 13$	5 , 12 , 13
7	$\frac{49 - 1}{2} = 24$	$\frac{49 + 1}{2} = 25$	7 , 24 , 25
9	$\frac{81 - 1}{2} = 40$	$\frac{81 + 1}{2} = 41$	9 , 40 , 41

Lesson 8 Geometric transformations

There are many types of geometric transformations.

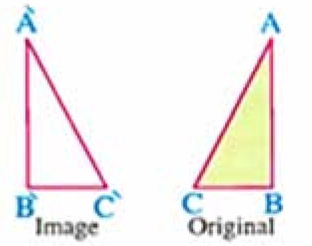
In this lesson, you will recognize the meaning of the geometric transformation, also recognize quickly three types of them :

1. Reflection.
2. Translation.
3. Rotation.

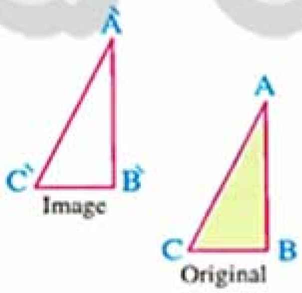
And you will study each of them in details next lessons.

The concept of the geometric transformation

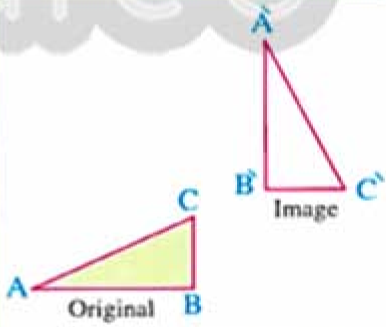
In each of the following figures , notice the image of △ ABC :



△ ABC has been **reflected**



△ ABC has been **translated**



△ ABC has been **rotated**

In each of the previous figures , notice that :

- The point A has been transferred to A`
- The point B has been transferred to B`

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• The point C has been transferred to $\grave{C}$

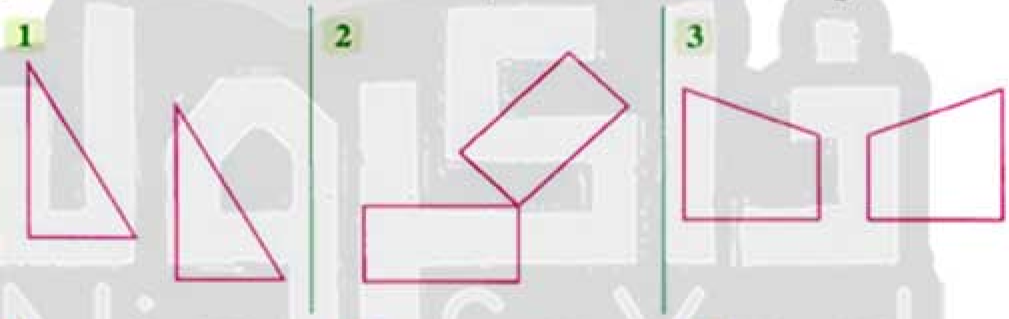
Thus , all the points of Δ ABC have been transferred to another position , then we say that Δ ABC has been transformed from a position to another position.

From the previous, we deduce that

If all the points of a geometrical figure have moved according to a certain system , then we will obtain an image in a new position to the same figure , then we say that this figure has been under the effect of a geometric transformation.

i.e. The geometric transformation maps each point P in the plane onto an image point $\grave{P}$ in the same plane.

Example 1 Write the geometric transformation which affected on the figure (reflection - translation - rotation) in each of the following :

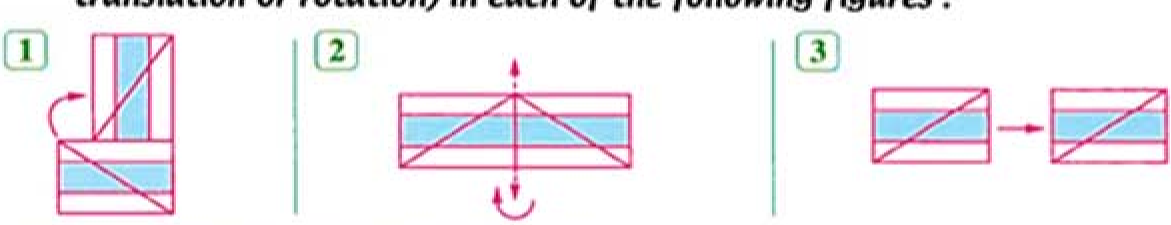


Solution

1 Translation.   2 Rotation.   3 Reflection

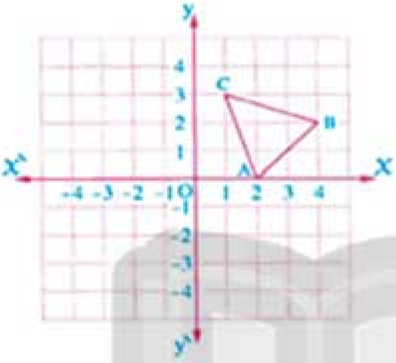
TRY 1 by yourself

Describe the type of the geometric transformation (reflection , translation or rotation) in each of the following figures :

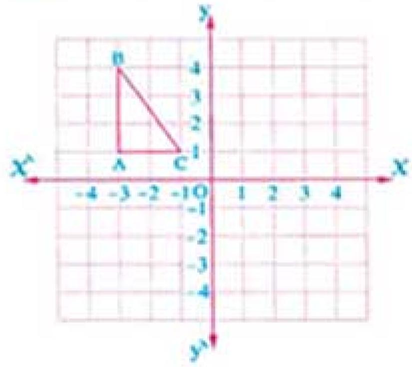


Example 2 Draw the image of each of the following figures according to the illustrated geometric transformation , then describe its type :

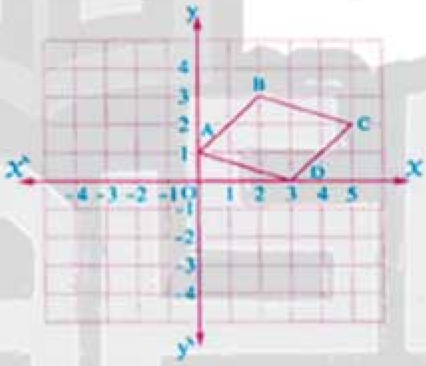
1 $(x , y) \longrightarrow (x - 4 , y + 1)$



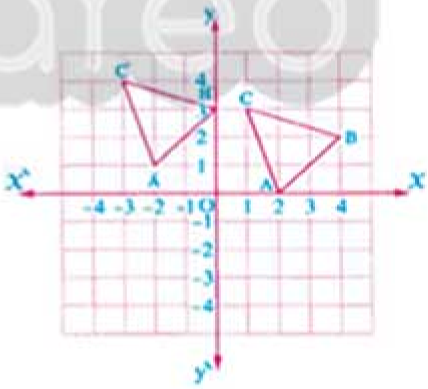
2 $(x , y) \longrightarrow (-x , -y)$



3 $(x , y) \longrightarrow (x , -y)$



Solution 1 $(x , y) \longrightarrow (x - 4 , y + 1)$



i.e. x transferred to $x - 4$ and

y transferred to $y + 1$, therefore we get :

$A (2 , 0) \longrightarrow \grave{A} (2 - 4 , 0 + 1)$

i.e. $\grave{A} (-2 , 1)$

$B (4 , 2) \longrightarrow \grave{B} (4 - 4 , 2 + 1)$

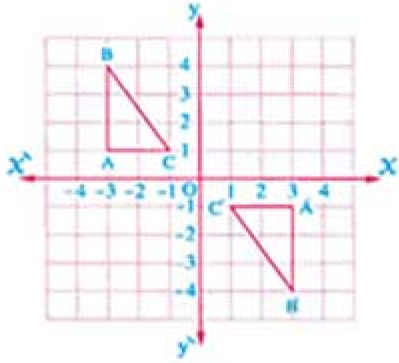
i.e. $\grave{B} (0 , 3)$

$C (1 , 3) \longrightarrow \grave{C} (1 - 4 , 3 + 1)$

i.e. $\grave{C} (-3 , 4)$

From the graph , it is shown that Δ ABC has been translated to become $\Delta \grave{A}\grave{B}\grave{C}$

2 $(x , y) \longrightarrow (-x , -y)$



i.e.

$A(-3 , 1) \longrightarrow \grave{A}(3 , -1)$

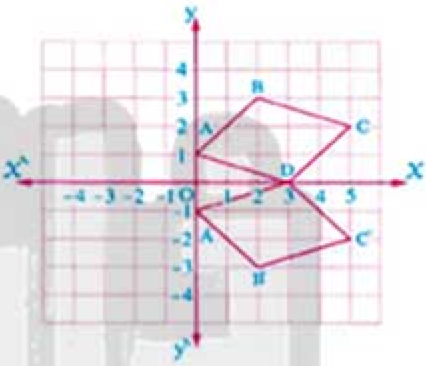
[Note : $-(-3) = 3$]

$B(-3 , 4) \longrightarrow \grave{B}(3 , -4)$

$C(-1 , 1) \longrightarrow \grave{C}(1 , -1)$

From the graph , it is shown that Δ ABC has been rotated to become Δ ÀB̀C̀

3 $(x , y) \longrightarrow (x , -y)$



$A(0 , 1) \longrightarrow \grave{A}(0 , -1)$

$B(2 , 3) \longrightarrow \grave{B}(2 , -3)$

$C(5 , 2) \longrightarrow \grave{C}(5 , -2)$

$D(3 , 0) \longrightarrow D(3 , 0)$

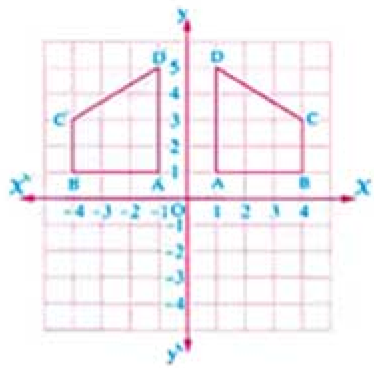
From the graph , it is shown that the shape ABCD has been reflected to become ÀB̀C̀D

Example 3 Draw the image of the quadrilateral ABCD where A (1 , 1) , B (4 , 1) , C (4 , 3) , D (1 , 5) according to each of the following transformations , then describe its type :

1. $(x , y) \longrightarrow (-x , y)$
2. $(x , y) \longrightarrow (-y , x)$
3. $(x , y) \longrightarrow (x , y - 5)$

Solution

1 $(x , y) \longrightarrow (-x , y)$



i.e.

$A(1 , 1) \longrightarrow \grave{A}(-1 , 1)$

$B(4 , 1) \longrightarrow \grave{B}(-4 , 1)$

$C(4 , 3) \longrightarrow \grave{C}(-4 , 3)$

$D(1 , 5) \longrightarrow \grave{D}(-1 , 5)$

The transformation is reflection.

2 $(x , y) \longrightarrow (-y , x)$

i.e.

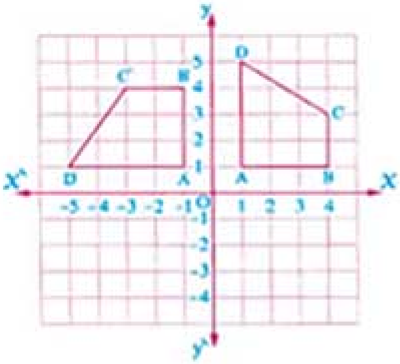
$A(1 , 1) \longrightarrow \grave{A}(-1 , 1)$

$B(4 , 1) \longrightarrow \grave{B}(-1 , 4)$

$C(4 , 3) \longrightarrow \grave{C}(-3 , 4)$

$D(1 , 5) \longrightarrow \grave{D}(-5 , 1)$

The transformation is rotation.



3 $(x , y) \longrightarrow (x , y - 5)$

i.e.

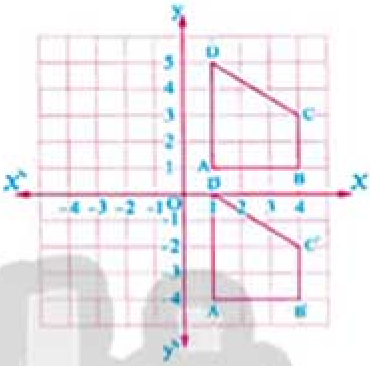
$A(1 , 1) \longrightarrow \grave{A}(1 , -4)$

$B(4 , 1) \longrightarrow \grave{B}(4 , -4)$

$C(4 , 3) \longrightarrow \grave{C}(4 , -2)$

$D(1 , 5) \longrightarrow \grave{D}(1 , 0)$

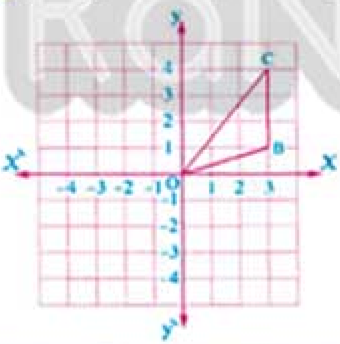
The transformation is translation.



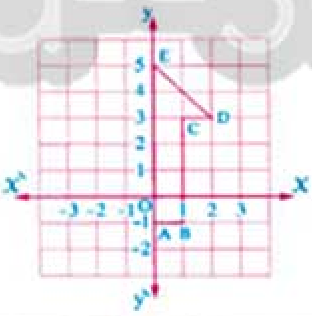
TRY 2 by yourself

Map each of the following shapes due to the geometric transformation above it , then describe its type :

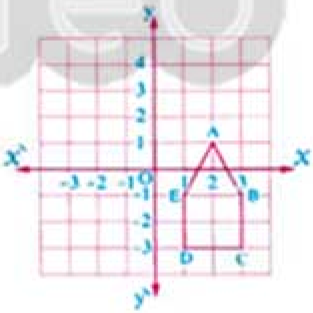
1 $(x , y) \longrightarrow (-x , -y)$



2 $(x , y) \longrightarrow (-x , y)$



3 $(x , y) \longrightarrow (x , y + 2)$



Answers of try by yourself

1 (1) rotation (2) reflection (3) translation

2 (1) rotation (2) reflection (3) translation

Lesson 9

Reflection in a straight line

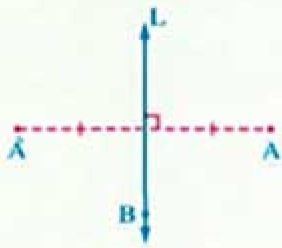
Prelude

If you stand in front of a plane mirror , then you will see your picture (image) reflected in the mirror in the same size and details and you will notice also that the distance between the image and the mirror equals the real distance between you and the mirror. If you approach the mirror , then you will find that your image approaches also the mirror.

Definition of reflection in a straight line

Reflection in the straight line L maps each point A to the point $\grave{A}$ in the same plane such that :

1. If $A \notin L$, then the straight line L is the perpendicular bisector to the line segment $\overline{A\grave{A}}$
2. If $B \in L$, then B is reflected onto itself
   i.e. $\grave{B}$ coincides B

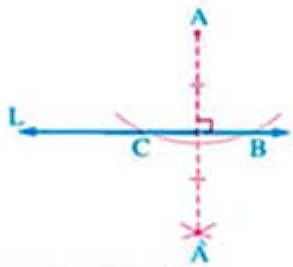


Finding the image of a point by reflection in a given straight line

• **To find $\grave{A}$ which is the image of A by reflection in the straight line L , we do as follows :**

1 Draw an arc of a circle with centre A to cut L at B and C

2 With the same radius length taking B and C as centres , draw two arcs in the other side of the straight line L to intersect at $\grave{A}$, then $\grave{A}$ is the image of A by reflection in L



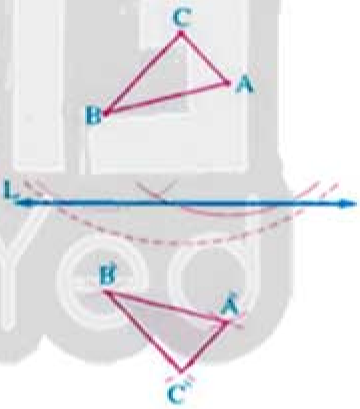
Check by measuring that :

$\overline{A\grave{A}} \perp$ L and L bisects $\overline{A\grave{A}}$

Finding the image of a polygon by reflection in a given straight line

• **To find the image of a polygon as $\triangle$ ABC by reflection in the straight line L , we do as follows :**

1 Find the image of each vertex of $\triangle$ ABC by reflection in the straight line L as we did before ($\grave{A}$ is the image of A , $\grave{B}$ is the image of B and $\grave{C}$ is the image of C)



2 Draw $\overline{\grave{A}\grave{B}}$, $\overline{\grave{B}\grave{C}}$ and $\overline{\grave{C}\grave{A}}$, then $\triangle \grave{A}\grave{B}\grave{C}$ is the image of $\triangle$ ABC by reflection in the straight line L

$\triangle ABC \equiv \triangle \grave{A}\grave{B}\grave{C}$

Check by measuring that :

• AB = $\grave{A}\grave{B}$, BC = $\grave{B}\grave{C}$ and CA = $\grave{C}\grave{A}$

• m ($\angle$ A) = m ($\angle \grave{A}$) , m ($\angle$ B) = m ($\angle \grave{B}$) and m ($\angle$ C) = m ($\angle \grave{C}$)

From the previous, we deduce that :

Reflection is a geometrical transformation which transforms the geometrical shape into another one congruent to it (equal to it in its side lengths and angle measures), but the direction of reading the shape is the opposite direction of reading the image.

Notice that :

The reading of $\triangle$ ABC is clockwise while the reading of $\triangle \grave{A}\grave{B}\grave{C}$ is anticlockwise.

Unit 3

Properties of reflection in a straight line

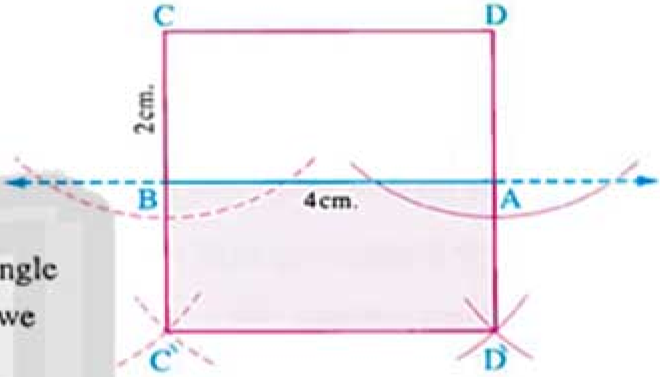
Illustrated example

Draw the image of the rectangle ABCD in which AB = 4 cm. , BC = 2 cm. by reflection in $\overleftrightarrow{AB}$

Solution

First

We draw the rectangle ABCD in which : AB = 4 cm. and BC = 2 cm.



Second

To find the image of the rectangle ABCD by reflection in $\overleftrightarrow{AB}$, we do as follows :

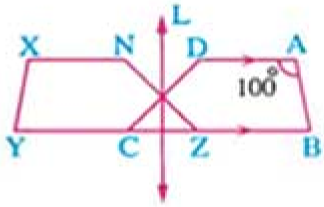
1. The images of A and B by reflection in $\overleftrightarrow{AB}$ are the same because they belong to $\overleftrightarrow{AB}$
2. We find the image of D by reflection in $\overleftrightarrow{AB}$, let it be $\grave{D}$, the image of C by reflection in $\overleftrightarrow{AB}$, let it be $\grave{C}$, then we get the rectangle $AB\grave{C}\grave{D}$ to be the image of the rectangle ABCD by reflection in $\overleftrightarrow{AB}$

Notice that :

1. $AD = A\grave{D}$, $DC = \grave{D}\grave{C}$, $CB = \grave{C}B$ and $\overline{AB}$ is a common side.	i.e.	Reflection in a straight line reserves the lengths of the line segments.
2. m (∠ BAD) = m (∠ $BA\grave{D}$) , m (∠ ABC) = m (∠ $AB\grave{C}$) , m (∠ C) = m (∠ $\grave{C}$) and m (∠ D) = m (∠ $\grave{D}$)	i.e.	Reflection in a straight line reserves the measures of the angles.
3. From the rectangle ABCD : $\overline{AD}$ // $\overline{BC}$, from the rectangle $AB\grave{C}\grave{D}$: $\overline{A\grave{D}}$ // $\overline{B\grave{C}}$ ∴ The images of the two parallel line segments are also two parallel line segments.	i.e.	Reflection in a straight line reserves the parallelism.
4. The reading of the rectangle ABCD is in the clockwise direction while the reading of the rectangle $AB\grave{C}\grave{D}$ is in anticlockwise direction.	i.e.	Reflection in a straight line doesn't reserve the orientation of the vertices of the figure.
5. If a point lies on $\overline{DC}$ and we find its image by reflection in $\overleftrightarrow{AB}$, we find its image lie on $\overline{\grave{D}\grave{C}}$	i.e.	Reflection in a straight line reserves the betweenness.

Example 1 **In the opposite figure :**

ABCD is a quadrilateral in which m (∠ A) = 100° and $\overline{AD}$ // $\overline{BC}$ If the figure XYZN is the image of the figure ABCD by reflection in the straight line L **Find :** m (∠ Y)



Solution

∵ The figure XYZN is the image of the figure ABCD by reflection in L

∴ X is the image of A by reflection in L

∴ m (∠ X) = m (∠ A) = 100°

(because reflection in the straight line reserves the measures of angles)

∵ $\overline{XN}$ is the image of $\overline{AD}$ and $\overline{YZ}$ is the image of $\overline{BC}$ by reflection in L

, ∵ $\overline{AD}$ // $\overline{BC}$

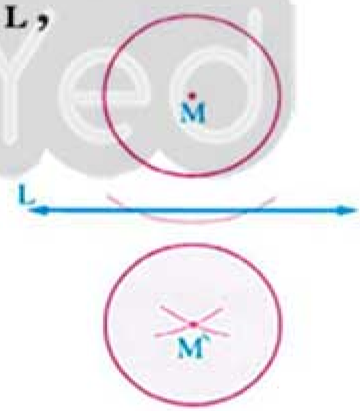
∴ $\overline{XN}$ // $\overline{YZ}$ (because reflection in a straight line reserves parallelism)

∴ m (∠ X) + m (∠ Y) = 180° (two interior angles in the same side of the transversal)

∴ m (∠ Y) = 180° − 100° = 80° (The req.)

Finding the image of a circle by reflection in a given straight line

- **To find the image of a circle M by reflection in the straight line L , we do as follows :**



1. Find the image of the centre M by reflection in L as we did before , say M̀
2. Use the compasses with radius length equal to the radius length of the circle M to draw a circle with centre M̀ that will be the image of the circle M by reflection in L

Optical illusion :

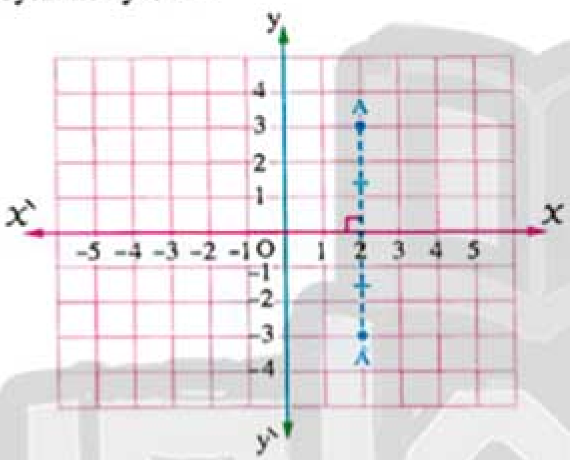
Look at the picture. What do you see ?

Unit 3

Reflection in the Cartesian coordinates plane

Reflection in the x-axis

- To find the image of the point A (2 , 3) by reflection in $\overleftrightarrow{XX'}$ (the X-axis) : draw $\overline{AA'}$ such that $\overleftrightarrow{XX'}$ is the line of symmetry of it.



Then we find that :

$A(2, 3) \longrightarrow A'(2, -3)$

i.e. The reflection in the X-axis changes the sign of the 2nd projection (y-coordinates)

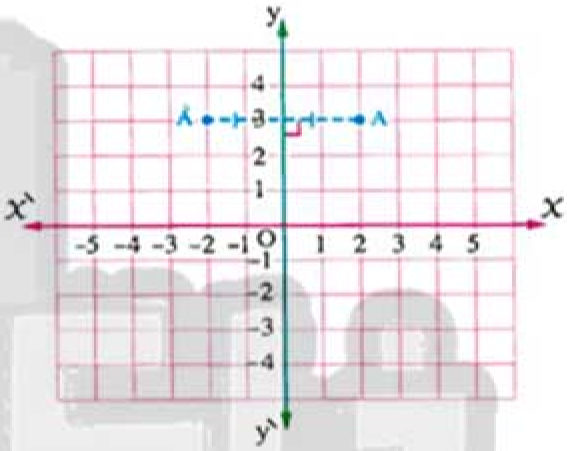
$$A(X, y) \xrightarrow[\text{the } X\text{-axis}]{\text{by reflection in}} A'(X, -y)$$

For example:

- $(2, 4) \xrightarrow[\text{the } X\text{-axis}]{\text{by reflection in}} (2, -4)$
- $(-2, -1) \xrightarrow[\text{the } X\text{-axis}]{\text{by reflection in}} (-2, 1)$
- $(-5, 3) \xrightarrow[\text{the } X\text{-axis}]{\text{by reflection in}} (-5, -3)$
- $(7, -2) \xrightarrow[\text{the } X\text{-axis}]{\text{by reflection in}} (7, 2)$

Reflection in the y-axis

- To find the image of the point A (2 , 3) by reflection in $\overleftrightarrow{yy'}$ (the y-axis) : draw $\overline{AA'}$ such that $\overleftrightarrow{yy'}$ is the line of symmetry of it.



Then we find that :

$A(2, 3) \longrightarrow A'(-2, 3)$

i.e. The reflection in the y-axis changes the sign of the 1st projection (X-coordinates)

$$A(X, y) \xrightarrow[\text{the y-axis}]{\text{by reflection in}} A'(-X, y)$$

For example:

- $(2, 4) \xrightarrow[\text{the y-axis}]{\text{by reflection in}} (-2, 4)$
- $(-2, -1) \xrightarrow[\text{the y-axis}]{\text{by reflection in}} (2, -1)$
- $(-5, 3) \xrightarrow[\text{the y-axis}]{\text{by reflection in}} (5, 3)$
- $(7, -2) \xrightarrow[\text{the y-axis}]{\text{by reflection in}} (-7, -2)$

Remarks

1. The image of the point $(x , 0)$ by reflection in the x-axis is itself because it lies on the x-axis
   For example: $(5 , 0) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (5 , 0)$
2. The image of the point $(0 , y)$ by reflection in the y-axis is itself because it lies on the y-axis
   For example: $(0 , -3) \xrightarrow[\text{the y-axis}]{\text{by reflection in}} (0 , -3)$
3. The image of the point $(0 , 0)$ by reflection in the x-axis or the y-axis is itself because it lies on both two axes.

TRY 1 by yourself

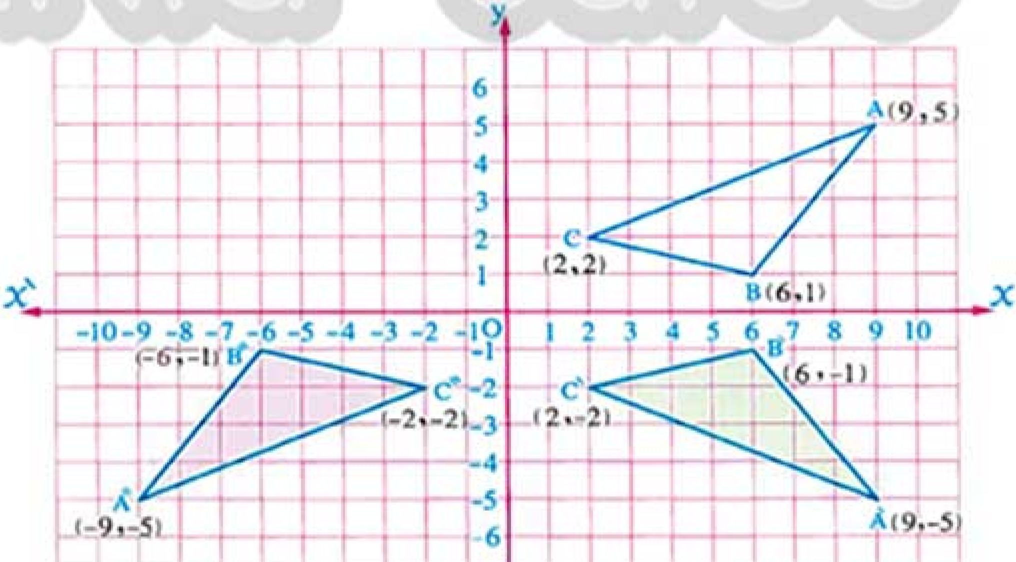
Complete the following table :

The point	(5 , 1)	(2 , −3)	(−1 , 4)	(−2 , −6)	(0 , −1)	(3 , 0)	(0 , 0)
Its image by reflection in the x-axis							
Its image by reflection in the y-axis							

Example 2 Draw on a square lattice the triangle ABC where A (9 , 5) , B (6 , 1) and C (2 , 2) :

1. Draw $\Delta A'B'C'$ which is the image of ΔABC by reflection in the x-axis
2. Draw $\Delta A''B''C''$ which is the image of $\Delta A'B'C'$ by reflection in the y-axis

Solution

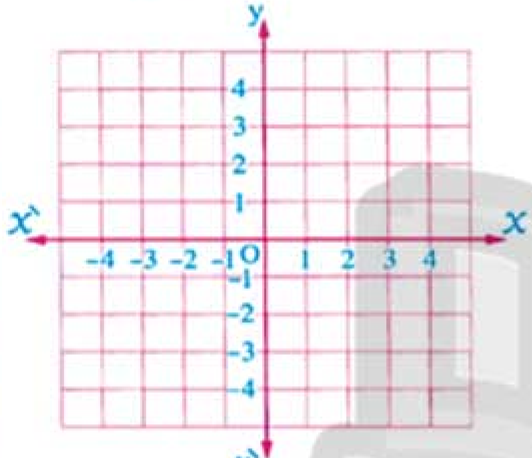


Unit 3

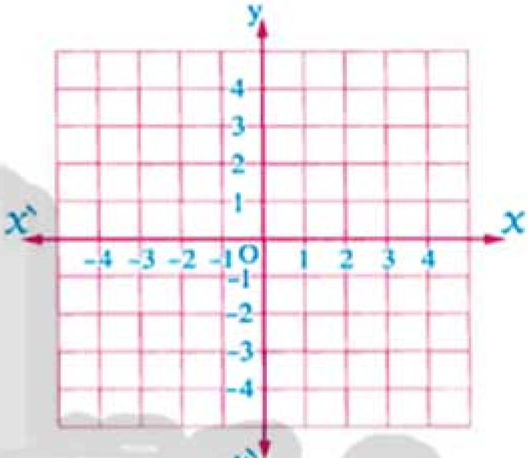
TRY 2 by yourself

Draw ΔABC ***where*** $A(1, 1)$, $B(4, 1)$ ***and*** $C(3, 3)$, ***then draw its image by reflection in :***

1 The x-axis

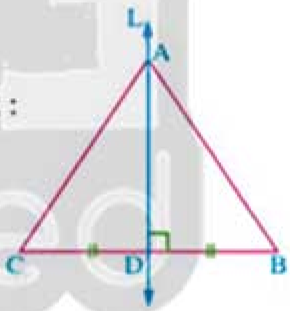


2 The y-axis



Symmetry

- **In the opposite figure :**



ABC is a triangle, $\overleftrightarrow{AD} \perp \overline{BC}$, D is the midpoint of $\overline{BC}$, we find that :

- The image of A by reflection in L is itself (A)
- The image of B by reflection in L is C
- The image of C by reflection in L is B

i.e. The image of ΔABC by reflection in L is ΔACB

We can say that ΔABC is transformed to itself by reflection in the straight line L, therefore the straight line L is called the axis of symmetry of ΔABC

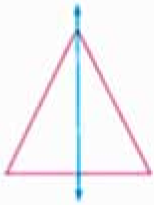
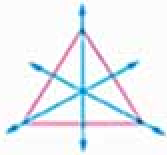
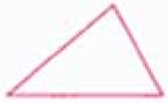
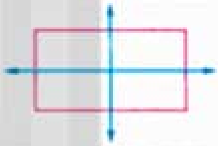
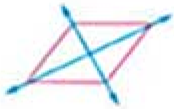
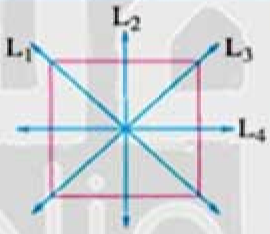
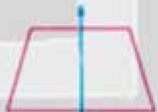
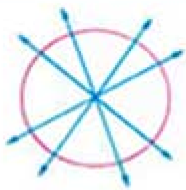
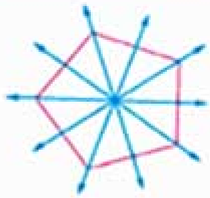
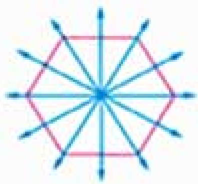
From the previous, we can deduce the definition of the axis of symmetry as follows :

If the reflection in a line transforms the figure to itself, then this line is called an axis of symmetry of the figure.

Remark

"The axis of symmetry divides the figure into two congruent figures."

The axes of symmetry of some geometric figures

The figure	 An isosceles triangle	 An equilateral triangle	 Scalene
Number of axes of symmetry	1	3	Zero (does not exist)
The figure	 Parallelogram	 Rectangle	 Rhombus
Number of axes of symmetry	Zero (does not exist)	2	2
The figure	 Square	 Trapezium	 An isosceles trapezium
Number of axes of symmetry	4	Zero (does not exist)	1
The figure	 The circle	 The regular pentagon	 The regular hexagon
Number of axes of symmetry	An infinite number	5	6

Example 3 On a square lattice , determine the point A , B and C in each case , then find the image of Δ ABC by reflection in $\overleftrightarrow{XX'}$ (X-axis).

Mention if $\overleftrightarrow{XX'}$ is an axis of symmetry of Δ ABC or not :

1 A (1 , 3) , B (3 , 1) , C (0 , – 1)

2 A (1 , 2) , B (4 , 0) , C (1 , – 2)

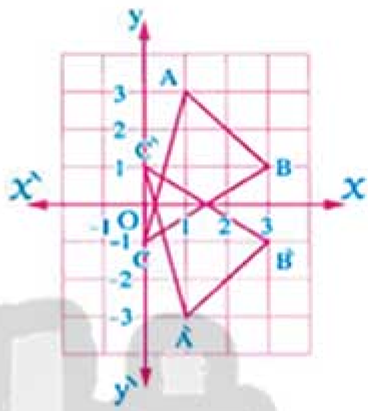
Solution

1 • A (1 , 3) $\longrightarrow$ A' (1 , – 3)

B (3 , 1) $\longrightarrow$ B' (3 , – 1)

C (0 , – 1) $\longrightarrow$ C' (0 , 1)

$\therefore$ Δ A'B'C' is the image of Δ ABC by reflection in X-axis



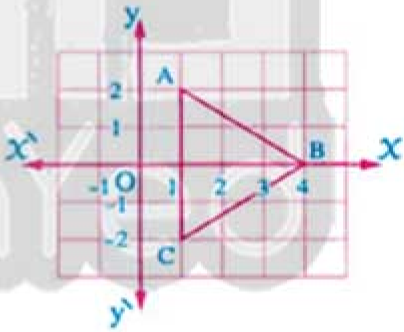
• $\overleftrightarrow{XX'}$ is not an axis of symmetry of Δ ABC because it does not transform the figure ABC to itself by reflection.

2 • A (1 , 2) $\longrightarrow$ C (1 , – 2)

B (4 , 0) $\longrightarrow$ B (4 , 0) itself

C (1 , – 2) $\longrightarrow$ A (1 , 2)

$\therefore$ Δ CBA is the image of Δ ABC by reflection in $\overleftrightarrow{XX'}$ (X-axis)

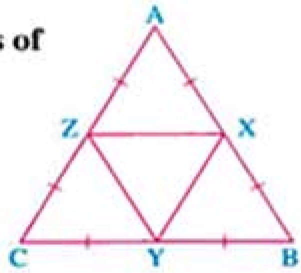


• $\overleftrightarrow{XX'}$ is the axis of symmetry of Δ ABC because it transforms Δ ABC by reflection in it to itself.

TRY 3 by yourself

In the opposite figure :

ABC is an equilateral triangle. X , Y and Z are the midpoints of its sides. Complete the following :



1. The image of Δ AXZ by reflection in $\overleftrightarrow{XZ}$ is ....
2. The image of the figure AZYX by reflection in $\overleftrightarrow{AY}$ is ....
3. Δ ABC is the image of Δ ACB by reflection in ....
4. The number of axes of symmetry of the figure ABYZ is ....
5. The number of axes of symmetry of Δ ABC is ....

Answers *of try by yourself*

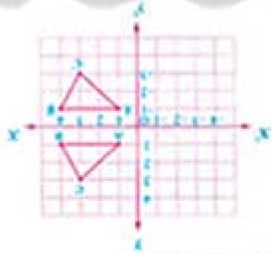
1 • The image by reflection in the X-axis :

(5 , −1) , (2 , 3) , (−1 , −4) , (−2 , 6) , (0 , 1) , (3 , 0) , (0 , 0)

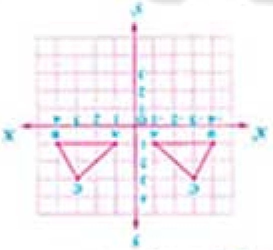
• The image by reflection in the y-axis :

(−5 , 1) , (−2 , −3) , (1 , 4) , (2 , −6) , (0 , −1) , (−3 , 0) , (0 , 0)

2 1 The X-axis



2 The y-axis



3 1 Δ YXZ   2 Figure AXYZ   3 $\overleftrightarrow{AY}$

4 1   5 3

Lesson 10 Reflection in a point

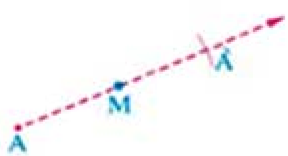
Definition of reflection in a point

Reflection in a point M maps each point A in the plane to the point $\grave{A}$ in the same plane where M is the midpoint of the line segment $\overline{A\grave{A}}$,
the point M is called the centre of reflection and the image of M by reflection in M is itself.

Finding the image of a point by reflection in a given point

- **To find the image of a point as A by reflection in M ,**
  we do as follows :

1. Draw $\overrightarrow{AM}$
2. Using the compasses with a radius length equals MA ,
   then use M as a centre and draw an arc to intersect $\overrightarrow{AM}$
   at a point as $\grave{A}$,
   then $\grave{A}$ is the image of the point A by reflection in the point M

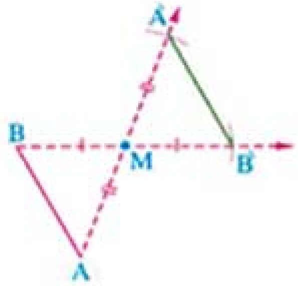


3. From the previous , we found that : $MA = M\grave{A}$

Finding the image of a line segment by reflection in a given point

- **To find the image of a line segment as $\overline{AB}$ by reflection in M , we do as follows :**

1. Find the image of A by reflection in M to be $\grave{A}$ as we mentioned before.
2. Similarly find the image of B by reflection in M to be $\grave{B}$
3. Draw $\overline{\grave{A}\grave{B}}$ to be the image of $\overline{AB}$ by reflection in the point M



Notice that :

$\grave{A}\grave{B} = AB$ and $\overline{\grave{A}\grave{B}} \parallel \overline{AB}$

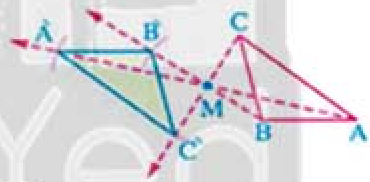
i.e. The image of a line segment by reflection in a point is a line segment parallel to the original one and its length equals the length of the original line segment.

Finding the image of a polygon by reflection in a given point

- **To find the image of a polygon as the triangle ABC by reflection in M , we do as follows :**

1. Find the image of each vertex of the vertices of the triangle ABC by reflection in the point M as we mentioned before to be :

   $\grave{A}$ is the image of A , $\grave{B}$ is the image of B and $\grave{C}$ is the image of C



2. Draw $\overline{\grave{A}\grave{B}}$, $\overline{\grave{B}\grave{C}}$ and $\overline{\grave{C}\grave{A}}$ to get $\Delta\, \grave{A}\grave{B}\grave{C}$ which is the image of Δ ABC by reflection in the point M

Notice that :

$\Delta ABC \equiv \Delta \grave{A}\grave{B}\grave{C}$, therefore it is said that the reflection in a point is **isometric**.

From the previous , we deduce that :

Reflection in a point is a geometric transformation that maps the geometric figure to another geometric figure congruent to it and has the same orientation of its vertices.

Unit 3

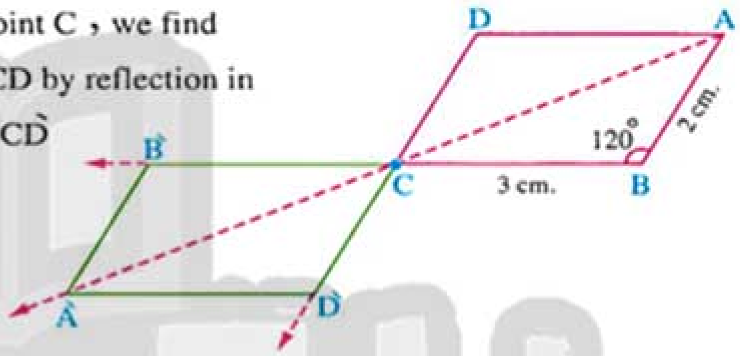
Properties of reflection in a point

Illustrated example

Draw the parallelogram ABCD in which AB = 2 cm. , BC = 3 cm. and m (∠ B) = 120° , then draw its image by reflection in the point C and show what you observe.

Solution

Finding the image of each vertex of the vertices of ▱ ABCD by reflection in the point C , we find the image of ▱ ABCD by reflection in the point C is ▱ A̖B̖CD̖

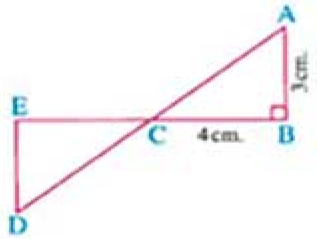


Notice that :

1 $\grave{A}\grave{B} = AB$, $\grave{B}C = BC$, $C\grave{D} = CD$ and $\grave{D}\grave{A} = DA$	i.e.	Reflection in a point reserves the lengths of the line segments.
2 m (∠ À) = m (∠ A) , m (∠ B̀) = m (∠ B) , m (∠ B̀CD̀) = m (∠ BCD) and m (∠ D̀) = m (∠ D)	i.e.	Reflection in a point reserves the measures of the angles.
3 From the parallelogram ABCD : $\overline{AB} \parallel \overline{DC}$, From the parallelogram ÀB̀CD̀ : $\overline{\grave{A}\grave{B}} \parallel \overline{\grave{D}C}$ ∴ The images of the two parallel line segments are also two parallel line segments.	i.e.	Reflection in a point reserves the parallelism.
4 The reading of the parallelogram ABCD is in the clockwise direction and the reading of the parallelogram ÀB̀CD̀ is in the clockwise direction also.	i.e.	Reflection in a point reserves the orientation of the vertices of the figure.
5 Putting a point belongs to $\overline{AB}$, we find its image by reflection in C belongs to $\overline{\grave{A}\grave{B}}$	i.e.	Reflection in a point reserves the betweenness.

Example 1 **In the opposite figure :**

ABC is a triangle in which m ($\angle$ B) = 90° ,
AB = 3 cm. , BC = 4 cm. and Δ DEC
is the image of Δ ABC by reflection in C



1 **Find :** The length of $\overline{DC}$

2 **Prove that :** $\overline{AB} // \overline{DE}$

Solution

In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (3)^2 + (4)^2 = 25$ (Pythagoras' theorem)

$\therefore$ AC = 5 cm.

, $\because$ Δ DEC is the image of Δ ABC by reflection in C

$\therefore$ DC = AC = 5 cm.

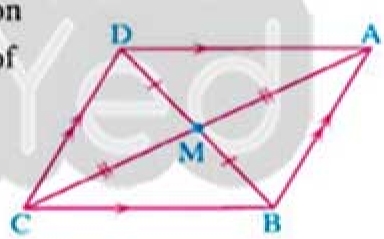
(because reflection reserves the lengths of the line segments) (First req.)

, $\because$ m ($\angle$ E) = m ($\angle$ B) = 90° (properties of reflection in a point) and they are alternate angles

$\therefore$ $\overline{AB} // \overline{DE}$ (Second req.)

Using reflection in a point to prove that a quadrilateral is a parallelogram

- We mentioned before that the image of a line segment by reflection in a point is a line segment parallel to it and has the same length of the original line segment.



If $\overline{CD}$ is the image of $\overline{AB}$ by reflection in the point M , then $\overline{AB} // \overline{DC}$ and AB = DC

We can prove that the quadrilateral ABCD is a parallelogram by several methods as follows :

1 $\because$ AB = DC and $\overline{AB} // \overline{DC}$

$\therefore$ The quadrilateral ABCD is a parallelogram.

2 $\because$ $\overline{CD}$ is the image of $\overline{AB}$ by reflection in the point M

$\therefore$ MA = MC and MB = MD

$\therefore$ The quadrilateral ABCD is a parallelogram.

Remember that

The quadrilateral in which two opposite sides are parallel and equal in length is a parallelogram.

Remember that

The quadrilateral whose diagonals bisect each other is a parallelogram.

3 $\because$ MA = MC and MB = MD

$\therefore$ $\overline{AD}$ is the image of $\overline{CB}$ by reflection in the point M

, $\because$ $\overline{CD}$ is the image of $\overline{AB}$ by reflection in the point M

$\therefore$ $\overline{AB}$ // $\overline{DC}$ and $\overline{AD}$ // $\overline{BC}$

$\therefore$ The quadrilateral ABCD is a parallelogram.

Remember that
The quadrilateral in which each two opposite sides are parallel is a parallelogram.

4 $\because$ $\overline{AD}$ is the image of $\overline{CB}$ and $\overline{CD}$ is the image of $\overline{AB}$ by reflection in the point M

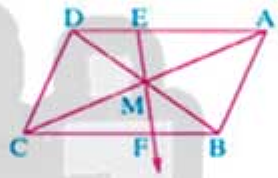
$\therefore$ AD = CB and CD = AB

$\therefore$ The quadrilateral ABCD is a parallelogram.

Remember that
The quadrilateral in which each two opposite sides are equal in length is a parallelogram.

Example 2 In the opposite figure :

ABCD is a parallelogram , M is the point of intersection of its diagonals , E $\in \overline{AD}$ and $\overrightarrow{EM} \cap \overline{BC} = \{F\}$

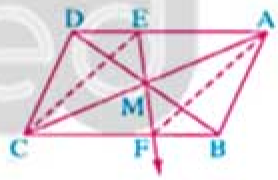


Prove that :

1 F is the image of E by reflection in M

2 The quadrilateral AFCE is a parallelogram.

Solution $\because$ ABCD is a parallelogram. $\therefore$ $\overline{AD}$ // $\overline{BC}$



$\therefore$ In $\Delta\Delta$ AME and CMF :

$\begin{cases} m(\angle DAC) = m(\angle BCA) \text{ (alternate angles)} \\ m(\angle AME) = m(\angle FMC) \text{ (V.O.A.)} \\ AM = MC \text{ (properties of parallelogram)} \end{cases}$

$\therefore$ Δ AME $\equiv$ Δ CMF , then we deduce that EM = FM

, $\because$ F $\in \overrightarrow{EM}$

$\therefore$ F is the image of E by reflection in the point M (Q.E.D. 1)

, $\because$ AM = CM and A $\in \overrightarrow{CM}$

$\therefore$ A is the image of C by reflection in the point M

$\therefore$ $\overline{AF}$ is the image of $\overline{CE}$ by reflection in the point M

$\therefore$ AF = CE and $\overline{AF}$ // $\overline{CE}$

$\therefore$ The quadrilateral AFCE is a parallelogram. (Q.E.D. 2)

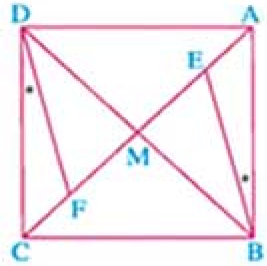
Example 3 **In the opposite figure :**

ABCD is a square whose diagonals intersect at M ,

$E \in \overline{AC}$ and $F \in \overline{AC}$ where m ($\angle$ ABE) = m ($\angle$ CDF)

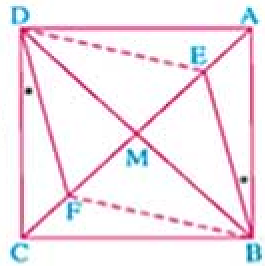
Prove that :

1. Δ ABE is the image of Δ CDF by reflection in M
2. The quadrilateral EBFD is a rhombus.



Solution

In $\Delta\Delta$ ABE and CDF :



$\begin{cases} AB = CD \text{ (properties of the square)} \\ m(\angle BAE) = m(\angle DCF) = 45^\circ \text{ (properties of the square)} \\ m(\angle ABE) = m(\angle CDF) \text{ (given)} \end{cases}$

$\therefore \Delta ABE \equiv \Delta CDF$, then we deduce that : AE = CF

, $\because$ AM = CM (properties of the square)

$\therefore$ AM – AE = CM – CF

$\therefore$ EM = FM , $E \in \overrightarrow{FM}$ $\therefore$ E is the image of F by reflection in M (1)

, AM = CM , $A \in \overrightarrow{CM}$ $\therefore$ A is the image of C by reflection in M

, BM = DM , $B \in \overrightarrow{DM}$ $\therefore$ B is the image of D by reflection in M (2)

$\therefore$ Δ ABE is the image of Δ CDF by reflection in M (Q.E.D. 1)

From (1) and (2) :

$\therefore$ $\overline{EB}$ is the image of $\overline{FD}$ by reflection in M

$\therefore$ $\overline{EB} \parallel \overline{FD}$ and EB = FD

$\therefore$ The quadrilateral EBFD is a parallelogram.

, $\because$ $\overline{DB} \perp \overline{EF}$ (properties of the square)

$\therefore$ The quadrilateral EBFD is a rhombus. (Q.E.D. 2)

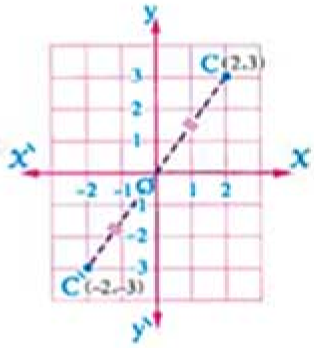
TRY 1 by yourself

Draw any triangle ABC , then draw its image $\grave{A}\grave{B}C$ by reflection in C , then prove that the quadrilateral $AB\grave{A}\grave{B}$ is a parallelogram.

Unit 3

Reflection in the origin point

- If the point C is a point in the Cartesian coordinates plane where C (2 , 3)
- To find the image of the point C by reflection in the origin point O using the same previous method , we will find it $\grave{C}$ (– 2 , – 3)



Notice that : The signs of the two projections of the ordered pair (2 , 3) have been changed , **hence we can define the reflection in the origin point as follows :**

Definition

If A (x , y) is a point in the Cartesian coordinates plane , then the image of the point A by reflection in the origin point O is $\grave{A}$ (– x , – y)

i.e. Reflection in the origin point converts the sign of each of the coordinates of the point.

$\therefore$ **The image of the point** $(x , y) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (-x , -y)$

For example:

- The image of the point $(2 , 3) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (-2 , -3)$
- The image of the point $(-4 , 1) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (4 , -1)$
- The image of the point $(5 , -2) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (-5 , 2)$
- The image of the point $(-3 , -6) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (3 , 6)$

Remark

The image of the point (0 , 0) by reflection in the origin point is itself.

Example 4 Draw ΔABC where $A(4, 1)$, $B(2, 4)$ and $C(-1, 3)$, then map draw image by reflection in the origin point.

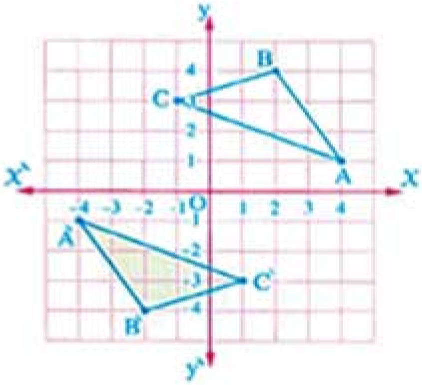
Solution

$\because (x, y) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (-x, -y)$

$\therefore A(4, 1) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \grave{A}(-4, -1)$

$, B(2, 4) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \grave{B}(-2, -4)$

$, C(-1, 3) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \grave{C}(1, -3)$



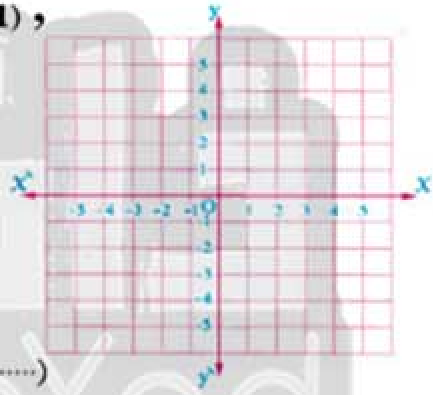
TRY 2 *by yourself*

Draw on a square lattice ΔABC, *where* $A(-2, 1)$, $B(4, -2)$ *and* $C(2, 3)$, *then draw its image by reflection in the origin point :*

$A(-2, 1) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \grave{A}(\dots\dots, \dots\dots)$

$, B(4, -2) \xrightarrow[\dots\dots\dots]{\dots\dots\dots} \grave{B}(\dots\dots, \dots\dots)$

$, C(\dots\dots, \dots\dots) \xrightarrow[\dots\dots\dots]{\dots\dots\dots} \grave{C}(\dots\dots, \dots\dots)$

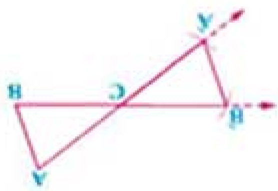


Answers *of try by yourself*

1

Prove by yourself

[Hint : $AB = \grave{A}\grave{B}$, $\overline{AB} \parallel \overline{\grave{A}\grave{B}}$]



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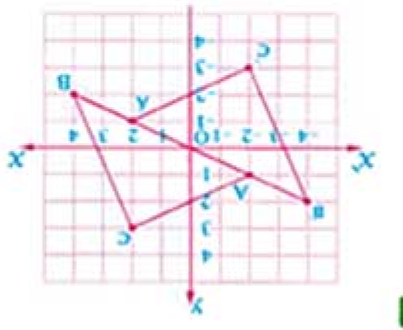


Lesson 11

Translation

Prelude

If a car moved a distance 25 metres in a straight line forward , then we say that :
The car translated for a distance 25 metres forward.

i.e. To determine the new position of the car after its movement , we should know two important elements which are :

1 The magnitude of the translation (25 metres).

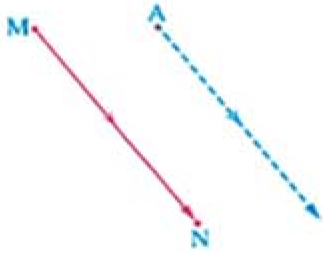
2 The direction of the translation (forward in a straight line).

According to this, we can say that :

Translation is a geometrical transformation which maps each point A in the plane to another point $\acute{A}$ in the same plane with a constant distance in a certain direction.

Translation in the plane

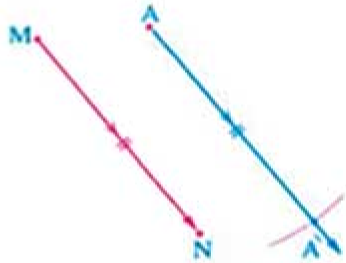
Finding the image of a point by a given translation



- **To find $\grave{A}$ which is the image of A by translation MN in the direction of $\overrightarrow{MN}$, we do as follows :**

  1 Draw from A a ray parallel to $\overrightarrow{MN}$ and in the same direction.

  2 By the compasses in A as a centre with radius = MN , draw an arc to intersect the ray drawn from A at the point $\grave{A}$ ($A\grave{A}$ = MN and $\overline{A\grave{A}} \parallel \overline{MN}$)



- **Then $\grave{A}$ is the image of A by translation of magnitude MN in the direction of $\overrightarrow{MN}$**

Finding the image of a line segment by a given translation

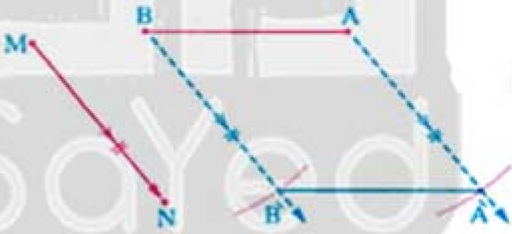
- **To find the image of $\overline{AB}$ by translation MN in the direction of $\overrightarrow{MN}$, we do as follows :**

  1 Find the image of the point A by translation MN in the direction of $\overrightarrow{MN}$ as we mentioned before , say $\grave{A}$

  2 Similarly , we find the image of the point B by translation MN in the direction of $\overrightarrow{MN}$, say $\grave{B}$

  3 Draw $\overline{\grave{A}\grave{B}}$ to be the image of $\overline{AB}$ by translation MN in the direction of $\overrightarrow{MN}$

  Check that : AB = $\grave{A}\grave{B}$ and $\overline{AB} \parallel \overline{\grave{A}\grave{B}}$

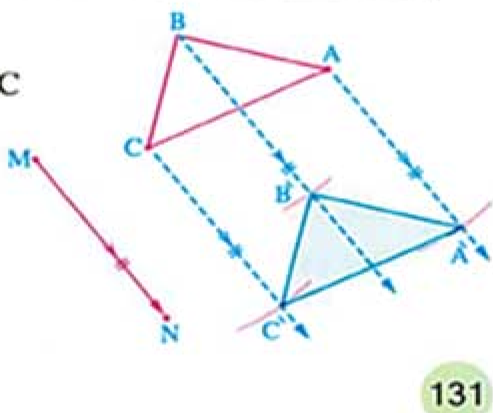


Finding the image of a polygon by a given translation

- **To find the image of a polygon as △ ABC by translation MN in the direction of $\overrightarrow{MN}$, we do as follows :**

  1 Find the image of each vertex of the vertices of △ ABC by translation MN in the direction of $\overrightarrow{MN}$ as we mentioned before (say $\grave{A}$ for A , $\grave{B}$ for B and $\grave{C}$ for C)

  2 Draw $\overline{\grave{A}\grave{B}}$, $\overline{\grave{B}\grave{C}}$ and $\overline{\grave{C}\grave{A}}$ then △ $\grave{A}\grave{B}\grave{C}$ is the image of △ ABC by translation MN in the direction of $\overrightarrow{MN}$



Unit 3

Check that :

- $AB = \grave{A}\grave{B}$, $BC = \grave{B}\grave{C}$ and $CA = \grave{C}\grave{A}$
- $m(\angle A) = m(\angle \grave{A})$, $m(\angle B) = m(\angle \grave{B})$, $m(\angle C) = m(\angle \grave{C})$

From the previous , we deduce that translation is a geometrical transformation which maps the geometrical figure to another geometrical figure congruent to it.

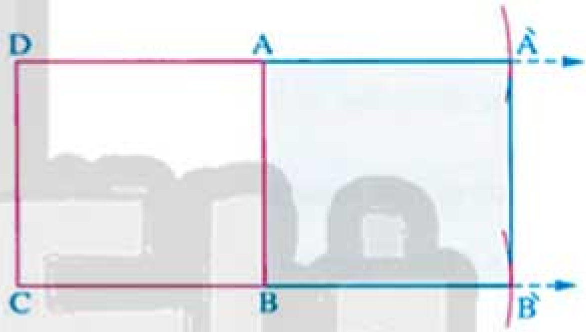
Properties of translation

Illustrated example

Draw the square ABCD whose side length is 3 cm., then draw its image by translation AB in the direction $\overrightarrow{DA}$

Solution

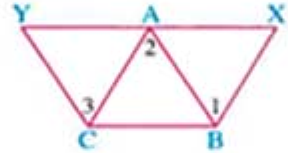
The square $\grave{A}\grave{B}BA$ is the image of the square ABCD by translation AB in the direction $\overrightarrow{DA}$



Notice that :

1 $\grave{A}\grave{B} = AB$, $AB = DC$	i.e.	Translation reserves the lengths of the line segments.
2 $m(\angle \grave{A}) = m(\angle BAD)$, $m(\angle \grave{B}) = m(\angle CBA)$	i.e.	Translation reserves the measures of the angles.
3 From the square ABCD : $\overline{AB} \parallel \overline{DC}$, from the square $\grave{A}\grave{B}BA$: $\overline{\grave{A}\grave{B}} \parallel \overline{AB}$ $\therefore$ The images of the two parallel line segments are also two parallel line segment.	i.e.	Translation reserves the parallelism.
4 The reading of the square ABCD is in the clockwise direction and the reading of the square $\grave{A}\grave{B}BA$ is in the clockwise direction also.	i.e.	Translation reserves the orientation of the vertices of the figure.
5 If you take a point lies on $\overline{AB}$ and find its image by the previous translation , you will find its image lies on $\overline{\grave{A}\grave{B}}$	i.e.	Translation reserves the betweenness.

Example 1 **In the opposite figure :**



ABC is a triangle, X is the image of B by translation of a distance CA in the direction of $\overrightarrow{CA}$,

Y is the image of C by translation of a distance BA in the direction of $\overrightarrow{BA}$

1 **Prove that :** $\Delta XAB \equiv \Delta AYC$

2 Determine the translation which makes ΔAYC the image of ΔXAB

Solution

$\because$ X is the image of B by translation of a distance CA in the direction $\overrightarrow{CA}$

$\therefore$ BX = CA and $\overline{BX} \parallel \overline{CA}$

$\therefore$ m ($\angle$ 1) = m ($\angle$ 2) (alternate angles) (1)

, $\because$ Y is the image of C by translation of a distance BA in the direction $\overrightarrow{BA}$

$\therefore$ CY = BA , $\overline{CY} \parallel \overline{BA}$

$\therefore$ m ($\angle$ 2) = m ($\angle$ 3) (alternate angles) (2)

From (1) and (2) : We deduce that : m ($\angle$ 1) = m ($\angle$ 3)

$\therefore$ In $\Delta\Delta$ XAB , AYC :

$\begin{cases} BX = CA \text{ (properties of translation)} \\ BA = CY \text{ (properties of translation)} \\ m(\angle 1) = m(\angle 3) \text{ (proved)} \end{cases}$

$\therefore \Delta XAB \equiv \Delta AYC$ (First req.)

The translation which makes ΔAYC is the image of ΔXAB is the translation of a distance BC in the direction of $\overrightarrow{BC}$ (Second req.)

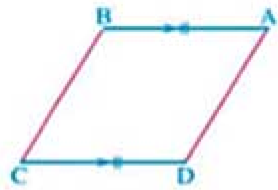
Using the properties of translation to prove that a quadrilateral is a parallelogram

We noticed that :

The image of a line segment by a translation is another line segment parallel to it and has the same length of the original line segment.

For example:

In the opposite figure :



If $\overline{AB}$ is the image of $\overline{DC}$ by a translation,

then : $\overline{AB} \parallel \overline{DC}$ and AB = DC

i.e. The figure ABCD is a parallelogram.

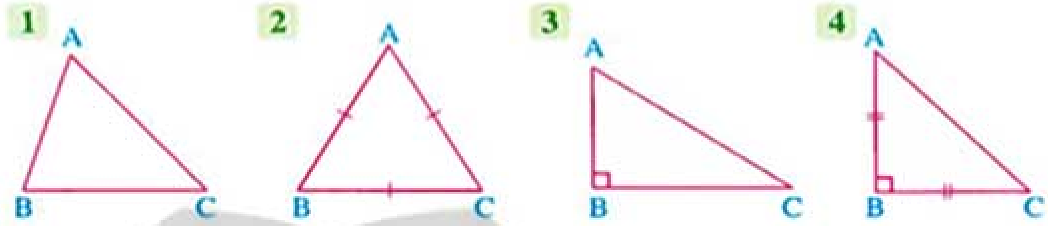
Remember that

The parallelogram is a quadrilateral in which two opposite sides are parallel and equal in length.

Unit 3

Example 2 **In each of the following figures :**

Draw $\Delta \grave{A}\grave{B}B$ as the image of ΔABC by translation of a distance CB in the direction of $\overrightarrow{CB}$, then show the kind of the figure $ABB\grave{}\grave{A}$ in each case :



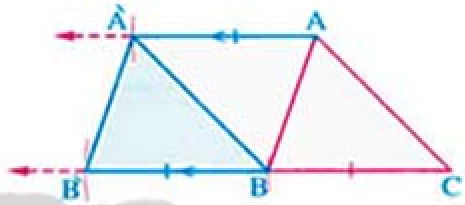
Solution

1 $\because \Delta \grave{A}\grave{B}B$ is the image of ΔABC

$\therefore \overline{\grave{A}\grave{B}}$ is the image of $\overline{AB}$

$\therefore \overline{\grave{A}\grave{B}} \mathbin{/\!/} \overline{AB}$, $\grave{A}\grave{B} = AB$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a parallelogram.

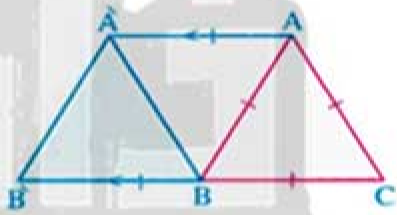


2 $\because \Delta \grave{A}\grave{B}B$ is the image of ΔABC

$\therefore \overline{\grave{A}\grave{B}}$ is the image of $\overline{AB}$

$\therefore \overline{\grave{A}\grave{B}} \mathbin{/\!/} \overline{AB}$, $\grave{A}\grave{B} = AB$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a parallelogram.



, $\because \Delta ABC \equiv \Delta \grave{A}\grave{B}B$

$\therefore \Delta \grave{A}\grave{B}B$ is an equilateral triangle

$\therefore \grave{A}\grave{B} = \grave{B}B$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a rhombus.

Remember that

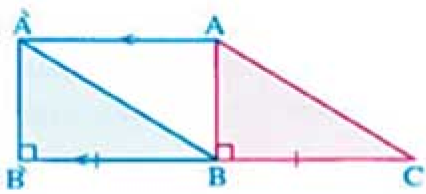
The rhombus is a parallelogram in which two adjacent sides are equal in length.

3 $\because \Delta \grave{A}\grave{B}B$ is the image of ΔABC

$\therefore \overline{\grave{A}\grave{B}}$ is the image of $\overline{AB}$

$\therefore \overline{\grave{A}\grave{B}} \mathbin{/\!/} \overline{AB}$, $\grave{A}\grave{B} = AB$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a parallelogram.



, $\because m(\angle \grave{B}) = m(\angle B) = 90°$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a rectangle.

Remember that

The rectangle is a parallelogram one of its angles is right.

4 $\because \Delta \grave{A}\grave{B}B$ is the image of ΔABC

$\therefore \overline{\grave{A}\grave{B}}$ is the image of $\overline{AB}$

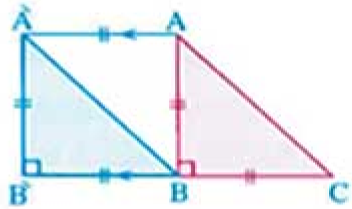
$\therefore \overline{\grave{A}\grave{B}} \parallel \overline{AB}$, $\grave{A}\grave{B} = AB$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a parallelogram.

$\because m(\angle \grave{B}) = m(\angle B) = 90^\circ$

, $\grave{A}\grave{B} = \grave{B}B$

$\therefore$ The figure $AB\grave{B}\grave{A}$ is a square.



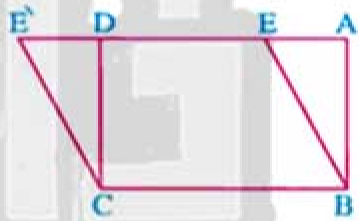
Remember that

The square is a parallelogram in which one of its angles is right and two adjacent sides are equal in length.

Example 3 Draw the rectangle ABCD. Take the point $E \in \overline{AD}$, then find the image of E by translation DA in the direction of $\overrightarrow{AD}$. Then prove that the figure $EBC\grave{E}$ is a parallelogram.

Solution

- We take $\grave{E} \in \overrightarrow{AD}$ such that $\grave{E}E = AD$, then $\grave{E}$ is the image of E by translation DA in the direction of $\overrightarrow{AD}$



$\because$ ABCD is a rectangle.

$\therefore \overline{AD} \parallel \overline{BC}$ and $AD = BC$

$\therefore$ C is the image of B by translation AD in the direction of $\overrightarrow{AD}$

, $\because \grave{E}$ is the image of E by the same translation.

$\therefore \overline{\grave{E}C}$ is the image of $\overline{EB}$ by translation AD in the direction of $\overrightarrow{AD}$

$\therefore EB = \grave{E}C$ and $\overline{EB} \parallel \overline{\grave{E}C}$

$\therefore$ The figure $EBC\grave{E}$ is a parallelogram.

TRY 1 by yourself

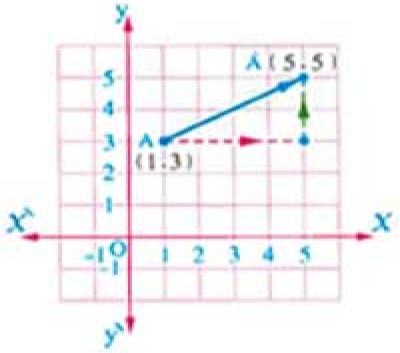
Draw the parallelogram ABCD , ***plot*** $E \in \overline{AD}$ ***where*** $\overline{BE} \perp \overline{AD}$, ***then draw the image of*** Δ ABE ***by translation*** AD ***in the direction of*** $\overrightarrow{AD}$:

1. Prove that the figure $EBC\grave{E}$ is a rectangle. (where $\grave{E}$ is the image of E by the previous translation)
2. Determine the distance and the direction of the translation which transforms $\overline{BC}$ to $\overline{E\grave{E}}$

Unit 3

Translation in the Cartesian plane

If A (1 , 3) is a point in the orthogonal coordinates plane and to find its image À by translation with magnitude 4 length units in the direction of $\overrightarrow{OX}$ followed by a translation with magnitude 2 length units in the direction of $\overrightarrow{OY}$



From the graph , we get À to be the point (5 , 5)

i.e. À (1 + 4 , 3 + 2)

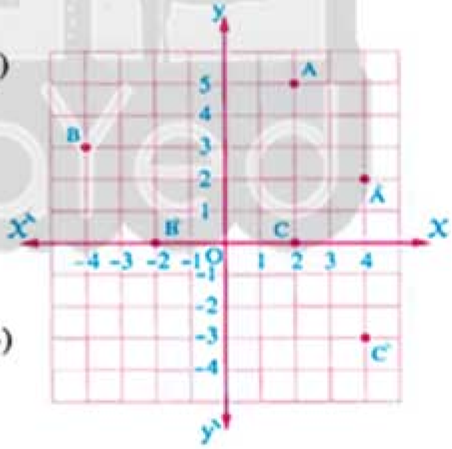
According to this :

Translation in the orthogonal Cartesian coordinates plane transforms each point by a displacement a in the direction of the X-axis followed by a displacement b in the direction of the y-axis

i.e. The image of the point A (X , y) ⟶ the point À (X + a , y + b)

Example 4 **Find the images of the points A (2 , 5) , B (– 4 , 3) and C (2 , 0) by translation (X , y) ⟶ (X + 2 , y – 3)**

Solution ∵ (X , y) ⟶ (X + 2 , y – 3) , then :



- The image of A (2 , 5) is À (2 + 2 , 5 – 3)

  i.e. À (4 , 2)

- The image of B (– 4 , 3) is B̀ (– 4 + 2 , 3 – 3)

  i.e. B̀ (– 2 , 0)

- The image of C (2 , 0) is C̀ (2 + 2 , 0 – 3)

  i.e. C̀ (4 , – 3)

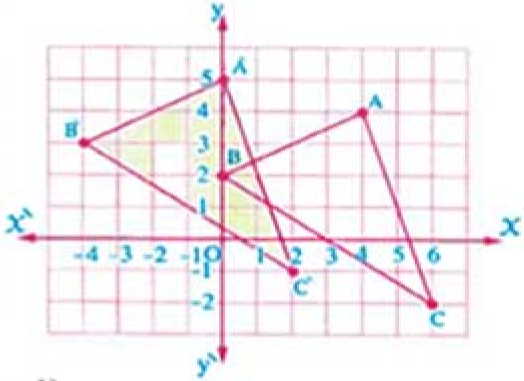
Notice that :

The translation (X , y) ⟶ (X + 2 , y – 3) transforms each point to another point by a right horizontal displacement of 2 units and a vertical displacement of 3 units downwards.

Example 5 **Draw on a square lattice Δ ABC where A (4 , 4) , B (0 , 2) , C (6 , – 2) , then find its image by translation (X , y) ⟶ (X – 4 , y + 1)**

Solution

The point	Its image by the translation
(x , y)	$(x - 4 , y + 1)$
$A (4 , 4)$	$A' (0 , 5)$
$B (0 , 2)$	$B' (-4 , 3)$
$C (6 , -2)$	$C' (2 , -1)$



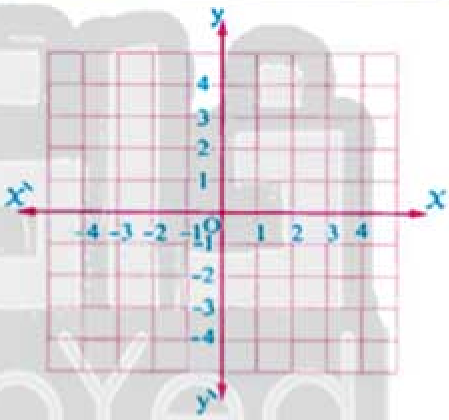
$\therefore \Delta A'B'C'$ is the image of ΔABC by translation $(x , y) \longrightarrow (x - 4 , y + 1)$

Remark

"The translation $(x , y) \longrightarrow (x + a , y + b)$ can be written as the translation (a , b) for example :
The translation $(x , y) \longrightarrow (x + 2 , y - 1)$ can be written as the translation $(2 , -1)$"

TRY 2 by yourself

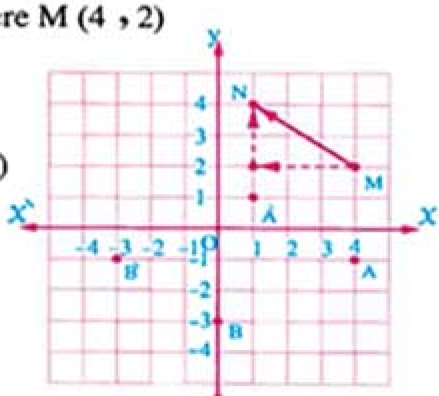
On a square lattice , draw ΔABC *where* $A (-3 , 2)$, $B (-1 , 1)$, $C (-2 , 0)$ *, then find its image by translation :*
$(x , y) \longrightarrow (x + 2 , y + 1)$



Example 6 Find the image of each of the two points $A (4 , -1)$ and $B (0 , -3)$ by translation with magnitude MN in the direction of $\overrightarrow{MN}$ where $M (4 , 2)$ and $N (1 , 4)$

Solution

- By noticing the opposite graph , we find that the translation with magnitude MN in the direction of $\overrightarrow{MN}$ where $M (4 , 2)$ and $N (1 , 4)$ is equivalent to :
- Horizontal displacement from 4 to 1
  i.e. a displacement = 3 units to the left (-3)
- Vertical displacement from 2 to 4
  i.e. a displacement = 2 units upwards (2)
  i.e. $(x , y) \longrightarrow (x - 3 , y + 2)$,



thus we get :

$A(4, -1) \longrightarrow A'(4-3, -1+2)$

i.e. $A'(1, 1)$

$B(0, -3) \longrightarrow B'(0-3, -3+2)$

i.e. $B'(-3, -1)$

Notice that :

The translation with magnitude MN in the direction of $\overrightarrow{MN}$ where $M(4, 2)$ and $N(1, 4)$ is equivalent to :

- A horizontal displacement (in the X-axis direction) from 4 to $1 = 1 - 4 = -3$
- A vertical displacement (in the y-axis direction) from 2 to $4 = 4 - 2 = 2$

i.e. The rule of translation is $(X, y) \longrightarrow (X - 3, y + 2)$

Example 7 **Draw the image of ΔABC where $A(5, 2)$, $B(4, 5)$ and $C(2, 2)$ by translation BC in the direction of $\overrightarrow{BC}$ and write the rule of the translation.**

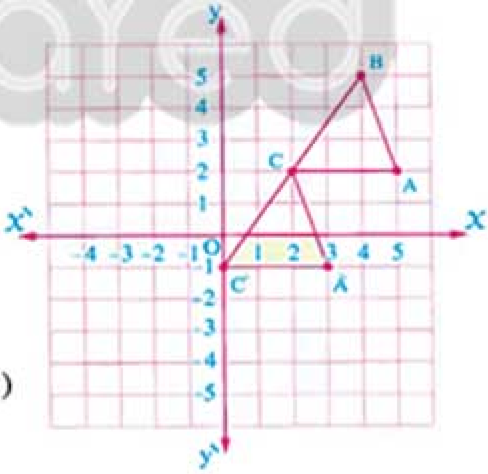
Solution

$\because B(4, 5), C(2, 2)$

$\therefore$ The translation BC in the direction of $\overrightarrow{BC}$ is equivalent to :

- Horizontal displacement $= 2 - 4 = -2$
- Vertical displacement $= 2 - 5 = -3$

Thus the rule of translation is $(X, y) \longrightarrow (X - 2, y - 3)$



Thus :

$A(5, 2) \longrightarrow A'(5-2, 2-3)$

i.e. $A'(3, -1)$

$B(4, 5) \longrightarrow B'(4-2, 5-3)$

i.e. $B'(2, 2)$ (Notice : B' coincides C)

$C(2, 2) \longrightarrow C'(2-2, 2-3)$

i.e. $C'(0, -1)$

i.e. $\Delta A'CC'$ is the image of ΔABC by translation BC in the direction of $\overrightarrow{BC}$

TRY 3 by yourself

***Draw the square* ABCD *where* A (4 , – 2) , B (4 , – 5) , C (1 , – 5) *and* D (1 , – 2) ,**
then find its image by translation CA in the direction of $\overrightarrow{CA}$

Example 8 **If the image of the point A (– 3 , 2) by translation in the Cartesian coordinates plane is $\grave{A}$ (2 , – 2) :**

1. Find the rule of translation.
2. Find the image of B (1 , – 3) by the same translation.

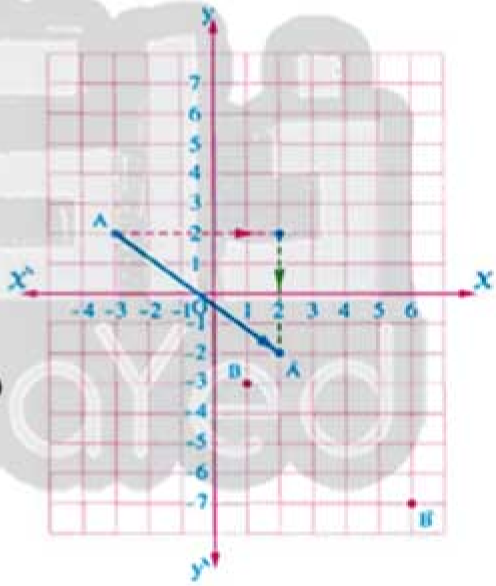
Solution

1. By noticing the following graph , we find that the translation which makes $\grave{A}$ (2 , – 2) the image of A (– 3 , 2) is equivalent to :

- Horizontal displacement of 5 units to the right side (5)
- Vertcial displacement of 4 units downwards (– 4)

∴ The rule of translation is

$(x , y) \longrightarrow (x + 5 , y - 4)$



2. B (1 , – 3) $\longrightarrow$ $\grave{B}$ (1 + 5 , – 3 – 4)

i.e. $\grave{B}$ (6 , – 7)

Unit 3

Example 9 **If $\grave{A}$ (7 , – 2) is the image of A by the translation whose rule is $(x , y) \longrightarrow (x - 3 , y + 1)$, find A**

Solution

Let A be (x , y)

$\therefore A (x , y) \longrightarrow \grave{A} (x - 3 , y + 1)$

$, \because \grave{A} (7 , -2)$

$\therefore (x - 3 , y + 1) = (7 , -2)$

$\therefore x - 3 = 7$

$\therefore x = 10$

$, y + 1 = -2$

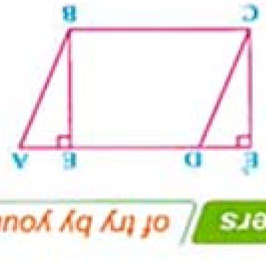
$\therefore y = -3$

$\therefore A (10 , -3)$

Notice that :
If $(x , y) = (a , b)$, then $x = a , y = b$

Answers of try by yourself

1

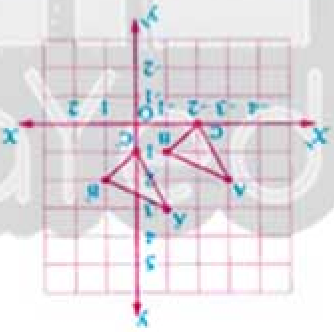


1 Prove by yourself

[Hint : BE = CÈ , $\overline{BE} \parallel \overline{CÈ}$, m (∠ BEÈ) = 90°]

2 Translation with magnitude BE , in the direction of $\overrightarrow{BE}$

2



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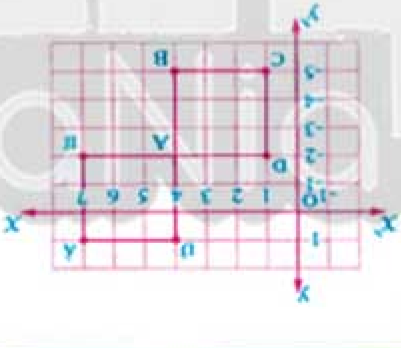


Lesson 12 Rotation

Prelude

If you stand in front of Ferris wheel in the funfair , you will find that the carriage moves in a circular motion around a fixed point in the direction of clockwise or in the direction of anticlockwise , this motion is called "rotation"

Definition of rotation

If M is a fixed point in the plane, then the rotation around M with an angle of measure θ is a geometric transformation transforming each point A in the plane to another point À in the same plane such that m (∠ AMÀ) = θ , MA = MÀ this rotation is denoted by R (M , θ) where :

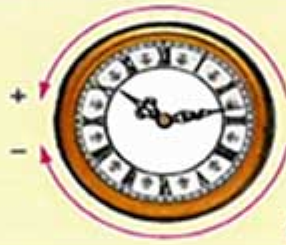
- M is the centre of rotation.
- θ is the measure of the angle of rotation.

According to this definition , the rotation is determined completely if we know the following elements

1. The centre of the rotation.
2. The measure of the angle of the rotation (θ)
3. The direction of rotation.

Remark

The measure of rotation angle is positive if the rotation is anticlockwise and it is negative if the rotation is clockwise.

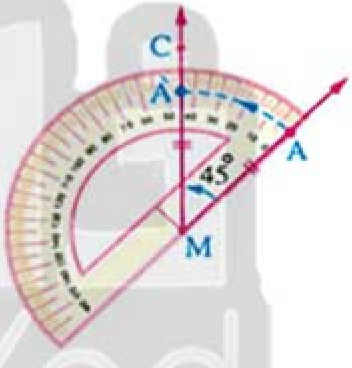


Rotation in the plane

Finding the image of a given point by a given rotation

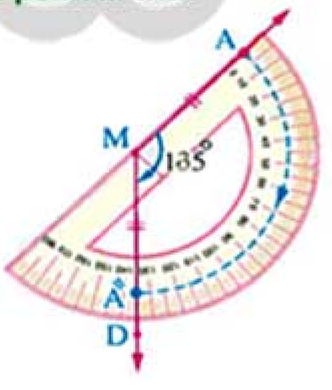
Firstly : Finding the image of the point A by rotation around the point M with an angle of measure 45° i.e. R (M , 45°) :

- Draw the ray $\overrightarrow{MA}$
- Put the protractor with its straight edge on $\overrightarrow{MA}$ and in the anticlockwise direction , then draw $\overrightarrow{MC}$ such that m (∠ AMC) = 45°
- Use the compasses at the point M as a centre with radius = MA , draw an arc to cut $\overrightarrow{MC}$ at À , then À is the image of the point A by rotation around M with an angle of measure 45°



Secondly : Finding the image of the point A by rotation around the point M with an angle of measure (– 135°) i.e. R (M , – 135°) :

- Repeat the same previous steps , then draw $\overrightarrow{MD}$ in the clockwise direction such that m (∠ AMD) = 135° , then determine on it the point À such that MÀ = MA , then À is the image of A by rotation around M with an angle of measure (– 135°)

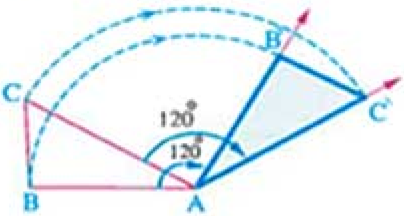


Remark

If À is the image of A by rotation around M with an angle θ , then A is the image of À by rotation around M with an angle of measure (– θ)

Finding the image of a polygon by a given rotation

The opposite figure shows how to find the image of ΔABC by the rotation R (A , – 120°) by finding the image of each vertex of its vertices , then $\Delta AB'C'$ is the image of ΔABC by rotation R (A , – 120°)



Notice that : $\Delta AB'C' \equiv \Delta ABC$

Remark

"From the previous figure , the image of the point A by rotation R (A , – 120°) is itself because it is the centre of rotation."

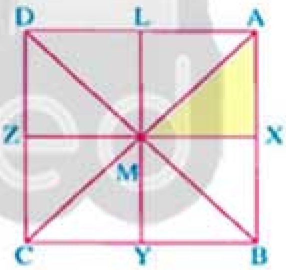
Properties of rotation

Through our study of rotation , we found that the rotation is a geometric transformation that maps the figure to another congruent figure to it.
Therefore it is said that the rotation in the plane is **isometric**

, thus we can deduce some of properties and add other properties through our study of the following illustrated example.

Illustrated example

In the opposite figure :
ABCD is a square whose diagonals intersect at M , X , Y , Z and L are the midpoints of its sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively. **Find :**



1. The image of ΔAXM by rotation R (M , 90°) , then mention what you observe.
2. The image of each of $\overline{AB}$ and $\overline{DC}$ by rotation R (M , – 90°) , then mention what you observe.
3. The image of each of B , Y and C by rotation R (M , 180°) , then mention what you observe.

Solution

1. ∵ D is the image of A by rotation R (M , 90°) , L is the image of X by rotation R (M , 90°) and M is the image of itself (The centre of rotation).
   ∴ ΔDLM is the image of ΔAXM by rotation R (M , 90°)

We notice that :	
• DL = AX , LM = XM and DM = AM	i.e. Rotation in the plane reserves the lengths of the line segments.
• m (∠ DLM) = m (∠ AXM), m (∠ LDM) = m (∠ XAM) and m (∠ DML) = m (∠ AMX)	i.e. Rotation in the plane reserves the measures of the angles.
• Reading Δ AXM is in the clockwise direction and reading its image Δ DLM is in the clockwise direction also.	i.e. Rotation in the plane reserves the orientation of the vertices of the figure.

2 ∵ B is the image of A by rotation R (M , – 90°) , C is the image of B by rotation R (M , – 90°)

∴ $\overline{BC}$ is the image of $\overline{AB}$ by rotation R (M , – 90°)

∵ A is the image of D by rotation R (M , – 90°) , D is the image of C by rotation R (M , – 90°)

∴ $\overline{AD}$ is the image of $\overline{DC}$ by rotation R (M , – 90°)

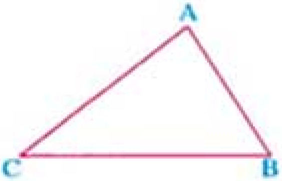
We notice that :	
• $\overline{AB}$ // $\overline{DC}$ and $\overline{BC}$ // $\overline{AD}$	i.e. Rotation in the plane reserves the parallelism.

3 • D is the image of B , L is the image of Y and A is the image of C by rotation (M , 180°)

We notice that :	
• Y ∈ $\overline{BC}$ and L (The image of Y) ∈ $\overline{AD}$	i.e. Rotation in the plane reserves the betweenness.
• B , Y , C are collinear , D , L , A are also collinear.	i.e. Rotation in the plane reserves the collinearity.

Example 1 **In the opposite figure :**

ABC is a triangle Find :



1 The point D as the image of B by rotation $R(A, m(\angle B))$

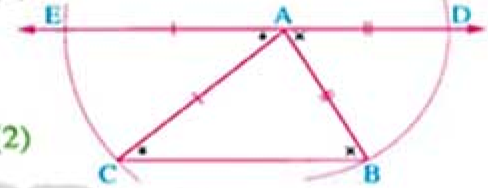
2 The point E as the image of C by rotation $R(A, -m(\angle C))$

Then prove that : The points D , A and E are collinear.

Solution

$\because$ D is the image of B by the rotation $R(A, m(\angle B))$

$\therefore m(\angle DAB) = m(\angle B)$ (1)



, $\because$ E is the image of C by the rotation $R(A, -m(\angle C))$

$\therefore m(\angle EAC) = m(\angle C)$ (2)

Adding (1) and (2), we deduce that :

$m(\angle DAB) + m(\angle EAC) = m(\angle B) + m(\angle C)$

Adding $m(\angle BAC)$ to both sides :

$\therefore m(\angle DAB) + m(\angle EAC) + m(\angle BAC) = m(\angle B) + m(\angle C) + m(\angle BAC)$

$\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore m(\angle B) + m(\angle C) + m(\angle BAC) = 180°$

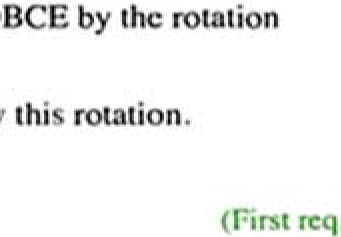
$\therefore m(\angle DAB) + m(\angle EAC) + m(\angle BAC) = 180°$

$\therefore$ D , A and E are collinear. (Q.E.D.)

Example 2 **In the opposite figure :**

If the figure XYBD is the image of the figure DBCE by the rotation R (A , 50°) ,

BC = 6 cm. , $\overline{DE} \parallel \overline{BC}$



1 **Find the length of :** $\overline{BY}$

2 **Prove that :** $\overline{DX} \parallel \overline{BY}$

Solution

$\because$ The figure XYBD is the image of the figure DBCE by the rotation R (A , 50°)

$\therefore$ B is the image of C and Y is the image of B by this rotation.

$\therefore \overline{BY}$ is the image of $\overline{CB}$ by this rotation.

$\therefore$ BY = CB = 6 cm. (First req.)

$\because$ The figure XYBD is the image of the figure DBCE by rotation R (A , 50°)

$\therefore$ $\overline{XD}$ and $\overline{YB}$ are the images of $\overline{DE}$ and $\overline{BC}$ by this rotation respectively.

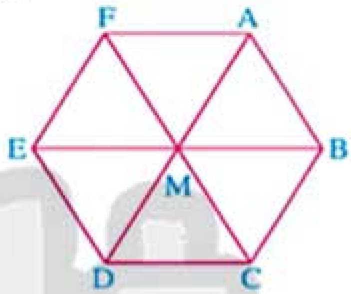
, $\because$ $\overline{DE}$ // $\overline{BC}$

$\therefore$ $\overline{XD}$ // $\overline{BY}$ (Second req.)

TRY 1 by yourself

In the opposite figure :

ABCDEF is a regular hexagon. Complete the following :

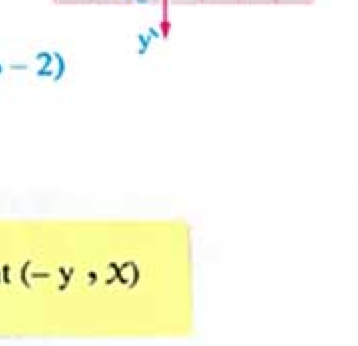


1. The image of the point A by rotation around M with an angle of measure 180° is ....
2. The image of $\overline{AB}$ by rotation around M with an angle of measure (– 60°) is ....
3. The image of Δ CMD by rotation around M with an angle of measure 120° is ....

Rotation in the Cartesian plane

First Rotation about the origin point (O) with an angle of measure 90°

The opposite figure shows the two images of the two points A (3 , 1) and B (– 2 , 1) by rotation R (O , 90°)



By noticing the figure , we find that :

- The image of the point A (3 , 1) $\xrightarrow[\text{R (O , 90°)}]{\text{by rotation}}$ the point À (– 1 , 3)
- The image of the point B (– 2 , 1) $\xrightarrow[\text{R (O , 90°)}]{\text{by rotation}}$ the point B̀ (– 1 , – 2)

From the previous , we deduce the following rule :

The image of the point (x , y) $\xrightarrow[\text{R (O , 90°)}]{\text{by rotation}}$ the point $(– y , x)$

Remarks

- The image of the point $(x , y) \xrightarrow[R(O , -90°)]{\text{by rotation}}$ the point $(y , -x)$

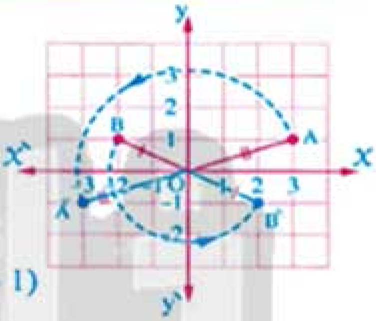
  For example: The image of the point $(2 , -3) \xrightarrow[R(O , -90°)]{\text{by rotation}}$ the point $(-3 , -2)$

- Rotation about the origin point with an angle of measure 270° is equivalent to rotation about the origin point with an angle of measure (– 90°)

  For example: The image of the point $(2 , -3) \xrightarrow[R(O , 270°)]{\text{by rotation}}$ the point $(-3 , -2)$

Second — Rotation about the origin point (O) with an angle of measure 180°

The opposite figure shows the two images of the two points A (3 , 1) and B (– 2 , 1) by rotation R (O , 180°)



By noticing the figure , we find that :

- The image of the point A $(3 , 1) \xrightarrow[R(O , 180°)]{\text{by rotation}}$ the point A` $(-3 , -1)$
- The image of the point B $(-2 , 1) \xrightarrow[R(O , 180°)]{\text{by rotation}}$ the point B` $(2 , -1)$

From the previous , we deduce the following rule :

The image of the point $(x , y) \xrightarrow[R(O , 180°)]{\text{by rotation}}$ the point $(-x , -y)$

Remarks

1. The image of the point A (x , y) by rotation R (O , 180°) is the same image of the point A by rotation R (O , – 180°)
2. The image of the point A (x , y) about the origin point with an angle of measure ± 360° is the same point A (x , y)
3. Rotation with an angle of measure 90° is called a $\frac{1}{4}$ turn.
4. Rotation with an angle of measure 180° is called a $\frac{1}{2}$ turn.
5. Rotation with an angle of measure 360° is called the identity rotation because it returns the figure to its original position.

Unit 3

Example 3 Complete the following table :

	The point	Its image by rotation R (O , ± 180°)	Its image by rotation R (O , 90°)
1	(3 , 2)	....	....
2	(− 3 , 4)	....	....
3	(− 2 , − 1)	....	....
4	....	(5 , − 2)	....
5	....	....	(6 , 0)

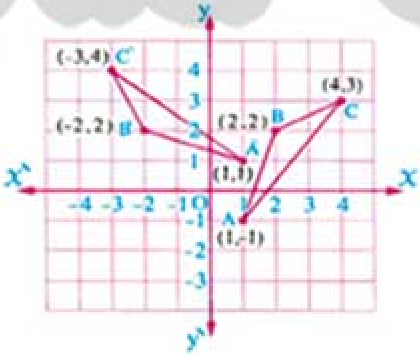
Solution

1. (− 3 , − 2) , (− 2 , 3)
2. (3 , − 4) , (− 4 , − 3)
3. (2 , 1) , (1 , − 2)
4. (− 5 , 2) , (− 2 , − 5)
5. (0 , − 6) , (0 , 6)

Example 4 On a square lattice , draw Δ ABC where A (1 , − 1) , B (2 , 2) and C (4 , 3) :

1. Draw Δ ÀB̀C̀ which is the image of Δ ABC by rotation R (O , 90°)
2. Draw Δ A̖B̖C̖ which is the image of Δ ABC by rotation R (O , 180°)

Solution

1


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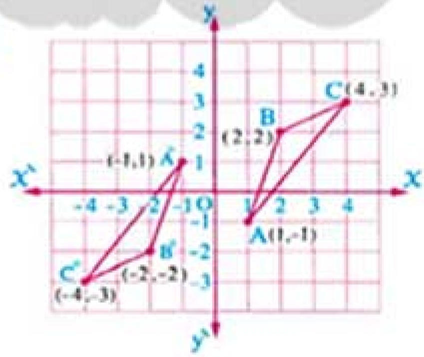
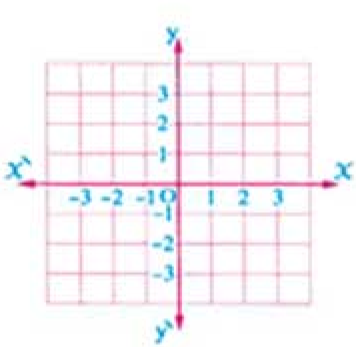


TRY 2 by yourself

In the opposite figure :

On the square lattice ,
draw $\overline{AB}$ where A (2 , 1) and B (1 , 3),
then draw its image by rotation :

1 R (O , 90°) **2** R (O , 180°)



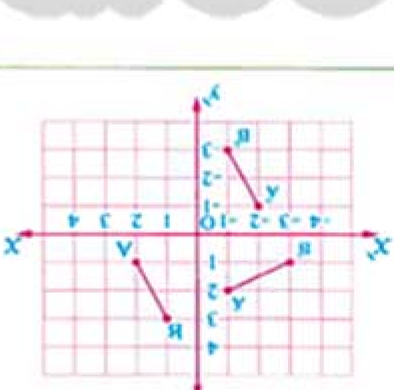
Optical illusion :

Look at the picture.
Turn the book with an angle of measure 180° and look at it again.
What do you notice ?

Answers of try by yourself

1 [1] Point D [2] $\overline{BC}$ [3] Δ AMB

2



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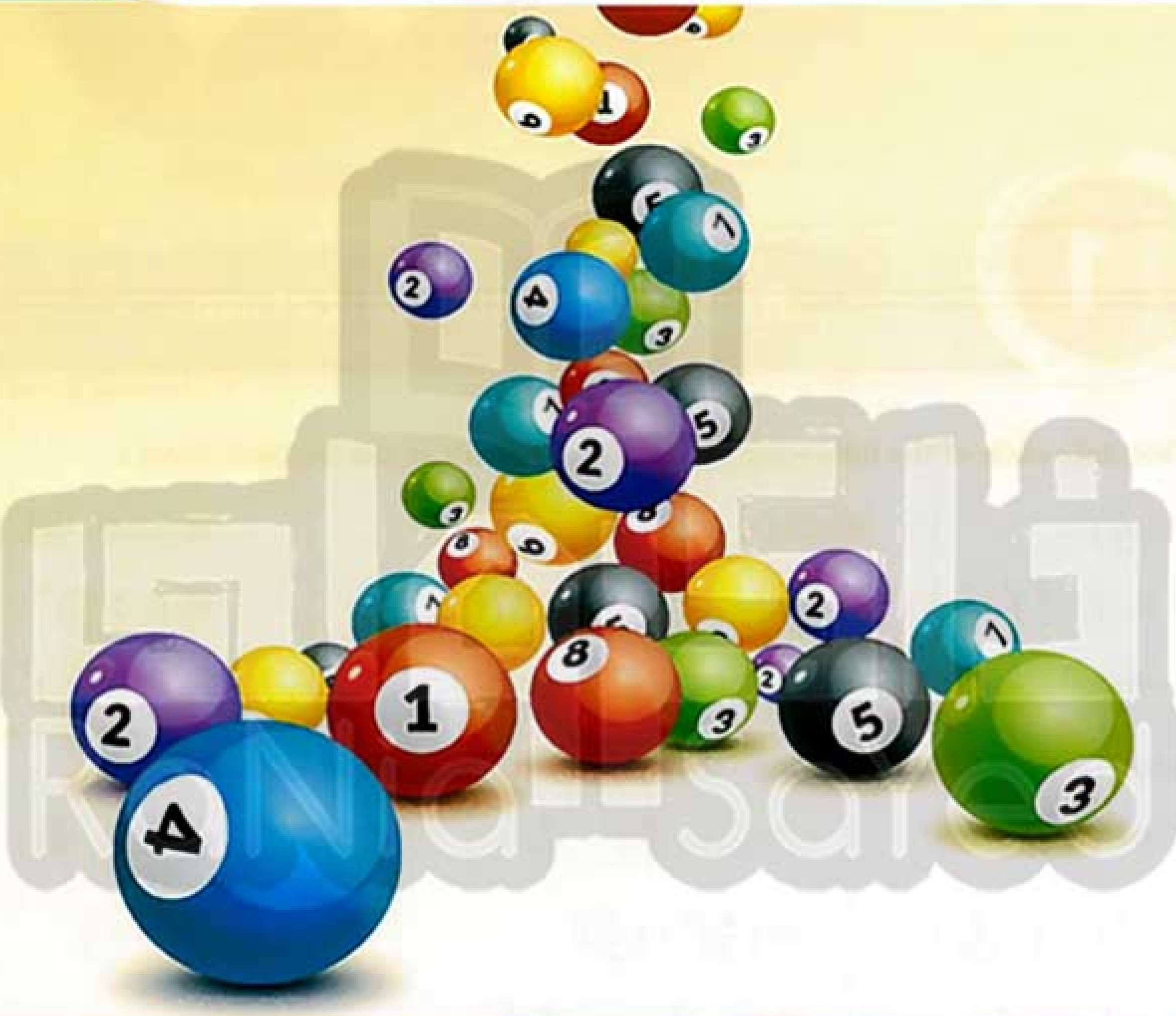
First Algebra and Statistics

UNIT 1

Numbers and Algebra



Exercises of the unit :

1. Repeated multiplication.
2. Non-negative integer powers.
3. Negative integer powers.
4. Scientific notation of the rational number.
5. Order of mathematical operations.
6. The square root of a perfect square rational number.
7. Solving equations in $\mathbb{Q}$.
8. Solving inequalities in $\mathbb{Q}$.

Exercise 1

Repeated multiplication

From the school book

1 Calculate each of the following , then put the result in the simplest form :

1. $\left(\frac{1}{2}\right)^3$
2. $\left(\frac{1}{3}\right)^4$
3. $\left(\frac{3}{5}\right)^2$
4. $\left(-\frac{1}{7}\right)^3$
5. $\left(-\frac{3}{4}\right)^4$
6. $\left(\frac{5}{9}\right)^0$
7. $\left(1\frac{1}{5}\right)^2$
8. $\left(-2\frac{1}{2}\right)^3$
9. $(0.04)^2$
10. $(1.5)^3$
11. $(-3.2)^2$
12. $\left(1-1\frac{2}{3}\right)^2$

2 Calculate each of the following , then put the result in the simplest form :

1. $8 \times \left(\frac{1}{2}\right)^3$
2. $\left(-\frac{3}{4}\right)^2 \times \frac{8}{27}$
3. $\left(-\frac{3}{5}\right)^3 \times \left(-\frac{25}{27}\right)$
4. $\left(\frac{3}{5}\right)^2 \div \left(-\frac{9}{125}\right)$
5. $\left(\frac{4}{3}\right)^2 \times \left(\frac{3}{2}\right)^3$
6. $\left(-\frac{5}{6}\right)^2 \div 3\frac{3}{4}$
7. $\left(2\frac{1}{2}\right)^2 \times \frac{4}{25}$
8. $2\frac{7}{9} \div \left(-1\frac{2}{3}\right)^2$

3 Calculate each of the following , then put the result in the simplest form :

1. $\left(\frac{4}{5}\right)^2 \times \frac{5}{16} \times \left(\frac{2}{3}\right)^0$
2. $\frac{3}{4} \times \left(-\frac{2}{3}\right)^3 \times \left(\frac{3}{2}\right)^2$
3. $\left(-\frac{5}{3}\right)^4 \times \left(-\frac{3}{5}\right)^3 \times (-1)^7$
4. $\left(-\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^3 \div \left(-\frac{2}{9}\right)^2$
5. $\left[\left(\frac{5}{2}\right)^3 \div \left(\frac{3}{2}\right)^4\right] \times \left(\frac{3}{5}\right)^3$
6. $\left(-\frac{1}{2}\right)^3 \div \left[8 \times \left(-\frac{1}{2}\right) \times \frac{3}{4}\right]$

4 **Choose the correct answer from those given :**

1 The multiplicative inverse of the number $\left(\frac{2}{5}\right)^0 =$ ....

(a) $\frac{5}{2}$ (b) $-\frac{2}{5}$ (c) 1 (d) 0

2 The additive inverse of the number $(-3)^0$ is ....

(a) 1 (b) -3 (c) 3 (d) $-(3)^0$

3 The multiplicative inverse of the number $(-1)^3$ is ....

(a) $(-1)^3$ (b) $(-1)^2$ (c) 1^3 (d) 1^2

4 The additive inverse of the number $\left(-\frac{2}{5}\right)^2$ is ....

(a) $\frac{4}{25}$ (b) $-\frac{4}{25}$ (c) $\frac{25}{4}$ (d) $-\frac{25}{4}$

5 $\left(\frac{1}{4}\right)^0 + \frac{1}{4} =$ ....

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) $\frac{2}{4}$

6 $\left(\frac{5}{3}\right)^2 \times \left(\frac{3}{5}\right)^0 =$ ....

(a) $\frac{5}{3}$ (b) $\frac{25}{9}$ (c) 0 (d) 1

7 If $x = y$, then $\left(\frac{3}{5}\right)^{x-y} =$ ....

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) 1 (d) 0

8 $\left(\frac{a}{b}\right)^2 \times \frac{b^2}{a^2} =$ .... (where $ab \neq 0$)

(a) ab (b) $\left(\frac{a}{b}\right)^4$ (c) $(ab)^0$ (d) $\frac{a}{b}$

9 If $x = -\frac{1}{2}$ and $y = 3$, then $x^y =$ ....

(a) $\frac{1}{8}$ (b) $-\frac{1}{8}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6}$

10 If $y^{26} + y^{27} = 0$, then $y =$ ....

(a) 1 (b) -1 (c) 2 (d) -2

5 **Complete the following :**

1 $\frac{8}{27} = \left(\frac{2}{3}\right)^{\cdots\cdots}$

2 $\frac{9}{16} = \left(\frac{3}{4}\right)^{\cdots\cdots}$

3 $-\frac{64}{125} = \left(-\frac{4}{5}\right)^{\cdots\cdots}$

4 $2\frac{1}{4} = \left(\frac{3}{2}\right)^{\cdots\cdots}$

5 $0.027 = \left(\frac{3}{10}\right)^{\cdots\cdots}$

6 $64\ \% = \left(\frac{4}{5}\right)^{\cdots\cdots}$

7 If $\frac{x}{y} = -\frac{2}{5}$, then $\left(\frac{x}{y}\right)^3 =$ ....

8 If $c = -3$ and $d = -5$, then $\left(\frac{c}{d}\right)^2 =$ ....

Unit 1

9 If $\frac{a}{b} = 0.2$, then $\left(\frac{a}{b}\right)^3 =$ ....

10 If $x = \frac{1}{2}$ and $y = \frac{2}{3}$, then $x^2 y^2 =$ ....

11 $\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 =$ ....

12 $2^2 + 2^2 = 2^{\cdots\cdots}$

13 $\frac{3}{4}$, $\frac{9}{16}$, $\frac{27}{64}$, ... , ... (in the same pattern)

14 The greater of the two numbers $\left(\frac{1}{4}\right)^2$ and $\left(-\frac{8}{3}\right)^5$ is ....

6 If $x = -\frac{2}{3}$ and $y = -\frac{1}{3}$, find the value of : $x^2 + y^3$ « $\frac{11}{27}$ »

7 If $a = \frac{2}{3}$ and $b = -\frac{4}{3}$, find the value of : $|a^3 \div b^3|$ « $\frac{1}{8}$ »

8 If $x = 0.5$, $y = -\frac{2}{3}$ and $z = -3$, find the value of : $9xy^2 - z^3$ « 29 »

9 If $a = -\frac{1}{2}$, $b = 2$ and $c = \frac{3}{4}$, find the numerical value of : $a^3 b^2 + b^2 c - 8abc$ « $8\frac{1}{2}$ »

10 **If $x = -\frac{3}{2}$, $y = \frac{1}{2}$ and $z = -\frac{4}{3}$, find the numerical value of each of the following in its simplest form :**

1 $x^2 y^2 z^2$ « 1 »

2 $x^2 \div z^2$ « $\frac{81}{64}$ »

3 $x^2 - yz^2$ « $\frac{49}{36}$ »

4 $\frac{x^2 y^2 z^2}{x + y}$ « -1 »

Geometric Application

11 If $V = \ell^3$ where V is the volume of a cube and ℓ is its edge length , then calculate the volume of the cube whose edge length is $1\frac{1}{2}$ cm. « $\frac{27}{8}$ cm.³ »

For excellent pupils

12 **Choose the correct answer from those given :**

1 If $y = \left(\frac{1}{2}\right)^x$ where $x \in \{0, 1, 2, 3\}$, then y takes its maximum value when $x =$ ....

(a) 0 (b) 1 (c) 2 (d) 3

2 If $y = \left(-\frac{2}{5}\right)^x$ where $x \in \{0, 1, 3, 4\}$, then y takes its minimum value when $x =$ ....

(a) 0 (b) 1 (c) 3 (d) 4

13 **Arrange the following numbers ascendingly without expanding :**

$\left(\frac{2}{3}\right)^2$, $\left(-\frac{2}{3}\right)^3$, $\left(-\frac{1}{3}\right)^2$, $\left(-\frac{1}{3}\right)^3$

Exercise 2 Non-negative integer powers

📖 From the school book

1 Calculate each of the following , then put the result in the simplest form :

1. 📖 $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2$
2. 📖 $\left(-\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2$
3. $\frac{1}{5} \times \left(-\frac{1}{5}\right)^4$
4. $\left(\frac{1}{6}\right)^9 \div \left(\frac{1}{6}\right)^8$
5. 📖 $\left(\frac{2}{7}\right)^5 \div \left(\frac{2}{7}\right)^3$
6. 📖 $\left(-\frac{3}{5}\right)^7 \div \left(\frac{3}{5}\right)^5$
7. $\left(-\frac{5}{2}\right)^2 \div 2\frac{1}{2}$
8. $\left(\frac{1}{2}\right)^2 \times \frac{1}{2} \times \left(\frac{1}{2}\right)^3$
9. $\left(\frac{4}{5}\right)^8 \div \left(\frac{4}{5}\right)^6 \times \frac{4}{5}$

2 Calculate each of the following , then put the result in the simplest form :

1. $\frac{3^7 \times 3^3}{3^6}$
2. $\frac{2^6 \times 2}{2^3 \times 2^4}$
3. $\frac{(-5)^4 \times 5^2}{5^3}$
4. $\frac{(-2)^5 \times 2^4}{(-2)^3 \times 2^2}$
5. $\frac{x^2 \times x^3 \times x^4}{x^7 \times x}$
6. $\frac{(-3)^5 \times (-2)^7}{(-3)^3 \times (-2)^5}$
7. $\frac{x^4 \times y^3 \times x^5}{x^6 \times y^2}$
8. $\frac{x y^2 \times x^2 y}{x^2 y^2}$
9. $\frac{\left(-1\frac{1}{2}\right)^5 \times \left(-1\frac{1}{4}\right)^8}{\left(-\frac{3}{2}\right)^4 \times \left(-\frac{5}{4}\right)^6}$

3 Find each of the following in the simplest form :

1. 📖 $\left(\frac{a\,b}{c}\right)^5$
2. $\left(\frac{5x}{3y}\right)^2$
3. $\left(-\frac{2\,a\,b}{3\,c}\right)^4$
4. 📖 $\left(\frac{x^2}{y^3}\right)^2$
5. $\left(\frac{a^3 b^2}{c^5}\right)^3$
6. 📖 $\left(-\frac{c^2}{d}\right)^3$

7 $\left(-\frac{x^3}{y^2}\right)^2$ | 8 $\frac{(4x^3y^2)^7}{(2x^2y)^7}$ | 9 $\frac{(2a)^3 \times (2a)^4}{(-2a)^6 \times a}$

4 Calculate each of the following , then put the result in the simplest form :

1 $[(\frac{1}{2})^2]^2$

2 $[(-\frac{3}{2})^2]^5$

3 $[(2\frac{1}{2})^3]^2$

4 $[(-1\frac{1}{3})^2]^3$

5 $(\frac{3}{5})^{10} \times (\frac{5}{3})^{10}$

6 $\left((\frac{2}{7})^2\right)^3 \times (\frac{7}{2})^6$

7 $(2\frac{1}{2})^2 \times (-\frac{2}{5})^2$

8 $\left(\frac{x^3}{y^2}\right)^{16} \div \left(\frac{x^2}{y^2}\right)^{16}$

5 Match each expression in column (A) with an equivalent expression in column (B) :

Column (A)	Column (B)
(1) $(x^2)^n$	(a) x^{n^2}
(2) $(x^n)^n$	(b) $\frac{3m^c}{2n^c}$
(3) $(xy^a)^b$	(c) $27x^{3a}$
(4) $\left(\frac{x}{y^a}\right)^b$	(d) $\frac{3^c m^c}{2^c n^c}$
(5) $(-3x^a)^3$	(e) x^{2n}
(6) $(3x^a)^3$	(f) $-27x^{3a}$
(7) $\frac{3}{2}\left(\frac{m}{n}\right)^c$	(g) $\frac{n^b}{y^{ab}}$
(8) $\left(\frac{3m}{2n}\right)^c$	(h) $x^b y^{ab}$
	(i) $\frac{x^b}{y^{ab}}$
	(j) xy^{ab}

6 **Choose the correct answer from those given :**

1. $3^2 \times 3^5 =$ ....
   (a) 3^7 (b) 3^3 (c) 3^{10} (d) 3^{25}
2. $5^2 + 5^2 =$ ....
   (a) 10^2 (b) 10^4 (c) 5^4 (d) 50
3. $3^5 \times 2^5 =$ ....
   (a) 5^{10} (b) 6^{10} (c) 6^5 (d) 6^{25}
4. $(5a)^0 =$ .... , $a \neq 0$
   (a) 5 (b) a (c) 5 a (d) 1
5. $3^{(2^3)} =$ ....
   (a) 3^6 (b) 3^5 (c) 3^8 (d) 3^{23}
6. $(5^2)^3 =$ ....
   (a) 5^6 (b) 5^5 (c) 5^{23} (d) 5
7. $3^{10} + 3^{10} + 3^{10} =$ ....
   (a) 3^{10} (b) 3^{30} (c) 9^{10} (d) 3^{11}
8. $4^x + 4^x + 4^x + 4^x =$ ....
   (a) 4^{x+4} (b) 4^{4x} (c) 4^{x+1} (d) $4x^4$
9. $\frac{(3^2)^5}{(3^5)^2} =$ ....
   (a) 3^{10} (b) 3^{52} (c) 3^{25} (d) 1
10. $\frac{(x^2)^3}{x^3} =$ .... , $x \neq 0$
   (a) x^6 (b) x^2 (c) x^3 (d) x
11. $(2y)^3 =$ ....
   (a) $2y^3$ (b) $8y$ (c) $8y^3$ (d) $23y$
12. $(b^3)^4 =$ ....
   (a) b^{34} (b) b^7 (c) $b^3 \times b^3 \times b^3$ (d) $b^4 \times b^4 \times b^4$
13. The quarter of the number $4^{20} =$ ....
   (a) 4^5 (b) 4^{10} (c) 4^{19} (d) 2^{10}

Unit 1

7 **Simplify to the simplest form :**

$\frac{(2y)^4 \times (3y)^2}{12y^5}$, then find the value of the result at $y = -\frac{1}{6}$ «-2»

8 If $a = \frac{1}{2}$, $b = \frac{3}{4}$ and $c = -\frac{2}{3}$, **find the numerical value of each of :**

1 $(c^2 b)^3$ 2 $(4a^3 c)^2$ 3 $(a^2 b c^2)^2$ «$\frac{1}{27}$, $\frac{1}{9}$, $\frac{1}{144}$»

9 If $a = \frac{5}{3}$, $b = -\frac{3}{2}$ and $c = \frac{2}{5}$, **find the numerical value of each of :**

1 $\frac{(a^2 c^2)^2}{b}$ 2 $\left(\frac{2ab}{5c}\right)^3$ «$-\frac{32}{243}$, $-\frac{125}{8}$»

10 If $x = -\frac{1}{2}$, $y = \frac{3}{4}$ and $z = -\frac{3}{2}$,

find the numerical value of each of the following in the simplest form :

1 $x^3 y^2$ 2 $y^3 x^2$ 3 $\frac{x^3}{y^2 z^2}$

«$-\frac{9}{128}$, $\frac{27}{256}$, $-\frac{8}{81}$»

11 **Complete the following :**

1 $\left(\left(\frac{7}{9}\right)^3\right)^4 = \frac{7^{12}}{3^{\cdots\cdots}}$ 2 If $\left(\frac{3}{4}\right)^5 \times x = \left(\frac{3}{4}\right)^7$, then $x = \cdots\cdots\cdots$

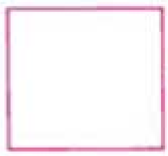
3 The greater number in the two numbers $((-3)^5)^3$ and $((-3)^2)^4$ is $\cdots\cdots\cdots$

4 $((-1)^5)^2 - ((-1)^3)^2 = \cdots\cdots\cdots$ 5 $\frac{4^4}{4^3} + \frac{4^3}{4^2} + \frac{4^2}{4} + 4 = 2^{\cdots\cdots\cdots}$

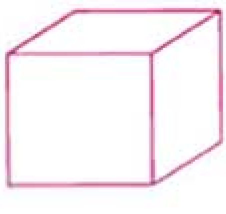
6 $2^{2x} \times 4^x = 4^{\cdots\cdots\cdots}$

Geometric Applications

12 Find the area of the square whose side length is $\frac{2x}{5}$ cm.



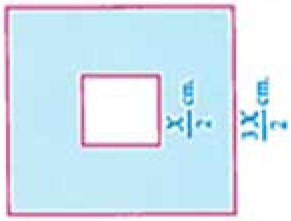
13 Find the volume of the cube whose edge length is $\frac{3a^3}{7}$ cm.



14 **In the opposite figure :**

A square is drawn inside another square.

Find the area of the shaded part.



Life Application

15 **From computer technology , we know that :**

1 kilobyte = 2^{10} bytes ,

1 megabyte = 2^{10} kilobytes ,

1 gigabyte = 2^{10} megabytes ,

1 terabyte = 2^{10} gigabyte ,

How many bytes are there in one terabyte ?

For excellent pupils

16 If four times a number is 4^3 , find $\frac{3}{4}$ this number. « 12 »

17 If $x = \left(\left(\frac{2}{3}\right)^5\right)^2$ and $y = 3\left(\frac{3}{2}\right)^9 - \left(\frac{3}{2}\right)^{10}$

, prove that the number x is the multiplicative inverse of the number y

18 If $x = \frac{1}{5}$ and y = 5 , find the value of : $x^{15}\,y^{14}$ « $\frac{1}{5}$ »

19 **Prove that :** 1 $5^{x+2} - 5^{x+1} = 20 \times 5^x$

2 $3^{15} + 3^{14}$ is divisible by 4

Exercise 3 Negative integer powers

(📖) From the school book

1 **Evaluate each of the following :**

1. 📖 4^{-1}
2. 📖 5^{-2}
3. $\left(\frac{1}{2}\right)^{-1}$
4. $\left(-\frac{2}{3}\right)^{-2}$
5. $(0.2)^{-2}$
6. $(1.2)^{-1}$

2 **Evaluate each of the following :**

1. 📖 $3^7 \times 3^{-3}$
2. $2^{-2} \times 2^{-3}$
3. 📖 $\frac{3}{3^{-2}}$
4. 📖 $\frac{6^{-2}}{6^{-3}}$

3 **Evaluate each of the following :**

1. 📖 $(5^{-1})^{-3}$
2. $(3^{-2})^2$
3. $(0.25)^{-2}$
4. $(2^{-1} \times 2^{-2})^3$
5. 📖 $\left(\frac{3^{-1}}{3}\right)^2$
6. 📖 $\left(\frac{8^4}{8^{-4}}\right)^0$

4 **Evaluate each of the following :**

1. 📖 $\frac{8 \times 8^{-2}}{8^{-3}}$
2. 📖 $\frac{7^{-2} \times 7^5}{7^3}$
3. $\frac{2^5 \times 2^{-2}}{2^{-4} \times 2^3}$
4. $\frac{2^3 \times 2^{-3}}{(2^2)^2}$
5. $\frac{(3^{-2})^3}{3^{-2} \times 3^{-6}}$
6. 📖 $\left(\frac{9^3 \times 9}{9^5}\right)^{-3}$
7. $\left(\frac{4^{-2} \times 3}{4^{-3}}\right)^{-3}$
8. $\left(\frac{2^5 \times 3^2}{3^4 \times 2^3}\right)^{-1}$
9. 📖 $(3^0 \times 2^{-2})^{-2}$
10. $(3^0 - 2^{-2})^{-2}$
11. $\frac{(10)^2 \times (0.01)^3}{(10)^{-3}}$

Exercise 3

5 Simplify each of the following and write the result in terms of positive exponents , where the denominator does not equal zero :

1. $7x^{-1}$
2. $x^{-1}y^2$
3. $a^{-2}b^{-3}$
4. $x^3 \times x^{-5}$
5. $x^3 \times x^{-2} \times x^{-1}$
6. $\frac{c^{-5}}{c^2}$
7. $x^7 \div x^{-5}$
8. $(a^{-2})^3$
9. $(b^{-1})^{-3}$
10. $(a^2 \times a^{-5})^2$
11. $(x^2)^{-3} \times (x^{-3})^{-2}$
12. $\left(\frac{y^5}{y^{-2}}\right)^{-3}$
13. $\frac{x^2 \times x^{-3}}{x^{-4} \times x}$
14. $\frac{(x^2)^{-3} \times (x^{-1})^2}{x^{-3} \times x^{-4}}$
15. $\left(\frac{x^{-2} \times y^{-1}}{y^{-3} \times x}\right)^{-1}$
16. $\frac{a^{-1}}{b^2}\left(\frac{a^{-1}}{2b^2}\right)^{-2}$
17. $(x + x^{-1})^2$

6 Complete the following :

1. $2^{-3} \times c^0 = \dots\dots$
2. $(b^{-1})^{-3} = b^{\dots}$
3. $2x^{-3} = \frac{2}{\dots}$
4. $(3x^{-1})^2 = 9x^{\dots} = \frac{9}{\dots}$
5. $(3y^{-2})^{-2} = \dots\dots$
6. $(3a^2)^{-1} = \frac{1}{\dots}$
7. $2x^{-2}y^{-3} = \frac{2}{\dots}$
8. $\frac{x^{-5}}{y^{-5}} = (\dots\dots)^5$
9. $(\frac{1}{2})^2 + 2^0 - (2)^{-2} = \dots\dots$
10. $(x^2)^{\dots} = \frac{1}{x^4}$
11. $2^{10} \times 2^{-10} = 3^{\dots}$
12. $a^{-5} + 1 = a^{-5}(\dots\dots + \dots\dots)$, where $a \neq 0$
13. If $x = \frac{1}{2}$, $y = \frac{1}{4}$, then $(x - y)^{-1} = \dots\dots$

7 Choose the correct answer from those given :

1. If $a^{-1} = \frac{2}{3}$, then $a = \dots\dots$
   (a) $-\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) 1
2. If $a = 7^x$ and $b = 7^{-x}$, then $a \times b = \dots\dots$
   (a) 7^{2x} (b) 49^{2x} (c) 1 (d) 0
3. $\frac{5^x}{5^{-y}} = \dots\dots$
   (a) 5^{x+y} (b) 5^{x-y} (c) 5^{x+y} (d) $-\frac{x}{y}$
4. $\frac{6a^2x^4}{2a^3x^3} = \dots\dots$
   (a) $3ax$ (b) $3a^5x^7$ (c) $\frac{3x}{a}$ (d) $\frac{3}{ax}$
5. $\frac{(-2s^2t)^3}{(-4st^2)^2} = \dots\dots$
   (a) $-\frac{s^3}{2t}$ (b) $-\frac{s^4}{2t}$ (c) $\frac{s^5}{2t^2}$ (d) $\frac{s^4}{t}$

Unit 1

6 $\left(\frac{m^2}{n^{-3}}\right)^{-1}\left(\frac{3\,m^{-2}}{n^{-2}}\right)^{-2} = \dots\dots$

(a) $\frac{9\,m^2}{n^7}$ (b) $\frac{m^2}{9\,n^7}$ (c) $\frac{m^2}{9\,n}$ (d) $\frac{9\,m^6}{n}$

7 $\frac{(2\,a\,b^{-2})^0}{3^0\,a^{-2}\,b} = \dots\dots$

(a) $\frac{a^3}{3\,b^3}$ (b) a^2 (c) 1 (d) $\frac{a^2}{b}$

8 If $a^x = 2$ and $a^{-y} = 3$, then $a^{x-y} = \dots\dots$

(a) 1 (b) -1 (c) $\frac{2}{3}$ (d) 6

9 If $x\,y^{-1} = \frac{1}{2}$, then $\frac{y}{x} = \dots\dots$

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) 2

10 $3^{-1} + 3^{-1} + 3^{-1} = \dots\dots$

(a) 3^{-3} (b) 3^3 (c) 9^{-3} (d) 1

11 The multiplicative inverse of 5^{-1} is $\dots\dots$

(a) $\frac{1}{5}$ (b) 5 (c) -5 (d) $-\frac{1}{5}$

12 $\left(\frac{3}{5}\right)^2 \times \left(\frac{5}{3}\right)^{-2} = \dots\dots$

(a) $\left(\frac{3}{5}\right)^4$ (b) 1 (c) $\left(\frac{3}{5}\right)^{-4}$ (d) 0

8 Complete each of the following by the suitable sign of (>) , (<) or (=) :

1 $2^{10} \dots\dots 2^{-10}$ 2 $3^{-20} \dots\dots 3^2$ 3 $5^{-15} \dots\dots 2^{-15}$

4 $(-7)^{-2} \dots\dots (-7)^{19}$ 5 $(-1)^{-6} \dots\dots (-1)^{-9}$ 6 $(-1)^{-20} \dots\dots (1)^{-10}$

9 Why b^{-3} is not defined when $b = 0$?

10 Calculate the value of $\left(-\frac{3}{5}\right)^x \times \left(\frac{3}{5}\right)^y$ in each of the following cases :

1 $x = -2$ and $y = 2$ « 1 » 2 $x = -1$ and $y = 2$ « $-\frac{3}{5}$ »

11 If $x = -\frac{1}{3}$, $y = \frac{2}{3}$, then find in the simplest form the numerical value of the expression : $\left(\frac{y}{x^2}\right)^{-2}$ « $\frac{1}{36}$ »

Life Applications

12 The flea can jump at a height of 200 times of its length.
If a flea of length 2^{-4} inches can jump at a height of 2^3 inches ,
what does this height represent according to
the length of the flea ?

13 The population of a city has been growing exponentially. It is estimated that in (t) years the population (p) will be : $p = 2\,(1.03)^t$ million.

1 What will the population be in 2 years ?
2 What is the population now ?
3 What was the population last year ?

For excellent pupils

14 **Simplify to the simplest form :** $\dfrac{2^{10} \times 3^4}{(12)^5}$ « $\frac{1}{3}$ »

15 **Simplify to the simplest form :**

$\dfrac{6^{2n+1} \times 4^{-n}}{2^n \times 3^{2n+1}}$, then find the value of the result when n = 3 « $\frac{1}{4}$ »

16 **If $2^n = 3$, find the value of :**

1 2^{n+1} 2 4^n 3 4^{-n} 4 2^{n-1} « 6 , 9 , $\frac{1}{9}$, $\frac{3}{2}$ »

17 **If a = 5 and b = 5^{-1} , find the value of :** $a^{51}\,b^{50}$ « 5 »

18 **Without expanding , arrange the following ascendingly by inspection :**

$(-2)^{-15}$, $(-5)^{20}$, $(-2)^{15}$, 2^{-20} , $(-5)^{15}$, $(-2)^{20}$

Exercise 4 Scientific notation of the rational number

From the school book

1 **Which of the following numbers are in the standard form :**

1 5.3×10^{7}	2 0.2×10^{-4}	3 0.025×10^{8}
4 7×10^{-4}	5 10×10^{-10}	6 4.25×10
7 33.9×10^{6}	8 -5.782×10^{2}	9 -0.0003×10^{3}

2 **Write each of the following numbers in the standard form :**

1 600 000	2 − 20 000	3 7 millions
4 19 millions	5 46 870 000	6 58

3 **Write each of the following numbers in the standard form :**

1 0.0006	2 0.000053	3 0.000864
4 0.421	5 25.0003	6 − 300.501

4 The area of the surface of the Earth is about 510 000 000 km^2.

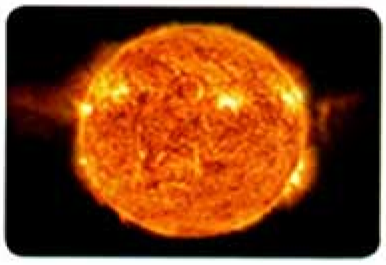
Write this number in the standard form.

5 The mass of the Hydrogen atom is about 0.00000000000000000000000167 gm.

Write this number in the standard form.

Hydrogen Atom

6 The light velocity is about 300 000 km./sec.
Express this velocity by m./sec.
in the standard form.



7 Dr. Ahmed Zewail discovered the femto second which is a millionth of a miliardth of a second.
Express it by the standard form.

8 At writting the number 2.74×10^{20} as a whole number, find the number of zeroes that are on the right of the digit 4

9 **Write the following numbers in the standard form :**

1. 68×10^{5}
2. 68×10^{-5}
3. 720×10^{6}
4. 750×10^{-9}
5. -32.4×10^{4}
6. -702.5×10^{-8}
7. 0.4×10^{-10}
8. 0.0005×10^{15}
9. 0.0036×10^{-4}
10. 0.0020205×10^{12}

10 **Put the suitable sign (<) or (>) :**

1. 6.4×10^{3} ☐ 4.6×10^{3}
2. 6.2×10^{4} ☐ 4.1×10^{5}
3. 0.0041 ☐ 3.2×10^{-2}
4. 4370 ☐ 3.41×10^{4}
5. 2.10×10^{-5} ☐ 1.82×10^{-5}
6. 9.1×10^{-4} ☐ 1.2×10^{-5}
7. 6.920×10^{5} ☐ 96230
8. 3.69×10^{-4} ☐ 0.0000623

11 **Arrange the following numbers in a descending order :**

3.6×10^{-3} , 5.2×10^{-5} , 1×10^{-2} , 8.35×10^{-2} , 6.08×10^{-8}

12 Choose the correct answer from those given :

1 $3.04 \times 10^7 =$ ....

(a) 340 000 (b) 304 000 (c) 3 400 000 (d) 30 400 000

2 $2.37 \times 10^{-4} =$ ....

(a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237

3 If $0.00079 = 7.9\ a$, then a = ...

(a) 10^3 (b) 10^{-3} (c) 10^{-4} (d) 10^4

4 If $0.0000503 = m \times 10^{-5}$, then m = ...

(a) 503 (b) 5.03 (c) 50.3 (d) 0.503

5 If the thickness of a sheet of paper is 0.012 cm. , then a ream of 400 sheets is of height ....

(a) 48×10^{-3} cm. (b) 48×10^{-2} cm. (c) 4.8×10^0 cm. (d) 48 cm.

6 Which of the following equals $\frac{1}{2}$ milliard ?

(a) 50×10^8 (b) 5×10^8 (c) 0.5×10^8 (d) 500×10^7

7 Which of the following is the greatest ?

(a) 6.3×10^5 (b) 9.8×10^4 (c) 5.2×10^5 (d) 7.3×10^4

8 Which of the following is the smallest ?

(a) 0.6×10^5 (b) 0.25×10^5 (c) 7×10^4 (d) 17.5×10^4

9 $6\,000 \times 50 =$ ....

(a) 300×10^2 (b) 30×10^5 (c) 3×10^5 (d) 30×10^4

10 $45 \times 900 =$ ....

(a) 4.05×10^2 (b) 4.05×10^3 (c) 4.05×10^4 (d) 45×10^2

11 $0.7 \times 0.005 =$ ....

(a) 3.5×10^3 (b) 3.5×10^{-2} (c) 3.5×10^2 (d) 3.5×10^{-3}

13 Write the result of each of the following in the standard form :

1 $(6.4 \times 10^8) \times (1.5 \times 10^5)$

2 $(8.2 \times 10^7) \times (2.1 \times 10^{-4})$

3 $(5.02 \times 10^{-4}) \times (0.1 \times 10^{-3})$

4 $(4.4 \times 10^3) \times (2 \times 10)^5$

5 $(3.8 \times 10^8) \div (1.9 \times 10^6)$

6 $(125.5 \times 10^{-3}) \div (5 \times 10^4)$

7 $(8.8 \times 10^{25}) \div (8.8 \times 10^{22})$

8 $(5 \times 10)^4 \div (2.5 \times 10^{-3})$

14 Write the result of each of the following in the standard form :

1 $(3.8 \times 10^5) + (4.6 \times 10^4)$

2 $(4.54 \times 10^4) + (3.76 \times 10^3)$

3 $(5.3 \times 10^8) - (0.8 \times 10^7)$

4 $(2.65 \times 10^{-2}) - (6.34 \times 10^{-3})$

15 **Write the result of each of the following in the standard form :**

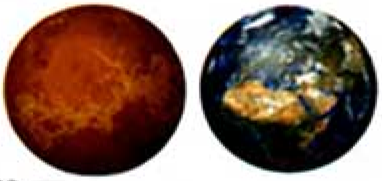
1. 5000×3000
2. 400×0.00007
3. $8000 \div 0.004$
4. $0.000033 \div 500$
5. $(20\,000)^3$
6. $(0.002)^2$
7. $(0.1)^{-8}$

16 **Find the value of n in each of the following :**

1. $800\,000 = 8 \times 10^n$
2. $0.00000006 = 6 \times 10^n$
3. $0.00052 = 5.2 \times 10^n$
4. $0.000357 = 3.57 \times 10^n$
5. $(0.004)^2 = 1.6 \times 10^n$
6. $76293 = n \times 10^4$

Life Applications

17 If the diameter of the Earth is about 1.27×10^4 km. long and the length of the diameter of Mars is about 6.79×10^3 km. Which of the two planets is the greater and what is the difference between the two diameter lengths in the standard form ?



18 If light travels at a speed of 3×10^8 m./s. :

(a) Calculate the distance from the Sun to the Earth if you know that the light of the Sun takes 8 minutes to reach the Earth.

(b) If the distance between planet Venus and the Sun is 108 million kilometres , calculate the elapsed time (in minutes) that light takes to reach Venus from the Sun.

For excellent pupils

19 **Find the result of the following in the standard form :** $\dfrac{9.02 \times 10^3 + 4.98 \times 10^4}{2.5 \times 10^{-5}}$

20 **Without using the calculator ,**
write each of the following numbers in the standard form :

1. $10^{29} - 10^{28}$
2. $2^{19} \times 5^{15}$

21 If $X = 5 + (3 \times 10) + (4 \times 10^2) + (6 \times 10^3) + (9 \times 10^4) + (4 \times 10^5) + (2 \times 10^6)$

Write X in the standard form without using the calculator.

Summary of the first part of unit 1
"From lesson 1 to lesson 4"

If a , b are two rational numbers, n, m are two integers, then :

The repeated multiplication

- $a^n = a \times a \times a \times \cdots \times a$, where a repeated n times.
- $(-a)^n = a^n$, where n is an even number.
- $(-a)^n = -a^n$, where n is an odd number.

The laws of powers

1. $a^n \times a^m = a^{n+m}$ (when multiplying like bases , we add their powers)
2. $a^n \div a^m = a^{n-m}$ (when dividing like bases , we subtract their powers)
3. $(a^n)^m = a^{n \times m}$
4. $(a \times b)^n = a^n \times b^n$
5. $(a \div b)^n = a^n \div b^n$ where $b \neq 0$

- If a is a rational number , $a \neq 0$, then $a^0 = 1$
- If a is a rational number , $a \neq 0$, n is a positive number , then $a^{-n} = \frac{1}{a^n}$, $a^n = \frac{1}{a^{-n}}$
- If $\frac{a}{b}$ is a rational number not equal to zero, n is a positive number , then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

The standard form of a rational number

The number is been written in the standard form as :

$a \times 10^n$, where $1 \leq |a| < 10$, $n \in \mathbb{Z}$

Exams on the first part of unit one
from lesson (1) to lesson (4)

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1. If a = b , then $\left(\frac{3}{7}\right)^{a-b} =$ ....

(a) zero (b) 1 (c) $\frac{3}{7}$ (d) $\frac{7}{3}$

2. If $a^{-1} = \frac{3}{4}$, then a = ...

(a) zero (b) 1 (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

3. $\frac{4\,a^2\,b^4}{2\,a^3\,b^3} =$ .... where ab ≠ zero

(a) 2 a b (b) $2\,a^5\,b^7$ (c) $\frac{2\,b}{a}$ (d) $\frac{2}{a\,b}$

4. $2^4 \times 3^4 =$ ....

(a) 5^4 (b) 6^4 (c) 6^8 (d) 6^{16}

5. Quarter of $4^{20} =$ ....

(a) 4^5 (b) 4^{10} (c) 4^{19} (d) 2^{10}

6. $3^{10} + 3^{10} + 3^{10} =$ ....

(a) 3^{10} (b) 3^{11} (c) 3^{20} (d) 3^{30}

2 **Complete the following :**

1. If $x = \frac{1}{4}$, $y = \frac{1}{8}$, then $(x - y)^{-1} =$ ....

2. The standard form of the number 0.000013 = ...

3. $\frac{8}{27} = \left(\frac{2}{3}\right)^{\cdots\cdots}$

4. 0.00023 = 2.3 × n , then n = ...

5. $5\,y^{-3} = \frac{5}{\cdots\cdots}$

3 [a] Find the value of the following expression in the simplest form : $\frac{5^{-2} \times 5^5}{5^3}$

[b] Write the result of each of the following in the standard form :

1. $(4.4 \times 10^3) \times (2 \times 10^5)$
2. $(5.3 \times 10^8) - (0.8 \times 10^7)$

4 [a] Find in the simplest form : $\left(\frac{3^3 \times 3^{-2}}{3^{-1} \times 3^4}\right)^{-2}$

[b] If $x = -\frac{1}{2}$, $y = \frac{3}{4}$, $z = -\frac{3}{2}$

Find in the simplest form : $\frac{x^3}{y^2 z^2}$

5 [a] Find the value of n in each of the following :

1. $0.00073 = 7.3 \times 10^n$
2. $4720000 = 4.72 \times 10^n$

[b] If $a = -\frac{1}{2}$, $b = 2$, $c = \frac{3}{4}$

Find the numerical value of the expression : $a^3 b^2 + b^2 c - 8 a b c$

Model 2

Answer the following questions :

1 Choose the correct answer from the given ones :

1. $(3 a b^2)^3 =$ ....

(a) $3 a^3 b^5$ (b) $9 a^3 b^6$ (c) $27 a^3 b^6$ (d) $9 a^3 b^5$

2. Twice the number $2^5 =$ ....

(a) 2^{10} (b) 2^6 (c) 4^5 (d) 4^{10}

3. $\frac{(3 x^{-2} y)^{zero}}{4^{zero} x^{-2} y} =$ .... where $x y = zero$

(a) $\frac{x^2}{y}$ (b) $\frac{y}{x^2}$ (c) $\frac{3 y}{4 x}$ (d) 1

4. $3.35 \times 10^{-3} =$ ....

(a) 0.000335 (b) 0.0335 (c) 335000 (d) 0.00335

5 The additive inverse of $(-\frac{2}{5})^2$ is ....

(a) $\frac{4}{25}$ (b) $\frac{-4}{25}$ (c) $\frac{25}{4}$ (d) $\frac{-25}{4}$

6 $x^9 \times x^{-4} =$ ....

(a) x^3 (b) x^5 (c) x^9 (d) x^{-3}

2 **Complete the following :**

1 If $74500000 = 7.45 \times 10^n$, then n = ...

2 $2\frac{1}{4} = (\frac{2}{3})^{\cdots\cdots}$

3 If $x^{-1} = \frac{1}{2}$, then $x =$ ....

4 If $x = \frac{1}{3}$, $y = \frac{3}{2}$, then $x^2 y^2 =$ ....

5 $(2x^{-1})^3 = \frac{8}{\cdots\cdots}$

3 **[a] Find in the simplest form :**

$\frac{a^5 \times a^8}{a^3 \times a^2 \times a^4}$ where $a \neq 0$

[b] If $x = -\frac{1}{4}$, $y = \frac{3}{4}$

, find in the simplest form : $\frac{x^4 y^3}{x^6 y^2}$

4 **[a] Write the result of each of the following in the standard form :**

1 $(3.4 \times 10^8) \div (1.7 \times 10^3)$

2 $(6.3 \times 10^5) + (2.8 \times 10^4)$

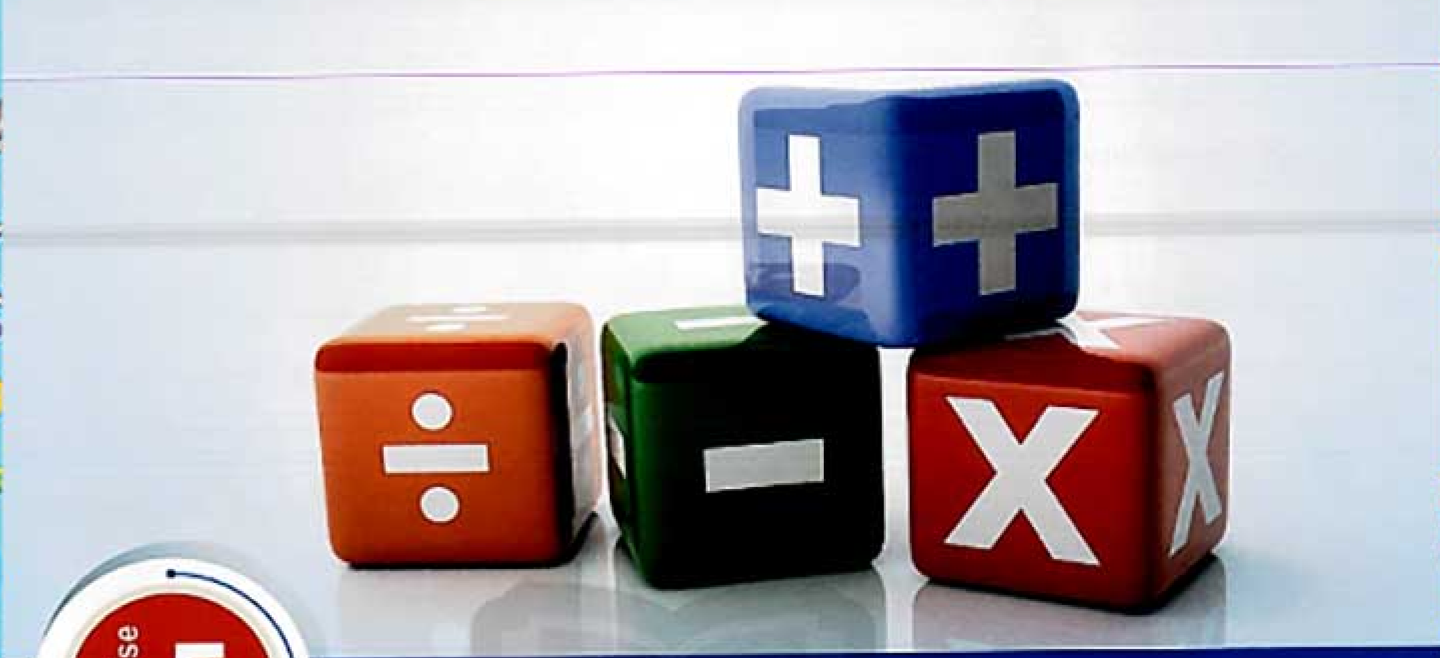
[b] Find in the simplest form : $\frac{8^2 \times 8^{-4}}{8^{-5} \times 8}$

5 **[a] Write each of the following in the standard form :**

1 0.00054 2 79400000

3 46.5×10^3

[b] Find the volume of a cube of edge length $\frac{3a^2}{5}$ cm.



Exercise 5 — Order of mathematical operations

📖 From the school book

1 Calculate the value of each of the following :

1. $3 + 12 \div 6$
2. $-5 + 2 \times 3$
3. 📖 $2 \times 6 - 4 \div 2$
4. 📖 $4 \times 7 - 3^2$
5. 📖 $4 \times 2^3 - 20$
6. 📖 $9 + 4 \times 3^2$
7. 📖 $144 - 8 \div 2^3$

2 Calculate the value of each of the following :

1. 📖 $196 \div (7 - 5)^2$
2. $18 \div (9 - 6) \times (1 + 2)$
3. $20 \div 5 + 8 - (4 - 1)$
4. $10 \times 4 - (2 \times 6 - 8)$
5. $(7 - 4) \times 2 \div (5 - 3)$
6. $(30 - 6) \div 6 \times 30 \div 3$
7. 📖 $7\,(6^2 \div 2 \times 3)$
8. 📖 $12\,(2^2) \div 24 + 3^2$
9. 📖 $9\,(4)^2 \div 2^2 \times 3$
10. $9 \times 10 + 20 \div 2 - 3$
11. $6 + 9 \div 3 + 2 \times 3^2 + 11 - 8$
12. $6 - 5 + 72 \div 9 + 24 \div 2^3 + 4 \times 1 + 10 \times 1 + 5$

3 Calculate the value of each of the following :

1. 📖 $2 - [(7 - 3) - 2]$
2. 📖 $[4 - (5 - 2)] - 1$
3. 📖 $3 + [5 + 2\,(8 \div 4)]$
4. 📖 $2^3 + [4 + (2 - 1)]$
5. $[(2 + 23 - 7) \times 2] \div 4$
6. $10 \times 3 \div [4 - (9 - 8)]$
7. $(26 + 1) \div [3\,(4 - 3)]$
8. $(15 \times 2) \div [5 - (9 - 7)]$
9. $2 + 3\,[4 + (6 \times 3 - 8)] \times 2$
10. 📖 $2\,[(5^2 + 1) - (4^2 - 1)]$
11. 📖 $5\,[(2^2 - 1) - (2^2 - 2)]$
12. $6 \div 3 + [7 + 20 \div (6 - 2^2)]$

4 Calculate the value of each of the following :

1. $[-6 \div (-3)] \times [-30 \div (-3)]$
2. $(-10+3) \div (-8+7)$
3. $7-[10-(-8)]-3$
4. $[(11-(-10)] \times 2 \div (-6)$
5. $2 \div (-1)-(-4)^2$
6. $-6-[-2-5]^2$

5 Calculate the value of each of the following :

1. $\frac{15+7}{15-4}$
2. $\frac{8+20-4}{8-4}$
3. $\frac{-4 \times (-10)}{-9+7}$
4. $\frac{1+15}{8-(2-2)}$
5. $\frac{11-(5-4)}{1+4}$
6. $(3-1)^3+\frac{7 \times 3}{-1-6}-\frac{2 \times 15}{6}$
7. $\frac{5^2-5 \times 2}{(15+3) \div 6}$
8. $\frac{5+2 \times 5}{2^2+1}+5^2-5$
9. $\frac{3^2 \times 6 \div 3}{2 \times 1+(3+1)^2}$

6 Calculate the value of each of the following :

1. $(\frac{3}{2} \times 3\frac{1}{2}) \div (\frac{6}{5}-1)$
2. $15 \div \frac{1}{3}-\frac{3}{4} \times 10^3+27$
3. $16+4 \div 2-3 \times 10^{-2}$
4. $9 \div \frac{1}{2} \times 2-3 \div \frac{1}{5}$

7 If $x=3$, what is the numerical value of the expression : $2\left(\frac{5x+3}{4x-3}\right)$ « 4 »

8 Evaluate the expressions when $t=2$ and $s=5$:

1. $(t+s)^2$
2. $(s-t)^3$
3. $(\frac{s}{t})^3$
4. $\frac{6^2}{s-1}$
5. $\frac{s-t}{s^3}$
6. $\frac{12}{4s^2}$

9 Evaluate : $16t \div (4s)+3st$, for $t=9$ and $s=6$ « 168 »

10 If $x=4(5+6)-6$ and $y=9(36 \div 12) \div 3$,
find the value of the expression : $2x+4y$ « 112 »

11 If $x=3(5+7)-4$ and $y=4(8+2) \div 5$,
find the numerical value of the expression : $x-4y$ « zero »

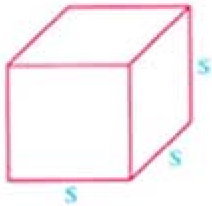
12 If $x=18-4 \times 2 \div 2+1$ and $y=8+9 \times 3-4^2+11$,
find the numerical value of the expression : $(\frac{y}{x})^{-3}$ « $\frac{1}{8}$ »

Unit 1

Geometric Applications

13 **In the opposite figure :**

The total area of a cube
is $T = 6\,s^2$, find T when :



1 $s = 3$ m.

2 $s = 0.8$ cm.

« 54 m², 3.84 cm² »

14 **In the opposite figure :**

The total area of a cuboid
is $T = 2\,(\mathcal{X}\,y + y\,z + z\,\mathcal{X})$ find T when :

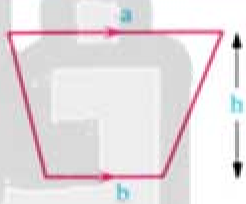
1 $\mathcal{X} = 2$ cm. , $y = 3$ cm. and $z = 5$ cm.

2 $\mathcal{X} = \frac{3}{5}$ m. , $y = 0.4$ m. and $z = \frac{1}{5}$ m.

« 62 cm², $\frac{22}{25}$ m² »

15 **In the opposite figure :**

The area of a trapezium
is $A = \frac{1}{2}\,h\,(a + b)$, find A when :



1 $h = 2$ metres , $a = \frac{3}{4}$ metre and $b = \frac{1}{4}$ metre.

2 $h = 4$ metres , $a = \frac{1}{2}$ metre and $b = \frac{1}{2}$ metre.

« 1 m², 2 m² »

For excellent pupils

16 **Put the parentheses in the place to make each of the following equalities true :**

1 $3 + 96 \div 12 \times 4 = 5$ 2 $3 + 96 \div 12 \times 4 = 35$ 3 $3 + 96 \div 12 \times 4 = 33$

Exercise 6

The square root of a perfect square rational number

From the school book

1 Find each of the following :

1. $\sqrt{16}$
2. $-\sqrt{25}$
3. $\pm\sqrt{2500}$
4. $\pm\sqrt{40000}$
5. $\sqrt{\frac{9}{49}}$
6. $-\sqrt{\frac{64}{25}}$
7. $\sqrt{0.81}$
8. $\pm\sqrt{1.44}$
9. $\sqrt{6\frac{1}{4}}$
10. $-\sqrt{1\frac{11}{25}}$
11. $-\sqrt{4^2}$
12. $\pm\sqrt{8^2}$
13. $\sqrt{(\frac{81}{100})^2}$
14. $\sqrt{(-\frac{3}{4})^2}$
15. $\pm\sqrt{\frac{576}{1225}}$
16. $-\sqrt{\frac{2.5}{40}}$
17. $-\sqrt{\frac{49\,a^4}{25\,b^6}}$
18. $\pm\sqrt{\frac{16\,b^8}{121\,h^2}}$
19. $\sqrt{\frac{49\,a^4\,b^2}{9}}$
20. $\sqrt{\frac{25\,x^2\,y^2}{36}}$

2 Find the two square roots of each of the following numbers :

1. 64
2. 144
3. $\frac{9}{25}$
4. $6\frac{1}{4}$
5. 0.25
6. 0.0049

3 Find each of the following :

1. $\sqrt{9}+\sqrt{16}$
2. $\sqrt{36+64}$
3. $\sqrt{25-9}$
4. $-\sqrt{225-81}$
5. $\sqrt{3^2+4^2}$
6. $-\sqrt{(10)^2-8^2}$
7. $\sqrt{\frac{9}{16}+1}$
8. $-\sqrt{\frac{1}{4}\left(1-\frac{3}{4}\right)}$
9. $\sqrt{\frac{5^4\times 5^3}{5^5}}$

10 $\sqrt{(\frac{1}{2})^2 \div (\frac{1}{5})^2}$ | 11 $\sqrt{(\frac{1}{2})^4 \times (\frac{1}{3})^4}$ | 12 $\sqrt{(\frac{1}{4})^2 \times (\frac{1}{4})^3}$

4 Complete the following :

1 $\frac{3}{4} \times \sqrt{\frac{16}{9}} =$ ....

2 $\sqrt{\frac{81}{49}} \times \frac{14}{27} =$ ....

3 $\sqrt{\frac{9}{4}} - \frac{3}{2} + (\frac{3}{2})^{zero} =$ ....

4 $\sqrt{36} + \sqrt{16} = \sqrt{\text{..........}}$

5 $\sqrt{(81)^2 - 81 \times 2 + 1} =$ ....

6 The multiplicative inverse of $\sqrt{\frac{4}{25}}$ in the simplest form equals ....

7 The multiplicative inverse of $\sqrt{0.49}$ in the simplest form equals ....

8 The multiplicative inverse of the rational number $\sqrt{\frac{10}{2.5}}$ equals ....

9 The additive inverse of the number $-\sqrt{\frac{9}{16}}$ in the simplest form equals ....

10 The rational number $6\frac{1}{4}$ in the form $(\frac{a}{b})^2$ is ....

11 $\sqrt{\frac{25}{64}} = \sqrt{(\frac{\text{.......}}{\text{.......}})^2} =$ ....

12 $\sqrt{(-3)^2} =$ ....

13 $\sqrt{a^4 b^8} =$ ....

14 If $a = \sqrt{\frac{1}{4}}$ and $b = 2$, then $ab =$ ....

15 If $a = -\frac{1}{2}$ and $b = -\frac{9}{8}$, then $\sqrt{ab} =$ ....

16 If $2x = \sqrt{36}$, then $x =$ ....

17 If $a = 0.000625$, then $\sqrt{a} = 2.5 \times 10^{\text{.....}}$

18 $\sqrt{(2009)^2 + 2(2009) \times 213 + (213)^2} =$ ....

5 Choose the correct answer from those given :

1 $\sqrt{1\frac{9}{16}} =$ ....

(a) $1\frac{3}{4}$ (b) $-1\frac{3}{4}$ (c) $1\frac{1}{4}$ (d) $-1\frac{1}{4}$

2 $\sqrt{10^2 - 6^2} =$ ....

(a) 4 (b) 8 (c) ± 4 (d) ± 8

3 $\sqrt{18 \times 10 \times 10 \times 18} =$ ....

(a) 18 (b) 180 (c) 10 (d) 100

4 $\sqrt{\sqrt{81}} = \dots\dots$

(a) 81 (b) 27 (c) 9 (d) 3

5 $\sqrt{2^2 + \sqrt{25}} = \dots\dots$

(a) 3 (b) -3 (c) 9 (d) -9

6 If $\frac{x}{2} = \frac{8}{x}$, then $x = \dots\dots$

(a) 4 (b) -4 (c) ± 4 (d) 16

7 If $x = \sqrt{\frac{1}{4}}$, then $x^3 = \dots\dots$

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{1}{64}$

8 $\sqrt{(a+b)^3(a+b)} = \dots\dots$

(a) $(a+b)^2$ (b) $a^4 + b^4$ (c) $-(a+b)^2$ (d) $\pm(a+b)^2$

9 $\sqrt{1}+\sqrt{4}+\sqrt{9}+\sqrt{16}+\sqrt{25}+\sqrt{36}+\sqrt{49}+\sqrt{64} = \dots\dots$

(a) 6 (b) $\sqrt{204}$ (c) $\sqrt{81}$ (d) 6^2

10 The side length of the square whose area is $16\,x^2$ cm.^{2} equals .... cm.

(a) $8\,x$ (b) $4\,x$ (c) $2\,x$ (d) $8\,x^2$

6 Simplify each of the following to the simplest form :

1 $(-\frac{1}{2})^3 \times \sqrt{\frac{64}{9}}$

2 $\sqrt{\frac{49}{4}} \times (\frac{2}{7})^{\text{zero}} \times (-\frac{2}{7})^2$

3 $\frac{2}{5} \times \sqrt{\frac{9}{16}} \div (-\frac{1}{2})^3$

4 $(-\frac{1}{3})^2 + \sqrt{\frac{64}{81}} - (\frac{3}{4})^{\text{zero}}$

5 $\frac{3}{4} \times (-\frac{2}{3})^3 \times \left(\frac{3}{\sqrt{4}}\right)^2$

6 $\sqrt{(\frac{25}{4})^2 \times (\frac{2}{5})^2}$

7 Simplify each of the following to the simplest form :

1 $\sqrt{16}+\sqrt{25}$

2 $\sqrt{\sqrt{16}+\sqrt{25}}$

3 $\sqrt{(\sqrt{16}+\sqrt{25})^2}$

8 Find two rational numbers lying between : $\sqrt{\frac{4}{9}}$ **and** $\frac{3}{4}$

9 Which is greater : $\frac{3}{5}$ or $\sqrt{\frac{4}{9}}$? Find the difference between them.

10 Which is smaller : $\sqrt{2\frac{1}{4}}$ or $(-\frac{2}{3})^2$? Find their difference.

11 Find each of the following :

1. $\sqrt{5^2 - 2 \times 5 + 1}$
2. $\sqrt{(\frac{1}{4})^2 - 2 \times \frac{1}{4} + 1}$
3. $\sqrt{20 \div 5 + 8 - (4 - 1)}$
4. $\sqrt{8 \times (5 + 11) \div (2 + 6)}$
5. $\sqrt{2 \times 8 + 10 - 3 + 12 + 11 \times 6 + 88 \div 2^3 + 99 \div 11}$
6. $\sqrt{6 + 3\sqrt{100} - \sqrt{121}}$

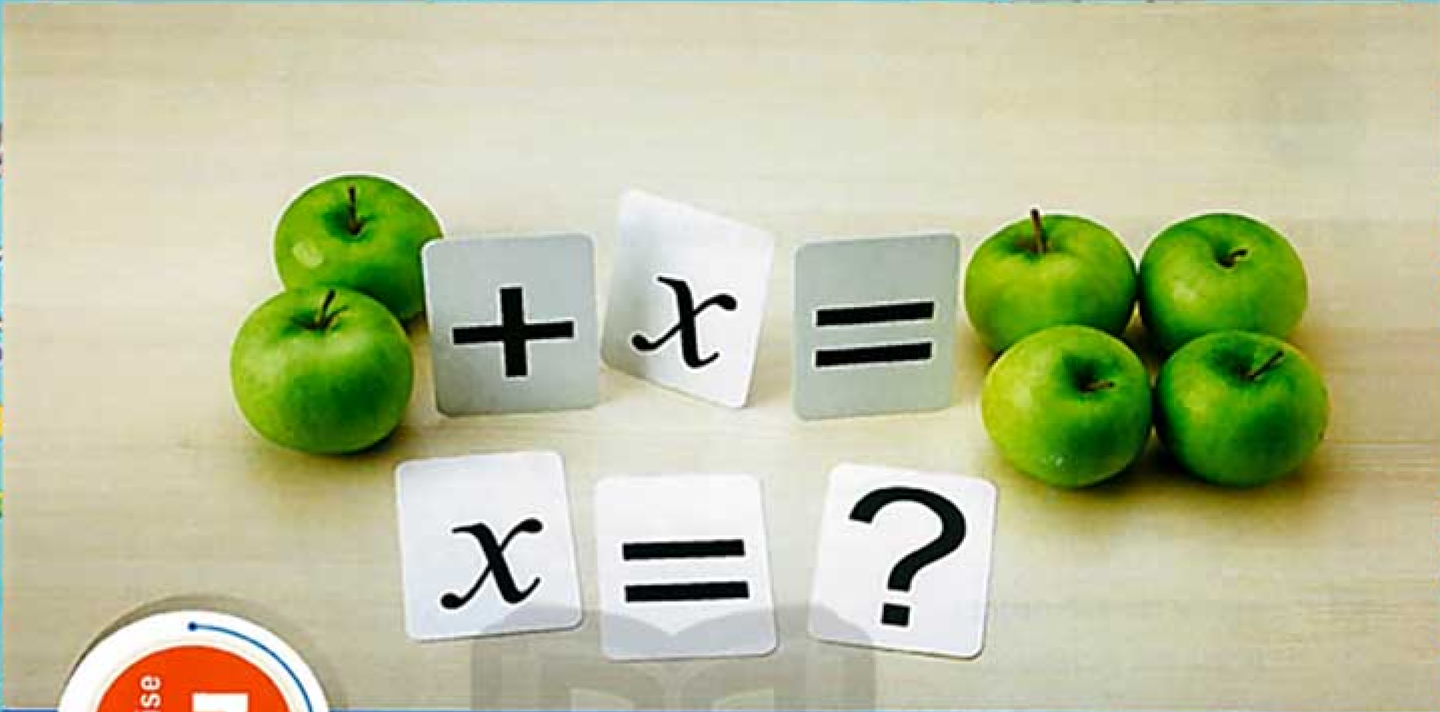
Geometric Applications

12

1. $\overline{XY}$ is a line segment where $(XY)^2 = 25$ cm.^{2} , E is the midpoint of $\overline{XY}$
   Find the length of : $\overline{XE}$ « 2.5 cm. »
2. If $(AB)^2 = 144$ cm.^{2} , $(BC)^2 = 625$ cm.^{2} and $B \in \overline{AC}$
   Find the length of : $\overline{AC}$ « 37 cm. »
3. The area of a square is 0.49 cm.^{2} Find its perimeter. « 2.8 cm. »
4. The area of a square is equal to the area of a triangle with base = 9 cm. long and its height = 8 cm. Find the side length of the square. « 6 cm. »
5. The area of a circle is 154 cm.^{2} Calculate its radius length $(\pi = \frac{22}{7})$ « 7 cm. »
6. The area of a circle is 78.5 cm.^{2} Calculate its diameter length $(\pi = 3.14)$ « 10 cm. »
7. The area of a circle is 616 cm.^{2} Calculate its circumference $(\pi = \frac{22}{7})$ « 88 cm. »
8. If three quarters of the area of a square is $1\frac{11}{64}$ m.^{2}
   Calculate the side length of the square. « $1\frac{1}{4}$ m. »
9. The length of a rectangle is twice its width and its area is 24.5 cm.^{2}
   Calculate each of its width and length. « 3.5 cm. , 7 cm. »

For excellent pupils

13 If $\frac{m}{n}$ is a rational number and $\frac{m^2}{n^2} = 0.16$, find the value of $(\frac{m}{n})^3$ « ± 0.064 »



Exercise 7

Solving equations in $\mathbb{Q}$

From the school book

1 **Find the solution set of each of the following equations :**

1. $x - 7 = 3$ where $x \in \mathbb{N}$
2. $x + 17 = 13$ where $x \in \mathbb{N}$
3. $5x = 20$ where $x \in \mathbb{Q}$
4. $\frac{2}{5}x = \frac{1}{5}$ where $x \in \mathbb{Q}$
5. $-4 + y = 13$ where $y \in \mathbb{N}$
6. $m - (-3) = 1$ where $m \in \mathbb{Z}$
7. $x - 7 = 0$ where $x \in \mathbb{Z}$
8. $y - (-5) = -3$ where $y \in \mathbb{Q}$
9. $x - 6\frac{1}{4} = 12\frac{1}{2}$ where $x \in \mathbb{Q}$
10. $8.91 + x = 11.09$ where $x \in \mathbb{Q}$

2 **Solve each of the following equations :**

1. $2x - 1 = 5$ where $x \in \mathbb{Q}$
2. $8x + 4 = 12$ where $x \in \mathbb{Q}$
3. $3x - 13 = 26$ where $x \in \mathbb{N}$
4. $2x + 14 = 14$ where $x \in \mathbb{N}$
5. $8 + 2x = 14$ where $x \in \mathbb{Z}$
6. $\frac{5}{6}x - 4 = 11$ where $x \in \mathbb{Q}$
7. $8 - 2x = -2$ where $x \in \mathbb{Z}$
8. $2 - 5x = 0$ where $x \in \mathbb{Q}$
9. $2x + 3x + 25 = 5$ where $x \in \mathbb{Z}$
10. $6x - 2x + 7 = 4$ where $x \in \mathbb{Z}$

3 **Solve each of the following equations in $\mathbb{Q}$:**

1. $2(x - 3) = 4$
2. $3x + 2(5x - 3) = 7$
3. $7(x - 2) - 3(x + 1) = 3$
4. $3(x + 2) + 7(x - 1) = 12$
5. $4(x - 1) - (x + 3) = 0$
6. $5(x - 2) + 2(x + 4) = -16$
7. $2(x - 3) + 3(x - 2) - 4x = -3$
8. $3y + 6(y + 3) - (8y - 16) = 60$

Unit 1

4 **Find in $\mathbb{Q}$ the solution set of each of the following equations :**

1 $2x + 5 = x + 9$

2 $5x - 4 = 2x + 11$

3 $x + 3 = 18 - 3x$

4 $3x + 6 = 30 - 5x$

5 $4(x + 1) = 2(x - 1)$

6 $3(x - 2) = 5x - 10$

7 $a + 5a - 2 = 2(3 - a)$

8 $3(2x - 8) - (2x + 2) = x - 3$

9 $\frac{x+1}{3} = \frac{x-1}{4}$

10 $\frac{5}{4+4x} = \frac{3}{1-2x}$

5 **Complete the following :**

1 If $x + 5 = 7$, then $x =$ ....

2 If $3t = 6$, then the value of : $6t =$ ....

3 If $2x = 5$, then the value of : $4x =$ ....

4 If $x + 9 = 11$, then the value of : $7x =$ ....

5 If $2t + 3 = 15$, then the value of : $\frac{1}{3}t =$ ....

6 If $z - 1\frac{1}{4} = 5\frac{1}{2}$, then the value of : $4z - 18 =$ ....

7 If $\frac{p}{4} = \frac{2}{3}$, then the value of : $\frac{p}{2} =$ ....

8 If the age of a man now is x years, then his age 5 years ago is ....

9 If the age of a man now is y years, then his age after 4 years is ....

10 If the age of a man after 5 years is x years, then his age now is ....

11 If the age of Youssef after 4 years is x years, then his age 2 years ago is ....

12 A rectangle with length equals triple its width. if the length = x cm. , then its width = ... cm.

13 The rectangle whose width = x cm. and its length is twice its width, then its perimeter = ... cm.

14 Two integers, their sum is 5, if one of them is x, then the other one is ....

15 Two integers, the difference between them is 2, if the small one is x, then the great one is ....

6 **Choose the correct answer from those given :**

1 If $2x = 2$, then $3x - 1 =$ ....

(a) 2 (b) 3 (c) 4 (d) 5

2 If $2x = 0$, then $x =$ ....

(a) 2 (b) 3 (c) 5 (d) zero

Exercise 7

3 If 2 a b = 10 , then 3 a b = ...

(a) 5 (b) 6 (c) 15 (d) 30

4 If 0.2 + a = 5 , then $\frac{a}{4}$ = ...

(a) 4.8 (b) 1.3 (c) 1.2 (d) 19.2

5 If $5X + 8X + 2X + 4X = 114$, then $5X + 3 =$ ....

(a) 33 (b) 35 (c) 47 (d) $8X$

6 The S.S. of the equation $\frac{2a}{3} = 8 + 4a$ in $\mathbb{Q}$ is ....

(a) $\{-2.4\}$ (b) $\{2.4\}$ (c) $\{-3\frac{1}{3}\}$ (d) $\{0\}$

7 Which of the following equations is equivalent to the equation $X + 3 = 12$?

(a) $X - 3 = -12$ (b) $X + (-3) = 12$

(c) $X - (-3) = 12$ (d) $X - (-3) = -12$

8 Which of the following equations is equivalent to the equation $X - 12 = 15$?

(a) $X + 12 = -15$ (b) $\frac{1}{3}X - 4 = 5$ (c) $X - 4 = -5$ (d) $X + 4 = 5$

Geometric Applications

7 Find the measure of each angle in each of the following triangles :

Fig. (1)

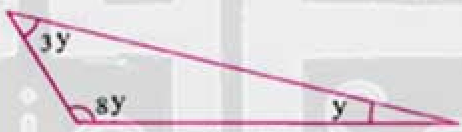
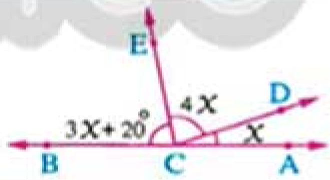


Fig. (2)

Fig. (3)

8 In the opposite figure :

If $C \in \overleftrightarrow{AB}$, find m (∠ DCE)



« 80° »

9 In each of the following figures find the value of X :

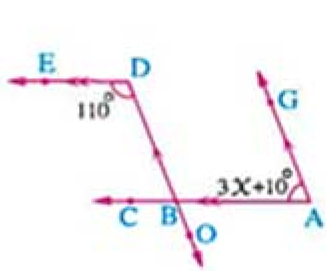


Fig. (1)

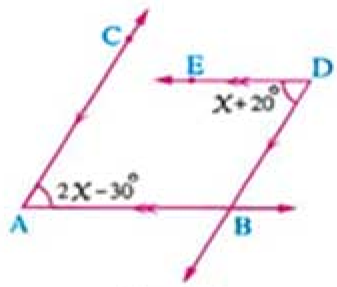


Fig. (2)

10 The length of a rectangle exceeds its width by 4 metres and its perimeter is 68 metres. Find the dimensions of the rectangle. « 19 m. , 15 m. »

11 The length of a rectangle is less than twice its width by 4 cm., if its perimeter equals the perimeter of a square of side length equals 7 cm. Find the dimensions of the rectangle. « 6 cm. , 8 cm. »

12 The length of a rectangle is twice its width. If the length decreases by 5 cm. and the width increases by 6 cm. , then the rectangle becomes a square. Find the area of the rectangle. « 242 $cm.^2$ »

Life Applications

13 Two integers , the smaller number is $2X$ and the greater number is $7X$, if the difference between them is 25 , find the two integers. « 10 , 35 »

14 Two natural numbers, one of them is twice the other and their sum is 108 Find the two numbers. « 36 , 72 »

15 The difference between two natural numbers is 5 and their sum is 21 What are the two numbers ? « 13 , 8 »

16 Find the number which if added to its triple the result is 32 « 8 »

17 Find the number which if we subtract 9 from its triple , the result will be 6 « 5 »

18 Three consecutive natural numbers their sum is 213 What are these numbers ? « 70 , 71 , 72 »

19 The sum of three consecutive even numbers is 966 , find them. « 320 , 322 , 324 »

20 Find three consecutive odd numbers if their sum is 357 « 117 , 119 , 121 »

21 A man's age now is three times his son's age and after two years , the sum of their ages will be 52 years. What is the age of each now ? « 12 years , 36 years »

22 Three brothers , Amgad , Bassim and Ayman , the sum of their ages is 89 years. If Amgad was born before Bassim by 2 years and Bassim was born before Ayman by 6 years , what is the age of each of them now ? « 25 years , 31 years , 33 years »

23 The price of one metre of wool exceeds by 2 pounds than the price of one metre of silk. If the price of 3 metres of wool and 4 metres of silk is 671 pounds. Find the price of one metre of each kind. « 97 pounds , 95 pounds »

For excellent pupils

24 **Find in $\mathbb{Q}$ the S.S. of each of the following equations :**

1. $5 - \frac{6}{x} = -1$
2. $-\frac{3}{5} + \frac{x}{10} = -\frac{1}{5} - \frac{x}{5}$

25 **Find in $\mathbb{Q}$ the S.S. of each of the following equations :**

1. $(x + 3)^2 - (x - 2)^2 = 15$
2. $(2x + 3)(2x - 1) - (2x - 1)^2 = 14$

26 If the S.S. of the equation $12x + 3 = 39$ in $\mathbb{Q}$ equals the S.S. of the equation $ax - 12 = a$ in $\mathbb{Q}$, find the value of a « 6 »

27 If a + 1 is a solution of the equation $(x + a)(x - a) = x^2 - ax + 3$ in $\mathbb{Q}$, find the value of a « 3 »

28 Three brothers were born in 1980, 1984 and 1986, the required is finding the year in which the sum of their ages became 41 years. « 1997 »

Exercise 8 Solving inequalities in $\mathbb{Q}$

From the school book

1 Which number would you add to each side of the inequality to obtain x in one side of it ?

1. $x + 5 > 9$
2. $x - 4 < 6$
3. $x - 7 < 3$
4. $x + 9 > 12$
5. $x - 1.5 \leq 3.2$
6. $4.8 \leq x + 0.6$
7. $1\frac{1}{2} > x - 2\frac{1}{2}$
8. $x + \frac{1}{3} > -\frac{1}{6}$

2 Find the solution set of the inequality $x + 3 \leq 6$ in each of the following cases :

1. $x \in \mathbb{Z}$
2. $x \in \mathbb{N}$

, then represent the solution set on the number line.

3 Find the solution set of each of the following inequalities in $\mathbb{Q}$:

1. $x + 2 > 5$
2. $x + 4 > 1$
3. $y - 5 > 7$
4. $19 < y + 14$
5. $-1 \geq x - 3$
6. $-5\frac{1}{2} > a + 1\frac{1}{4}$
7. $-2x < 12$
8. $\frac{2}{3}x \geq 1$
9. $-\frac{1}{4}x \leq \frac{1}{4}$

4 Solve each of the following inequalities in $\mathbb{Q}$:

1. $3x - 2 < 1$
2. $2x + 3 < 9$
3. $4x + 2 \geq -10$
4. $3x - 2 \geq 5$
5. $3x - 9 < 0$
6. $1 + 2x \leq -3$
7. $9 - 6x < 15$
8. $2 - 3x \leq 4$
9. $\frac{3x - 2}{5} \geq \frac{1}{2}$
10. $8x - 3x + 1 \leq 29$
11. $4n - 2(n - 1) \geq 0$
12. $-3m + 6(m - 4) > 9$

5 Solve each of the following inequalities in $\mathbb{Q}$:

1. $6d + 1 \le 5d - 3$
2. $6x + 2 \ge 14 + 5x$
3. $3x - 2 < 5x - 8$
4. $8 - 2x \le 5x$
5. $5x + 1 \ge 2(x + 2)$
6. $3(x + 2) < -x + 4$
7. $3(x + 2) \ge -2(x + 1)$
8. $2 - 3(x - 5) \ge x + 7$
9. $3(7y - \frac{1}{3}) \le 20y - 1$
10. $\frac{x}{2} + 3 \le 2x + 1$

6 Find the S.S. of each of the following inequalities :

1. $9 \le 4x + 1 \le 17 \quad , x \in \mathbb{Z}$
2. $9 \le 3x + 2 < 12 \quad , x \in \mathbb{Q}$
3. $9 > x + 6 > 2 \quad , x \in \mathbb{N}$

7 Complete :

1. If $x > y$, then $x + z$ ...... $y + z$
2. If $x < y$, then $x + z$ ...... $y + z$
3. If $x < y$ and $y < z$, then $x <$ ......
4. If $z > y$ and $y > x$, then $z >$ ......
5. If $a - 3 < 0$, then ...... $>$ ......
6. If $a + 5 > 0$, then ...... $>$ ......
7. If $b < 0$, then $b + 3$ 3
8. If $x > y$ and z is positive $(z > 0)$, then $x z$ ...... $y z$
9. If $x < y$ and z is negative $(z < 0)$, then $x z$ ...... $y z$

8 Choose the correct answer from those given :

1. If $-x < 5$, then ....
   (a) $x > 5$ (b) $x > -5$ (c) $x < 5$ (d) $x < -5$
2. If $x \in \mathbb{N}$, then the S.S. of the inequality $-x > 3$ is ....
   (a) $\{4 , 5 , ...\}$ (b) $\{-4 , -5 , ...\}$ (c) $\{-3\}$ (d) $\varnothing$
3. $\frac{x}{3} < 4$ is equivalent to ....
   (a) $x > \frac{4}{3}$ (b) $x < \frac{4}{3}$ (c) $x > 12$ (d) $x < 12$
4. If $x \in \mathbb{Z}$, then the S.S. of the inequality $20 < 5x < 25$ is ....
   (a) $\{4\}$ (b) $\{5\}$ (c) $\{4 , 5\}$ (d) $\varnothing$
5. The S.S. of the inequality $-2x <$ zero in $\mathbb{Q}$ is ....
   (a) $\varnothing$ (b) $\mathbb{Q}_+$ (c) $\mathbb{Q}_-$ (d) $\mathbb{Z}_+$
6. The number of solutions of the inequality $\frac{1}{5} < x < \frac{2}{3}$, where $x \in \mathbb{Q}$ is ....
   (a) zero (b) 1 (c) 2 (d) an infinite number.

Unit 1

7 If $x > y$, then $\frac{1}{x}$ .... $\frac{1}{y}$, where $x \neq 0$, $y \neq 0$

(a) > (b) < (c) = (d) ≥

8 The number 2 belongs to the S.S. of the inequality .... where x is an integer.

(a) $x > 2$ (b) $x < 2$ (c) $-x > -3$ (d) $-x > 3$

9 If $x > 5$, then $-x$ ....

(a) < -9 (b) ≥ -5 (c) < -5 (d) > -5

9 Show by using examples that if $a > b$ and $c > d$, then it is not always correct that $a - c > b - d$

10 **Put (✓) for the correct statement and (✗) for the incorrect statement , when a statement is false, give an example that shows why it is false (given that $x > y$) :**

1 $y < x$ ( )	2 $x > 0$ ( )	3 $y^2 \geq 0$ ( )	4 $y^2 > y$ ( )
5 $xy > 0$ ( )	6 $x + y > y$ ( )	7 $y^2 > x$ ( )	8 $y^2 < xy$ ( )
9 $xy < x^2$ ( )	10 $x^3 < y^2$ ( )		

Life Application

11 Hany wants to buy a pair of shoes and some shirts , if Hany has L.E. 200 , the price of the pair of shoes is L.E. 70 and the price of one shirt is L.E. 40

What is the greatest number of shirts Hany can buy ?

« 3 »

For excellent pupils

12 If the S.S. of the inequality $a \leq 3x - 5 \leq b$ in $\mathbb{Q}$ is $\{x : x \in \mathbb{Q} , 2 \leq x \leq 5\}$, find the values of a and b « 1 , 10 »

13 **If $-4 \leq x \leq 5$ and $2 \leq y \leq 7$, where $x \in \mathbb{Q}$ and $y \in \mathbb{Q}$, find :**

1 The greatest possible value of the expression $x + y$ « 12 »

2 The greatest possible value of the expression $y - x$ « 11 »

3 The smallest possible value of the expression xy « – 28 »

4 The smallest possible value of the expression $x^2 + y^2$ « 0 »

Summary of the second part of unit 1
"From lesson 5 to lesson 8"

- **The order of performing the mathematical operations :**

1. Perform the operations within parentheses (interior parentheses then exterior ones).
2. Evaluate the powers.
3. Perform multiplications and divisions in order from left to right.
4. Perform additions and subtractions in order from left to right.

- The square root of the perfect square rational number "a" is the number whose square equals "a"
- $\sqrt{\text{zero}} = \text{zero}$
- In the set of rational numbers it is meaningless to find $\sqrt{a}$ if a is a negative rational number because there is no rational number if it is multiplied by itself, the result will be negative.
- $\sqrt{a^2} = |a|$
- If $x^2 = a$ where $a \geq 0$, then $x = \pm\sqrt{a}$
- Solving the equation or the inequality is to find the values of the variable which satisfies the equation or the inequality.
- **Let a , b and c are three rational numbers :**
  - If $a < b$, then $a + c < b + c$
  - If $a < b$, then $a - c < b - c$
  - If $a < b$, c is a positive number , then $a\,c < b\,c$
  - If $a < b$, c is a negative number , then $\frac{a}{c} > \frac{b}{c}$
  - If $a < b$, c is a negative number , then $a\,c > b\,c$
  - If $a < b$, c is a negative number , then $\frac{a}{c} > \frac{b}{c}$
  - If $a > b$, $b > c$, then $a > c$

Exams on the second part of unit one
from lesson (5) to lesson (8)

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1. $\sqrt{10^2 - 6^2} = \cdots\cdots$
   (a) 4 (b) 8 (c) -4 (d) ± 8
2. $4 \times 2^3 - 20 = \cdots\cdots$
   (a) -48 (b) 4 (c) 12 (d) 16
3. If $2X = 4$, then $3X + 1 = \cdots\cdots$
   (a) 13 (b) 4 (c) 15 (d) 7
4. If $-X < 7$, then $\cdots\cdots$
   (a) $X > 7$ (b) $X > -7$ (c) $X < 7$ (d) $X < -7$
5. The multiplicative inverse of the number $\sqrt{\frac{9}{25}}$ is $\cdots\cdots$
   (a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{25}{9}$ (d) $\frac{9}{25}$
6. The S.S. of the equation : $3X = -9$ in $\mathbb{N}$ is $\cdots\cdots$
   (a) $\{-3\}$ (b) $\{-6\}$ (c) zero (d) $\varnothing$

2 **Complete the following :**

1. If the age of a man now is X years , then his age 9 years ago is $\cdots\cdots$ years.
2. $\sqrt{\frac{49\,a^2\,b^2}{9}} = \cdots\cdots$ (in the simplest form)
3. $144 - 8 \div 2^3 = \cdots\cdots$
4. The S.S. of the inequality : $-3X < 0$ in $\mathbb{Q}$ is $\cdots\cdots$
5. If $X + 9 = 11$, then $7X = \cdots\cdots$

3 **[a] Find the S.S. of the following inequality in $\mathbb{Q}$: $3 - 2X \geq 1$**

[b] Simplify : $\sqrt{\frac{49}{4}} \times \left(\frac{3}{11}\right)^{zero} \times \left(-\frac{2}{7}\right)^2$

4 **[a] Find :**

1. $\frac{5 + 2 \times 5}{2^2 + 1} + 5^2 - 5$
2. $2^3 \times [4 + (2 - 1)]$

[b] A rectangle in which the length is twice the width and its perimeter is 24 cm. Find the dimensions of the rectangle.

5 **[a] Find in $\mathbb{Q}$ the S.S. of the equation : $x + 3 = 18 - 3x$**

[b] Find in $\mathbb{Q}$ the S.S. of the inequality : $9 \le 3x + 2 < 12$

Model 2

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1. If $3y = 15$, then $5y =$ ....
   (a) 5 (b) 15 (c) 25 (d) 125
2. The S.S. of the inequality : $3 < x \le 4$ in $\mathbb{N}$ is ....
   (a) $\{3\}$ (b) $\{4\}$ (c) $\{3, 4\}$ (d) $\varnothing$
3. The side length of a square whose area : $9x^2$ cm^2 is ....
   (a) $3x$ (b) $3x^2$ (c) $9x$ (d) $9x^2$
4. If $x > y$, $z >$ zero , then xz .... yz
   (a) $>$ (b) $<$ (c) $=$ (d) $\le$
5. $\sqrt{9 + 16} = 3 +$ ....
   (a) 4 (b) 2 (c) 25 (d) 22
6. If $\frac{26}{x} + 1 = 14$, then $x =$ ....
   (a) 2 (b) 10 (c) 13 (d) 15

2 **Complete the following :**

1. $20 \div 5 - 2^2 =$ ....
2. If $x = 0.0009$, then $\sqrt{x} =$ ....
3. If $3x + 1 \ge 10$, then $x \ge$ ....

Unit 1

4 If $\frac{6x}{5} = -2$, then $3x =$ ....

5 An equilateral triangle of side length x cm.

, then its perimeter = ... cm.

3 **[a] Find :** $2[(5^2 + 1)] - (4^2 - 1)]$

[b] Simplify : $(\frac{-3}{7})^{zero} \times (\frac{-2}{5})^2 \times \sqrt{6\frac{1}{4}}$

4 **[a] Find in $\mathbb{Q}$ the S.S. of the equation :**

$(3x + 2) + 5 = 12$

[b] Find in $\mathbb{Q}$ the S.S. of the inequality :

$2x - 7 \le 3$

5 **[a] Find in $\mathbb{Q}$ the S.S. of the inequality :**

$6x + 2 \ge 14 + 4x$

[b] Find the number if added to its three times the result will be 28

UNIT 2 Statistics and Probability

Exercises of the unit :

9. Samples :
   - Systematic sample.
   - Random sample.
10. Probability :
   - Experimental probability.
   - Theoretical probability.

Exercise 9

Samples

From the school book

1 A factory's canteen service wanted to find the preferences of their 427 employees during their 15 – minute break. Each employee was given a number from 1 to 427

A 10% sample of the 427 were to be surveyed and asked to select a preference from :

- Hot beverage.
- Hot soup with bread.
- Cold drink with biscuits.
- Fruit with fresh water.

The sample were determined by selecting 43 sample numbers in the range using calculator. Identify the sample numbers using a calculator.

2 A school makes a study about how the pupils come to school. If the number of pupils in the school is 320 , each pupil is given a number from 1 to 320 A sample of 10% from this number is selected as a sample to ask them how they come to school :

- On foot
- Public transport
- Taxi
- Bike
- Private car

Determine the numbers of the sample using the calculator.

3 A company makes a study about the best places which the workers in the company prefer to spend their annual holiday among :

- Port Said
- Alexandria
- Matrouh
- The North Coast
- Ismailia

If the number of the workers in the company is 250 workers and a sample of 10% from the number of workers is selected to make a survey on it , determine the numbers of the sample using the calculator.

4 It is noticed that 230 persons use a public bus daily and the public transport authority wanted to collect some informations concerning with the using daily of this service. It is necessary to form a random sample representing 10% from the users of this bus to make a survey on them. Determine the numbers of this sample using the calculator.

Exercise 10

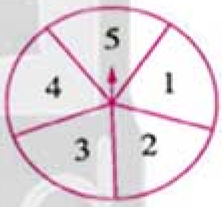
Probability

From the school book

First Problems on experimental probability :

1 In the experiment of spinning game , roll the disc 50 times. In each time , record the number at which the pointer stops in the following table :

	1	2	3	4	5	Total
The tally sign						
Frequency						50



Calculate : 1 The probability that : The pointer stops at the number 2

2 The probability that : The pointer stops at the number 5

2 (a) Draw six parallel lines with a distance of 2 cm. between each of them on an A_4 sheet of paper.

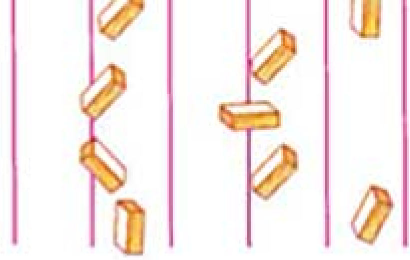
(b) Bring a piece of wood of length 2 cm.

(c) Slightly toss the piece of wood in the air so that it falls from a suitable height onto the A_4 sheet.

(d) Repeat the trial 50 times.

(e) Record the number of times that the piece of wood falls across the line and also between the lines.

(f) Deduce the probability of the piece of wood falling between the lines.

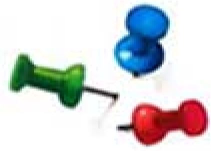


	Across	Between	Total
Tally			
Frequency			50

3 (a) Drop a drawing pin 100 times from a suitable height.

(b) Record the number of times it lands with its point up and its point down :

	Up	Down	Total
Tally			
Frequency			100



(c) Deduce the probability of the drawing pin landing point "UP" and point "Down"

4 **In the spinner game :**

The disc is divided into two unequal parts X and Y The pointer is rolled 800 times.

It stands 197 times at the X-zone

In which of the following figures the pointer points to X-zone ?

(a) (b) (c) (d)

Second Problems on theoretical probability :

1 **As throwing a fair die and observing the upper face , complete the following :**

1. The probability of appearance a number greater than 2 = ...
2. The probability of appearance a number less than 3 = ...
3. The probability of appearance an even number = ...
4. The probability of appearance the number 4 = ...
5. The probability of appearance the number 7 = ...
6. The probability of appearance a number less than or equal to 6 = ...
7. The probability of appearance a prime number = ...
8. The probability of appearance a prime even number = ...
9. The probability of appearance a number divisible by 5 = ...
10. The probability of appearance the number 5 or the number 6 = ...

2 **Complete the following :**

1. The probability of occuring the impossible event = ... and the probability of occuring the certain event = ...
2. If a coin is flipped once , then the probability of appearance of a head = ...

3 10 cards numbered from 1 to 10. If a card is drawn randomly , then the probability that the card is numbered by an odd number = ...

4 A box has 5 white balls , 7 red balls , 3 blue balls. If a ball is drawn randomly from the box , then the probability that the ball is blue = ...

5 In the experiment of throwing a fair die once and observing the upper face , the probability that the apparent number is less than 1 = ...

6 If one of the digits of the number 867742231 is selected randomly , then the probability that the selected number is even equals ....

7 A box contains 48 oranges and 4 oranges of them are bad. If an orange is drawn randomly , then the probability that the drawn orange is bad = ... and the probability that the drawn orange is good = ...

8 If the probability of occuring an event is $\frac{5}{8}$, then the probability that the event doesn't occur = ...

9 An activity room has 3 doors numbered from 1 to 3. If a student went out using one of them , then the probability that the student went out using the door number 2 is ....

10 If the probability that a person get infected (in a city whose number of inhabitants is 200000) with a disease is 0.003 , then the expected number of infected persons with the disease in this city is .... persons.

3 Choose the correct answer from those given :

1 Which of the following is the probability of occurrence of an event ?

(a) 1.2 (b) – 0.4 (c) 315% (d) 75%

2 As throwing a fair die once, the probability of appearance of a number greater than 4 is ....

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

3 A basket contains cards numbered from 1 to 20 If a card is drawn randomly , what is the probability that the number written on it is divisible by 6 ?

(a) $\frac{3}{20}$ (b) $\frac{4}{20}$ (c) $\frac{5}{20}$ (d) $\frac{6}{20}$

4 A bag has 5 red balls and 3 white balls. If the balls are similar and a person draws a ball randomly , then the probability that the drawn ball is white = ...

(a) $\frac{3}{5}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{5}{3}$

5 A letter is selected randomly from the name "ZAMALEK". The probability of selecting the letter A is ....

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

6 Rashad is in grade 7 in a class of 36 students. 16 of them are girls. If a student is selected randomly from the class , what is the probability that the student is a boy ?

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) $\frac{5}{9}$ (d) $\frac{1}{36}$

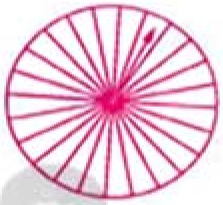
7 A class has 25 boys and 20 girls. A pupil of them is selected randomly , then the probability that the pupil is a girl = ...

(a) $\frac{1}{20}$ (b) $\frac{4}{9}$ (c) $\frac{1}{25}$ (d) $\frac{5}{9}$

8 If a die is tossed once , then the probability of getting a number satisfying the inequality $2 < X < 3$ equals ....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) zero

9 The opposite figure shows a spinner with 24 sectors.



When someone spins the arrow it is likely equal to stop on any sector. $\frac{1}{8}$ of sectors are blue , $\frac{1}{3}$ are red , $\frac{1}{2}$ are orange and $\frac{1}{24}$ are purple. If a person spins the arrow , at which colour of sectors does the arrow least likely to stop ?

(a) Blue. (b) Purple. (c) Orange. (d) Red.

10 If the probability of success of a student is 70% , then the probability of his failure = ...

(a) 0.7 (b) 0.07 (c) 0.3 (d) 0.03

4 A card is drawn from a bag of 25 cards numbered from 1 to 25
Calculate the probability that the drawn card carries :

1 A number divisible by 5
2 A number ≥ 20
3 A perfect square number.

5 One card is selected randomly from 8 cards numbered from 1 to 8
Write down the sample space. Then find the probability of each of the following events :

1 Getting an even number.
2 Getting an odd number.
3 Getting a number greater than or equal to 6
4 Getting a number divisible by 3

6 A letter is selected randomly from the word "SAMEH"
Calculate the probability of selecting the letter :

1 S 2 E 3 R

Unit 2

7 A bag contains 5 red balls , 3 yellow balls and 2 black balls. If all balls are alike and a ball is drawn from the bag randomly , find :

1 The probability that the drawn ball is yellow.
2 The probability that the drawn ball is yellow or red.
3 The probability that the drawn ball is not yellow.

8 A card is chosen randomly from ten cards numbered from 1 to 10 What is the probability that the chosen card shows :

1 An odd number.
2 A prime number.
3 An even number.
4 An odd number greater than 3

9 If a fair die is tossed once , what is the probability of each of the following events :

1 Appearance of an even number less than or equal to 4
2 Appearance of a number between zero and 10
3 Appearance of a number divisible by 7
4 Appearance of a number not divisible by 2

10 A fair die is rolled once and the number of dots on the upper face is observed. Write down the sample space , then find the probability of each of the following events :

1 Getting a number greater than 6
2 Getting a number satisfying the inequality : $1 \le X \le 6$
3 Getting a number satisfying the inequality : $2 < X < 4$

11 8 cards , numbered by the opposite numbers , are put in a bag. Bassim drew a card from these cards randomly. Find :

1 The probability that the card carries a number whose tens digit is even.
2 The probability that the card carries a number whose units digit is odd.
3 The probability that the card carries a number multiple of 4

12 A cube is designed such that each two opposite faces carry one of the digits 1 , 2 and 3 The cube is rolled and the apparent face is observed :

1 Write down the sample space.
2 What is the probability such that the number on the upper face is 2 ?
3 What is the probability such that the number on the upper face is odd ?

13 A bag contains 30 similar marbles. Hani drew a marble randomly and he found it red. If the probability of drawing a red marble = $\frac{2}{5}$, find the number of red marbles in the bag.

14 A box contains 80 similar balls. Some of them are red and the rest is blue. If the probability of drawing a red ball is $\frac{1}{4}$, find the number of blue balls.

15 The set $\{2 , 3 , 5\}$ is used in writing a 2-digit number. Find the probability of each of the following events :

1 The tens digit is odd.
2 The units digit is odd.
3 The sum of the two digits is 7
4 The product of the two digits is 15

16 Wael has a bag containing 22 marbles , 12 of them are black and the rest is red. If two marbles of them are drawn without returning them to the bag and they were red. Then he drew a third marble without looking at it. What is the probability that the last marble is black ?

17 A class has 50 students. The number of girls is less than the number of boys by 10 If one student is chosen randomly , find the probability that the student is a boy.

18 **Choose the correct answer from those given :**

1 A bowl contains 32 coloured beads. All beads are of the same size. Some of them are blue , some are green , some are red and the rest is yellow. The probability of drawing a blue bead is $\frac{3}{8}$, how many blue beads are in the bowl ?

(a) 4 (b) 8 (c) 12 (d) 16

2 A bag contains 3 white balls , 2 black and one red. If a ball is drawn randomly from the bag , then the probability that the drawn ball is not black equals ....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$

3 A bag has a number of similar balls. Half of them is red , the third is black and the rest is white. A ball is drawn randomly , then the probability that the drawn ball is white = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) zero

4 A box contains coloured balls (red , green , blue and yellow). If the box contains 20 yellow balls and the probability of drawing a yellow ball from the box randomly is $\frac{1}{4}$. What is the number of all balls in the box ?

(a) 5 (b) 25 (c) 60 (d) 80

5 The number of pupils in a class (7 grade) is 36 pupils. The probability of choosing a pupil whose age is less than or equal to 13 is $\frac{1}{6}$. What is the number of pupils whose ages are more than 13 years old ?

(a) 23 (b) 24 (c) 30 (d) 32

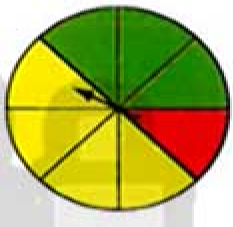
6 In a mixed school , if the ratio between the number of boys to the number of girls is 7 : 9 A student is selected randomly from the students of this school.
The probability that the selected student is a boy equals ....

(a) zero (b) $\frac{7}{16}$ (c) $\frac{9}{16}$ (d) 7

7 A small box contains 25 tickets numbered from 1 to 25 A large box contains 50 tickets numbered from 1 to 50 Without looking at them , a ticket is picked from one of the two boxes. Which box would give the larger chance of picking a ticket with the number 17 ?

(a) The larger box. (b) The smaller box.
(c) Both would give the same chance. (d) The given information is not enough.

19 **The opposite figure represents the spinning game. Find :**



1 The probability that the pointer stops at :
(a) Red colour.
(b) Green colour.
(c) Yellow colour.

2 The probability that the pointer does not stop at the red colour.

20 The opposite spinning game is divided into 8 sectors of the same area. $\frac{1}{8}$ of the sectors is coloured in red , and $\frac{1}{4}$ of the sectors is coloured in green , $\frac{3}{8}$ of the sectors is coloured in blue and the rest in yellow.
If the pointer of the spinner is spinned , what is the probability that the pointer stops at the yellow or the red colour ?

21 A class contains 40 students , 30 of them succeeded in maths , 24 succeeded in science and 20 succeeded in both.
A student is chosen randomly.
Find the probability that this student :

1 Succeeded in maths.
2 Succeeded in science.
3 Failed in science.
4 Failed in both maths and science.

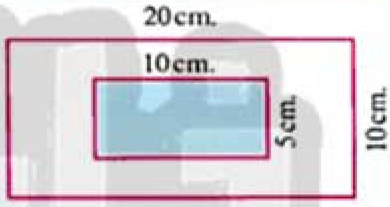
Exercise 10

22 Two players play in a football team.
During training , one of them shot 21 penalty kicks and he scored 18 goals and the other shot 32 penalty kicks and he scored 25 goals. Which of them should you choose to shoot a penalty kick ? Why ?

23 Maryam and Souad played together with two dice.
If the product of the two apparent numbers on the upper face is even , then Souad wins the game.
If the product of those numbers is odd , then Maryam wins :

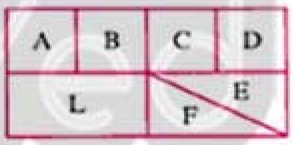
1. On your opinion , is this system of the game fair ? Why ?
2. If it is not fair , determine which one of the two girls has the greater chance to win ? Why ?

24 **In the opposite figure :**
If a person shot towards the drawn board , find the probability of shooting the shaded part.



For excellent pupils

25 In the game of the target and the arrow , the target was in the shape of a rectangle divided into parts as shown in the opposite figure :



1. Find the probability of shooting the part E
2. Find the probability of shooting the part formed from A , B and C together.

26 A bag contains a number of similar balls , 5 of them are white and the rest are red.
If the probability of drawing a red ball is $\frac{2}{3}$
Find the total number of balls.

27 A card is drawn from a group of cards numbered from 1 to n.
If the probability that the drawn card carries a number greater than 8 is $\frac{1}{3}$, then find the value of n

28 When you throw a regular die two successive times and notice the upper face.
Find the probability of appearance of the number 3 in the two times.

Summary of unit 2

- The sample is a small part from a large society that looks like this society and represents it well and is selected randomly.

  1. **Systematic sample :**

     It is a sample whose elements are selected from the elements of a society distributed randomly by following a certain system or method in selection.

  2. **Random sample :**

     It is a sample whose elements are selected from the elements of a society distributed randomly by following a random and irregular method or system of selecting.

- The experimental probability : depends on performing an experiment , then we record the results and use them to calculate the value of probability of an event occurrence using the rule :

  $$\text{Experimental probability} = \frac{\text{Number of trials in which the outcome occurs}}{\text{Total number of trials}}$$

- **Random experiment :**

  It is an experiment in which we can specify all its possible outcomes before carrying it out but we cannot determine certainly which of them will occur.

- **Sample space :**

  It is the set of all possible outcomes of a random experiment and it is denoted by S

- **Event :** The event is a subset of the sample space.

- The probability of any event occurrence $A \subset S$ is denoted by P (A) and it is given by using the relation :

  $$P(A) = \frac{\text{The number of elements of the event «A»}}{\text{The number of elements of sample space «S»}} = \frac{n(A)}{n(S)}$$

- The probability of the impossible event = zero

- The probability of the certain event = 1

- $0 \leq$ The probability of an event occurrence ≤ 1

Exam on unit two

Answer the following questions :

1 Choose the correct answer from those given :

1 In the experiment of throwing a fair die once , the probability of appearing a number greater than 4 is ....

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

2 Which of the following is the probability of occurrence of an event ?

(a) 1.25 (b) – 0.3 (c) 215 % (d) 23 %

3 A class has 15 boys , 20 girls. A pupil is selected randomly , then the probability that the pupil is a girl = ...

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

4 If the probability of success of a student is 75% , then the probability of his failure = ...

(a) 0.75 % (b) 0.25 (c) 75 (d) 25

5 As throwing a fair die once , then the probability of having a number satisfies the inequality : $3 < X < 6$ is ....

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

6 If a letter is selected randomly from the word "NOVEMBER" , then the probability of selecting the letter E is ....

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{4}$ (d) 1

2 Complete the following :

1 The probability of the impossible event = ...

2 If a coin is flipped once , then the probability of appearance of a tail = ...

3 If the probability of occurrence of an event is $\frac{3}{7}$, then the probability of non-occurrence of this event = ...

4 A garden has 5 doors numbered from 1 to 5. A man got out from one of these doors , then the probability that he got out of door number 5 is ....

5 As throwing a fair die once , then the probability of appearing a number divisible by 5 = ...

3 A bag contains 5 red balls , 7 yellow balls and 3 black balls. If all balls are alike and a ball is drawn randomly from the bag , **find :**

1 The probability that the drawn ball is red.

2 The probability that the drawn ball is not yellow.

3 The probability that the drawn ball is not green.

4 The probability that the drawn ball is black or yellow.

4 **If a fair die is rolled once , then find the probability of appearing :**

1 A prime number.

2 An even prime number.

3 A number less than 6

4 A number greater than 6

5 A number less than 10

5 From the set of numbers $\{1 , 3 , 4\}$ a 2-digit number is formed.

What is the probability of each of the following events :

1 The event of the tens digit is odd.

2 The event of the units digit is even.

3 The event of the sum of the two numbers is 5

TIMSS Problems

Accumulative basic skills

1 **Choose the correct answer from the given ones :**

1. $3x^2 + 2x + 2x^2 - x =$ ....
   (a) $6x$ (b) $6x^2$ (c) $5x^2 + x$ (d) $7x^2 - x$

2. If $y = \frac{a+b}{c}$, $a = 8$, $b = -6$ and $c = -2$, then $y =$ ....
   (a) -1 (b) 1   (c) -7 (d) 7

3. At dividing 113 + 113 + 113 + 113 by 4 , then the remainder = ...
   (a) zero   (b) 1   (c) 4   (d) 13

4. $4(3 + x) =$ ....
   (a) $12 + x$ (b) $7 + x$ (c) $12 + 4x$ (d) $12x$

5. $\frac{4}{10} + \frac{3}{100} =$ ....
   (a) 0.34   (b) 0.43   (c) 4.3   (d) 3.4

6. If three times of a number equals 27 , then $\frac{1}{9}$ of this number is ....
   (a) 1   (b) 3   (c) 9   (d) 27

7. Which of the following is equal to $\frac{3}{5}$?
   (a) 6 %   (b) 60 %   (c) $\frac{9}{10}$ (d) 0.53

8. If the fractions $\frac{4}{14}$ and $\frac{x}{21}$ are equal, then $x =$ ....
   (a) 6   (b) 7   (c) 11   (d) 14

9. $2 \times 4 \times 6 \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{6} =$ ....
   (a) 48   (b) 2304   (c) 1   (d) zero

10. A worker cut a part of a pipe equals $\frac{1}{5}$ of this pipe, if the length of the cut part equals 3m., then the length of the pipe completely = ...
    (a) 8 m.   (b) 12 m.   (c) 15 m.   (d) 18 m.

Basic skills

11 Which of the following does express the number 36 as the product of its prime factors ?

(a) 6×6 (b) 4×9 (c) $4 \times 3 \times 3$ (d) $2 \times 2 \times 3 \times 3$

12 $5 \times 4 \times 3 \times 2 \times 1 \times 0 =$ ....

(a) 120 (b) 60 (c) 20 (d) zero

13 Double of the square of the number (half) is ....

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) 2

14 If the number of boys in one of the parties is m and the number of girls is n , if each person has 2 balloons, then which of the following expression does express the number of balloons in this party ?

(a) $2(m + n)$ (b) $2 + (m + n)$ (c) $2m + n$ (d) $m + 2n$

15 The smallest number of the following numbers is ....

(a) 0.52 (b) 0.5 (c) 0.056 (d) 0.562

2 Complete the following :

1 $24.65 + 5.748 =$ ....

2 $-2\frac{3}{4} \div 1\frac{3}{8} =$ ....

3 Third of the third = ...

4 $\frac{(19)^2 - 9 \times 19 + 19}{19} =$ ....

5 $\frac{3x}{8} + \frac{x}{4} + \frac{x}{2} =$ .... (in the simplest form)

6 If $y = 100 - \frac{100}{1 + m}$, at $m = 9$, then $y =$ ....

7 If $a + b = 25$, then $2a + 2b =$ ....

8 $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, ... (in the same pattern)

9 If $3y = 6$, then $5y =$ ....

10 If $\frac{1}{2}x = 5y = 10$, then $xy =$ ....

11 If $x + y = yx = 5$, then $x^2y + y^2x =$ ....

12 If $x - y = 3$, $x + y = 5$, then $x^2 - y^2 =$ ....

13 If $x^2 = 16$, $y^2 = 9$, $xy = 12$, then $(x - y)^2 =$ ....

14 If the degree of the algebraic term $5x^ny^2$ is 5 , then $n =$ ....

15 A piece of wood of length 40 cm., it is cut into three parts of lengths $2x - 5$, $x + 7$ and $x + 6$ in centimetres, then the length of the longest part = ... cm.

Second Geometry and Measurement

UNIT 3
Geometry and Measurement

Exercises of the unit :

1. Deductive proof.
2. The polygon.
3. The parallelogram and its properties.
4. The special cases of the parallelogram.
5. The triangle : Theorem (1) , exterior angle of the triangle.
6. Theorem (2) , theorem (3).
7. Pythagoras' theorem.
8. Geometric transformations.
9. Reflection in a straight line.
10. Reflection in a point.
11. Translation.
12. Rotation.

Exercise 1 Deductive proof

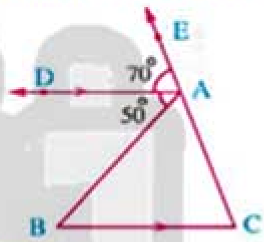
From the school book

1 **In the opposite figure :**

$\overrightarrow{AD} \parallel \overline{BC}$, m (∠ DAB) = 50° and m (∠ DAE) = 70°

Find the measures of the angles of Δ ABC

Complete the following table by writting the reason of each step of the solution steps :

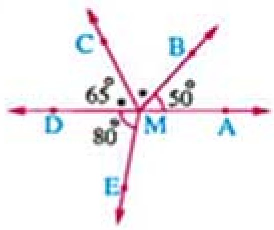


Mathematical Statement	The reason
m (∠ DAB) = 50° , m (∠ DAE) = 70°	1 ....
m (∠ CAB) = 180° − (50° + 70°) = 60°	2 ....
$\overrightarrow{AD} \parallel \overline{BC}$	3 ....
m (∠ C) = m (∠ DAE) = 70°	4 ....
m (∠ B) = m (∠ DAB) = 50°	5 ....

2 **In the opposite figure :**

m (∠ AMB) = 50° , m (∠ EMD) = 80° , $\overrightarrow{MC}$ bisects ∠ BMD and m (∠ CMD) = 65°

Complete the following proof to find m (∠ AME)



Given ....

R.T.F. ....

Proof ∵ $\overrightarrow{MC}$ bisects ∠ .... (given)

∴ m (∠ BMC) = m (∠ ....) = ...°

∵ m (∠ AMB) + m (∠ BMC) + m (∠ CMD) + m (∠ DME) + m (∠ AME) = ...°

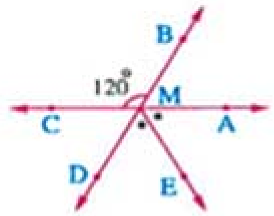
∴ m (∠ AME) = ...° − ....° = ...° (The req.)

Unit 3

3 **In the opposite figure :**

$\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$, m (∠ BMC) = 120°

and $\overrightarrow{ME}$ bisects ∠ AMD



Complete the following solution steps to find m (∠ EMC)

Given ....

R.T.F. ....

Proof

∵ $\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$

∴ m (∠ BMC) = m (∠ ....) (V.O.A.)

∴ m (∠ ....) = 120°

∵ $\overrightarrow{ME}$ bisects ∠ AMD

∴ m (∠ ....) = m (∠ ....)

∴ m (∠ EMD) = $\frac{\text{..........}}{\text{..........}}$ = ...°

∵ M ∈ $\overleftrightarrow{BD}$

∴ m (∠ BMC) + m (∠ ....) = 180°

∴ m (∠ DMC) = ...° − ....° = ...°

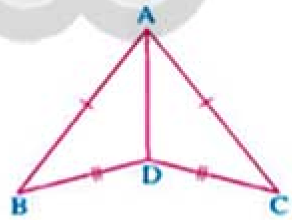
∵ m (∠ EMC) = m (∠ ....) + m (∠ ....)

∴ m (∠ EMC) = ...° + ...° = ...° (The req.)

4 **In the opposite figure :**

AB = AC , BD = CD

Complete the following proof to prove that $\overrightarrow{AD}$ bisects ∠ BAC



Given ....

R.T.P. ....

Proof

∵ In Δ Δ ADB , ... :

{ AB = ... (given)
.... = CD (given)
$\overline{AD}$ ....

∴ Δ ADB ≡ Δ .... , then we deduce that :

m (∠ ....) = m (∠ ....)

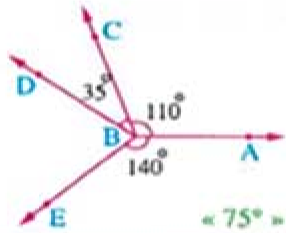
∴ $\overrightarrow{AD}$ bisects ∠ .... (Q.E.D.)

5 In the opposite figure :

m (∠ ABC) = 110° , m (∠ CBD) = 35°

and m (∠ ABE) = 140°

Find : m (∠ EBD)



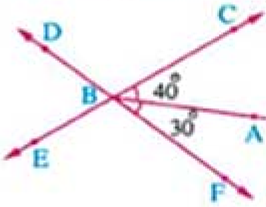
« 75° »

6 In the opposite figure :

$\overleftrightarrow{CE} \cap \overleftrightarrow{FD} = \{B\}$,

m (∠ ABC) = 40° and m (∠ ABF) = 30°

Find : m (∠ DBC)



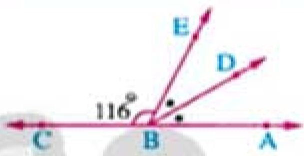
« 110° »

7 In the opposite figure :

$B \in \overleftrightarrow{AC}$, m (∠ CBE) = 116°

and $\overrightarrow{BD}$ bisects ∠ ABE

Find : m (∠ ABD)



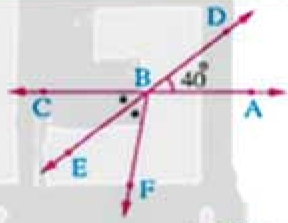
« 32° »

8 In the opposite figure :

$\overleftrightarrow{AC} \cap \overleftrightarrow{DE} = \{B\}$, m (∠ ABD) = 40°

and $\overrightarrow{BE}$ bisects ∠ CBF

Find : m (∠ ABF)



« 100° »

9 In the opposite figure :

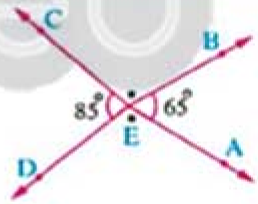
$\overrightarrow{EA} \cap \overrightarrow{EB} \cap \overrightarrow{EC} \cap \overrightarrow{ED} = \{E\}$,

If m (∠ BEC) = m (∠ AED)

, m (∠ AEB) = 65° , m (∠ CED) = 85°

Find : m (∠ BEC)

Are A , E , and C on the same straight line ? Why ?

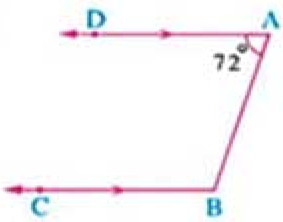


« 105° »

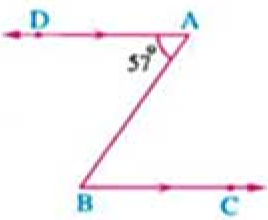
10 In each of the following figures ,

If $\overrightarrow{AD} \parallel \overrightarrow{BC}$, **Find :** m (∠ ABC) , giving reason.

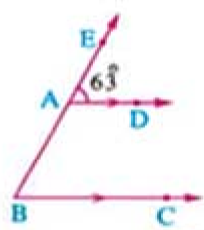
1



2



3



Unit 3

11 **In the opposite figure :**

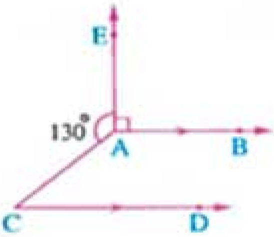
$\overrightarrow{AB} \parallel \overrightarrow{CD}$

, m (∠ EAC) = 130°

and m (∠ EAB) = 90°

Find : 1 m (∠ BAC)

2 m (∠ C)



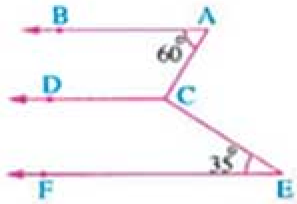
« 140° , 40° »

12 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{AB} \parallel \overrightarrow{EF}$

, m (∠ A) = 60° and m (∠ E) = 35°

Find : m (∠ ACE)



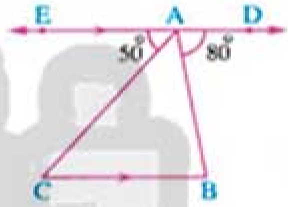
« 95° »

13 **In the opposite figure :**

$\overleftrightarrow{DE} \parallel \overline{BC}$, A ∈ $\overleftrightarrow{DE}$, m (∠ DAB) = 80°

and m (∠ EAC) = 50°

Find the measures of the angles of Δ ABC



« m (∠ BAC) = 50° , m (∠ B) = 80° , m (∠ C) = 50° »

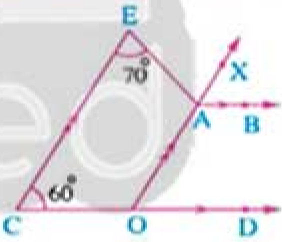
14 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{OD}$, $\overrightarrow{OA} \parallel \overrightarrow{CE}$, X ∈ $\overrightarrow{OA}$

, O ∈ $\overline{CD}$, m (∠ E) = 70°

and m (∠ C) = 60°

Find : m (∠ BAE)



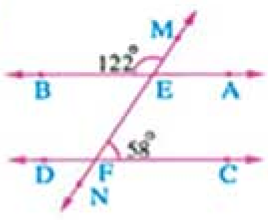
« 130° »

15 **In each of the following figures ,**

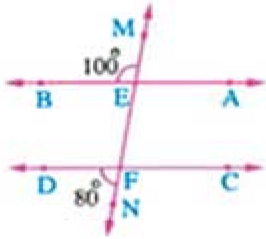
If $\overleftrightarrow{MN}$ intersects $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ at E and F respectively ,

Prove that : $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

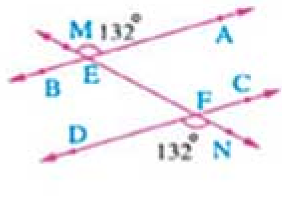
1



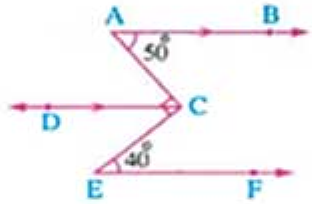
2



3



16 **In the opposite figure :**

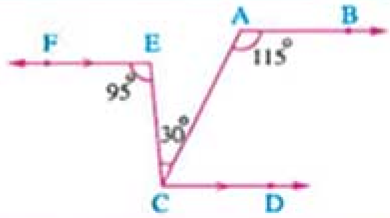


$\overrightarrow{AB} // \overrightarrow{CD}$, m (∠ A) = 50° ,

∠ ACE is right and m (∠ E) = 40°

Prove that : $\overrightarrow{AB} // \overrightarrow{EF}$

17 **In the opposite figure :**

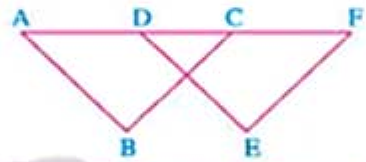


$\overrightarrow{EF} // \overrightarrow{CD}$, m (∠ CEF) = 95° ,

m (∠ ACE) = 30° , m (∠ BAC) = 115°

Prove that : $\overrightarrow{AB} // \overrightarrow{EF}$

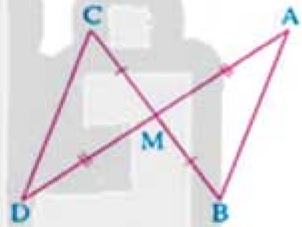
18 **In the opposite figure :**



The two triangles are congruent

Prove that : $\overline{BC} // \overline{EF}$

19 **In the opposite figure :**



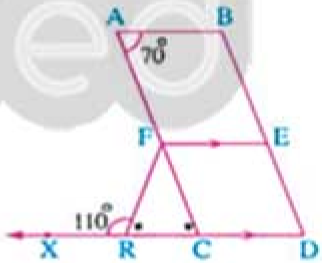
$\overline{AD} \cap \overline{BC} = \{M\}$,

MA = MD and MB = MC

Prove that :

1. AB = CD
2. $\overline{AB} // \overline{CD}$

20 **In the opposite figure :**



$\overline{FE} // \overline{CD}$, m (∠ FRC) = m (∠ FCR),

m (∠ A) = 70° and m (∠ FRX) = 110°

1. **Prove that :** $\overline{CD} // \overline{AB}$
2. **Find in three different ways :** m (∠ AFE)

« 110° »

21 **Prove that :**

1. A straight line which is perpendicular to one of two parallel lines is also perpendicular to the other.
2. A straight line that is parallel to one of two parallel lines is also parallel to the other.

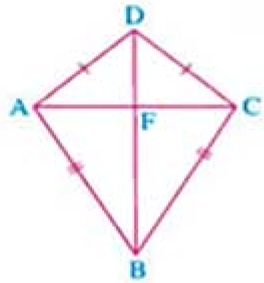
Unit 3

22 **In the opposite figure :**

AD = CD and AB = BC

Use the properties of congruent triangles to show that :

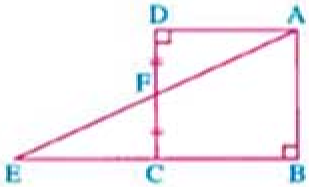
1. $\overrightarrow{DB}$ bisects ∠ ADC
2. $\overline{AC}$ and $\overline{DB}$ are perpendicular to each other.



23 **In the opposite figure :**

ABCD is a square in which F is the midpoint of $\overline{CD}$ and $\overrightarrow{AF} \cap \overrightarrow{BC} = \{E\}$

Prove that : CE = CB

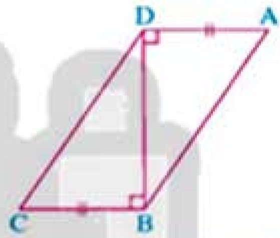


24 **In the opposite figure :**

AD = BC and m (∠ ADB) = m (∠ DBC) = 90°

Prove that :

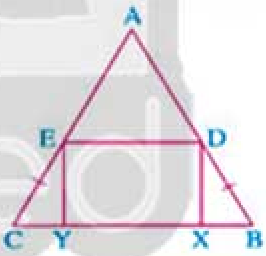
1. AB = CD
2. $\overline{AB}$ // $\overline{CD}$



25 **In the opposite figure :**

EC = DB and DXYE is a rectangle.

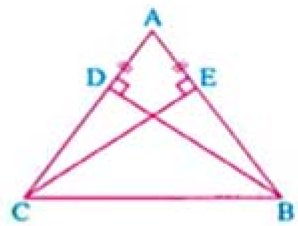
Prove that : m (∠ ADE) = m (∠ AED)



26 **In the opposite figure :**

$\overline{BD} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$ and AD = AE

Prove that : BD = CE

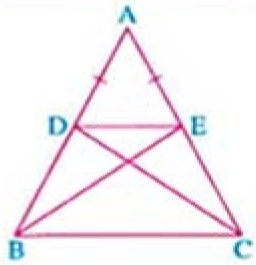


27 **In the opposite figure :**

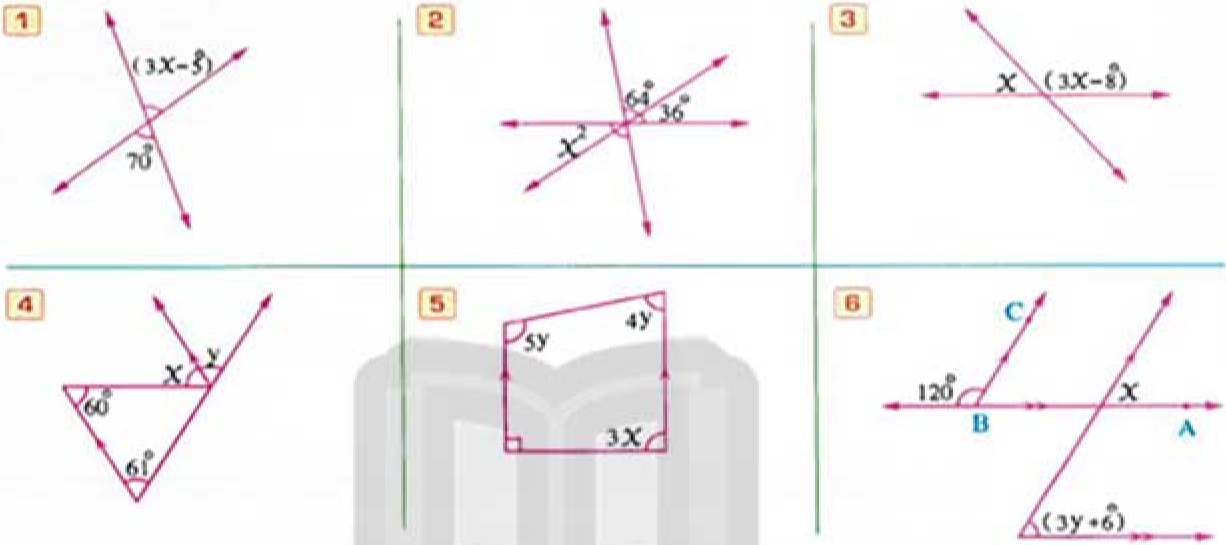
AD = AE and m (∠ ADC) = m (∠ AEB)

Show that :

1. BE = CD
2. BD = CE



28 Find the values of x and y in each of the following :



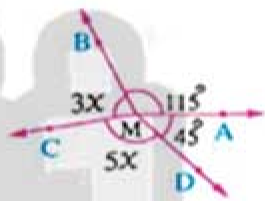
29 In the opposite figure :

m (∠ AMB) = 115° , m (∠ AMD) = 45° ,

m (∠ BMC) = 3 x and m (∠ CMD) = 5 x

What is the value of x ?

Are A , M and C collinear ? Why ?



« 25° »

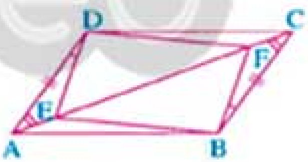
For excellent pupils

30 In the opposite figure :

1 Is Δ ADE congruent to Δ CBF ? Give your reason(s).

2 **Prove that :**

First : Δ DEF ≡ Δ BFE **Second :** Δ ABE ≡ Δ CDF

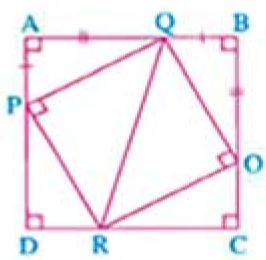


31 In the opposite figure :

1 Is Δ PAQ congruent to Δ QBO ? Give your reason (s).

2 **Show that :**

First : Δ PQR ≡ Δ OQR **Second :** Δ PDR ≡ Δ RCO



Exercise 2

The polygon

From the school book

1 Complete the following :

1. The regular polygon is the one in which :
   (a) .... (b) ....
2. The sum of measures of the interior angles of the quadrilateral = ... °
3. The sum of measures of the interior angles of the pentagon = ...°
4. The sum of measures of the interior angles of the hexagon = ...°
5. The sum of measures of the interior angles of the heptagon = ...°
6. The measure of the interior angle of the regular pentagon = ... °
   and the measure of the interior angle of the regular heptagon = ... °
7. The sum of measures of the exterior angles of the hexagon equals .... °
8. If the perimeter of a regular hexagon is 30 cm. , then its side length = ... cm.
   and the measure of each interior angle in it = ... °
9. If the perimeter of a regular polygon = 80 cm. and its side length = 10 cm. ,
   then the measure of each interior angle in it = ... °

2 Choose the correct answer from those given :

1. The sum of measures of the interior angles of a polygon of n sides = ...
   (a) $n \times 180°$ (b) $(n-2) \times 180°$ (c) $\frac{(n-2) \times 180°}{2}$ (d) $\frac{(n-2) \times 180°}{2\,n}$
2. The measure of the interior angle of a regular polygon of n sides equals ....
   (a) $\frac{(n-2) \times 90°}{n}$ (b) $\frac{(n-2) \times 180°}{2}$ (c) $\frac{(n-2) \times 180°}{n}$ (d) $180° \times (n-1)$
3. The measure of the interior angle of the regular polygon of 10 sides equals ....
   (a) 72° (b) 108° (c) 144° (d) 150°

4 The measure of the interior angle of a regular polygon of 18 sides equals ....
(a) 130° (b) 140° (c) 150° (d) 160°

5 If the measure of an interior angle of a regular polygon is 135° , then the number of its sides is ....
(a) 6 (b) 4 (c) 7 (d) 8

6 The sum of measures of the exterior angles of the triangle equals ....
(a) 90° (b) 180° (c) 360° (d) 720°

7 In the quadrilateral ABCD , if m (∠ A) = 2 m (∠ B) = m (∠ C) = 96° , then m (∠ D) = ...
(a) 96° (b) 48° (c) 120° (d) 144°

3 Find the number of the diagonals of each of the following figures :

1 Triangle.
2 Quadrilateral.
3 Pentagon.

Hint : The number of diagonals of the polygon of n sides $= \frac{n(n-3)}{2}$

4 In each of the following , find the measure of the angle marked by (?) :

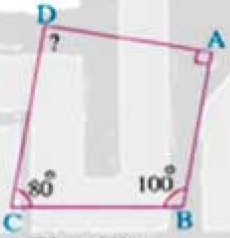


Fig. (1)

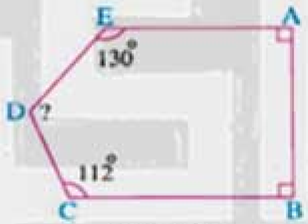
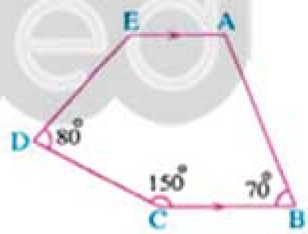


Fig. (2)

« 90° , 118° »

5 In the opposite figure :



$\overline{AE} \parallel \overline{BC}$, m (∠ B) = 70° , m (∠ C) = 150° and m (∠ D) = 80°

Complete the following proof to find m (∠ E)

Given ....

R.T.F. ....

Proof ∵ $\overline{AE} \parallel \overline{BC}$ and $\overleftrightarrow{\text{..........}}$ is a transversal to them.

∴ m (∠ A) + m (∠ ....) = 180°

(Two interior angles in the same side of the transversal)

∵ m (∠ ....) = 70°

∴ m (∠ A) = ... − 70° = ...°

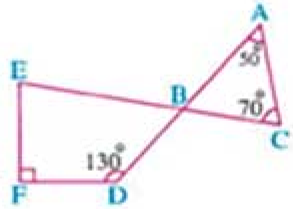
∵ ABCDE is a pentagon.

∴ The sum of measures of its interior angles

= (5 − ....) × .... = ... °

∴ m (∠ E) = ... − (70° + 150° + 80° + 110°) = ... ° (The req.)

6 **In the opposite figure :**



$\overline{CE} \cap \overline{AD} = \{B\}$, m ($\angle$ A) = 50°

, m ($\angle$ C) = 70° , m ($\angle$ D) = 130° and

m ($\angle$ F) = 90°

Complete the following proof to find m ($\angle$ E)

Given ....

R.T.F. ....

Proof $\because$ The sum of measures of the interior angles of the triangle = ...°

$\therefore$ In Δ ABC : m ($\angle$ ABC) = 180° – (.......... + ...) = ...°

$\because$ $\overline{CE} \cap \overline{AD} = \{B\}$

$\therefore$ m ($\angle$ ....) = m ($\angle$ ....) = ...° (V.O.A.)

$\because$ The sum of measures of the interior angles of the quadrilateral = ...°

$\therefore$ m ($\angle$ E) = ... – (.......... + ... + ...) = ...° (The req.)

7 **In the opposite figure :**

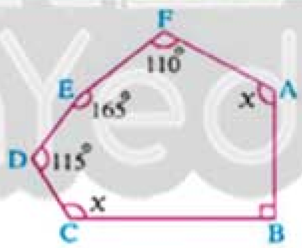
F $\in \overrightarrow{YZ}$, m ($\angle$ L) = 70° ,

m ($\angle$ Y) = 90° and m ($\angle$ LZF) = 120°

Find : m ($\angle$ X)

« 140° »

8 **In the opposite figure :**



ABCDEF is a hexagon.

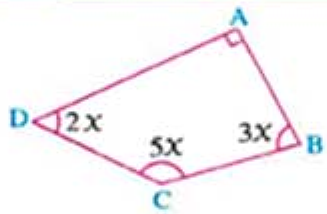
m ($\angle$ FAB) = m ($\angle$ DCB)

Find :

The value of x

« 120° »

9 **In the opposite figure :**



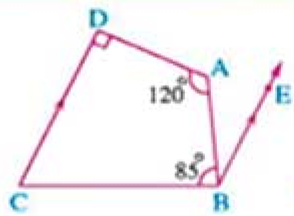
ABCD is a quadrilateral

in which : m ($\angle$ A) = 90°

Find : The value of x

« 27° »

10 **In the opposite figure :**



m ($\angle$ A) = 120° , m ($\angle$ D) = 90° ,

m ($\angle$ ABC) = 85° and $\overrightarrow{BE}$ // $\overline{CD}$

Find : m ($\angle$ ABE)

« 30° »

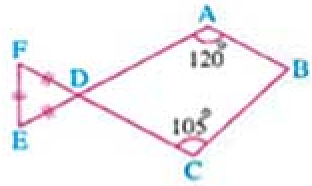
11 **In the opposite figure :**

$\overline{AE} \cap \overline{CF} = \{D\}$,

Δ DEF is an equilateral triangle ,

m ($\angle$ A) = 120° and m ($\angle$ C) = 105°

Find : m ($\angle$ B)



« 75° »

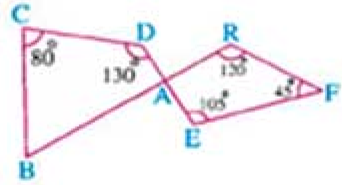
12 **In the opposite figure :**

$\overline{ED} \cap \overline{RB} = \{A\}$, m ($\angle$ F) = 45° ,

m ($\angle$ R) = 120° , m ($\angle$ E) = 105° ,

m ($\angle$ D) = 130° and m ($\angle$ C) = 80°

Find : m ($\angle$ B)



« 60° »

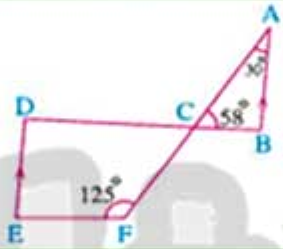
13 **In the opposite figure :**

$\overline{BD} \cap \overline{AF} = \{C\}$, $\overline{AB}$ // $\overline{ED}$,

m ($\angle$ A) = 30° and m ($\angle$ ACB) = 58° ,

m ($\angle$ CFE) = 125°

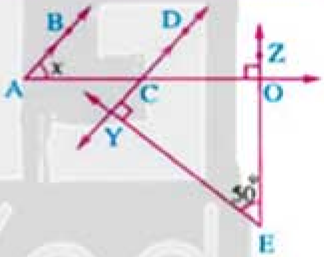
Find : m ($\angle$ E)



« 85° »

14 **In the opposite figure :**

Find : The value of x



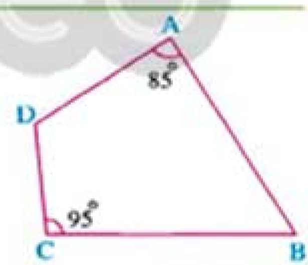
« 50° »

15 **In the opposite figure :**

m ($\angle$ A) = 85° , m ($\angle$ C) = 95°

and m ($\angle$ B) = $\frac{1}{2}$ m ($\angle$ D)

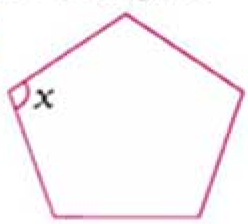
Find the measure of each of them.



« 60° , 120° »

16 **In each of the following , if the polygon is regular , find the measures of the unknown angles :**

1



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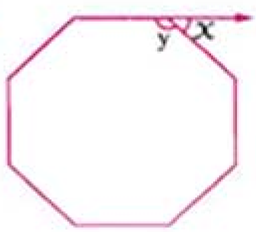
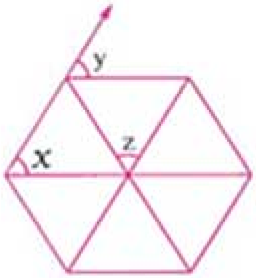


Unit 3

3


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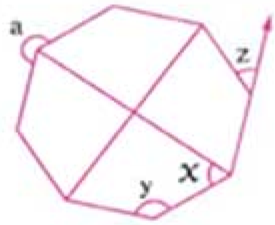
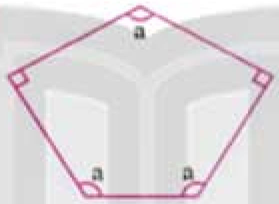
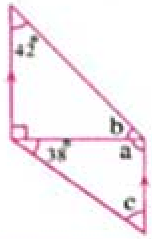
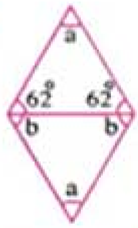


17 In each of the following , find the values of the unknown symbols :

1


2


3


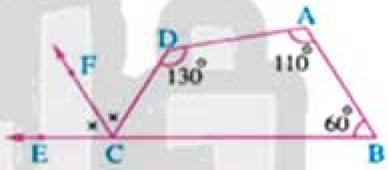
4


18 **In the opposite figure :**

m (∠ A) = 110° , m (∠ B) = 60° ,

m (∠ D) = 130° , $\overrightarrow{CF}$ bisects ∠ DCE and C ∈ $\overrightarrow{BE}$

Prove that : $\overrightarrow{CF} \,/\!/\, \overline{AB}$

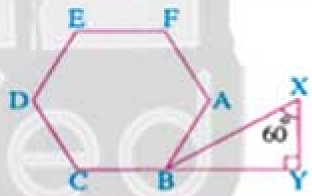


19 **In the opposite figure :**

ABCDEF is a regular hexagon ,

Y ∈ $\overrightarrow{CB}$, $\overline{XY} \perp \overline{YB}$ and m (∠ X) = 60°

Prove that : $\overrightarrow{BX}$ bisects ∠ ABY



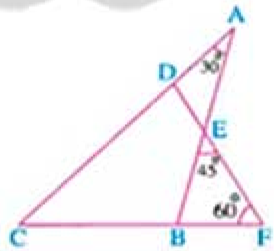
20 **In the opposite figure :**

ABC is a triangle in which : m (∠ A) = 30° ,

$\overrightarrow{DE} \cap \overrightarrow{CB} = \{F\}$, m (∠ F) = 60° and

m (∠ BEF) = 45°

Find the measure of each angle of the figure DEBC



« m (∠ EBC) = 105° , m (∠ C) = 45° , m (∠ BED) = 135° , m (∠ EDC) = 75° »

21 If the ratio among the measures of the angles of a pentagon is 3 : 3 : 2 : 3 : 4 , find the greatest measure of the angles of this pentagon. « 144° »

22 If the measure of the exterior angle of a regular polygon is 30° , how many sides does it have ? What is the sum of the measures of its interior angles ? « 12 , 1800° »

23 Is it possible that a regular polygon has an interior angle of measure 100° ? Why ?

24 A polygon of 9 sides. The sum of measures of eight angles of it is 1140°

1 Find the measure of the remained angle. « 120° »

2 Is it possible that this polygon is regular ? Explain your answer.

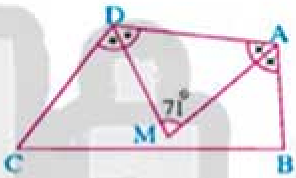
25 A polygon has 15 sides :

1 Calculate the sum of the measures of its interior angles. « 2340° »

2 If the sum of the measures of five of its exterior angles is 200°, calculate the sum of the measures of the ten interior angles which are not adjacent to the five exterior angles. « 1640° »

For excellent pupils

26 **In the opposite figure :**

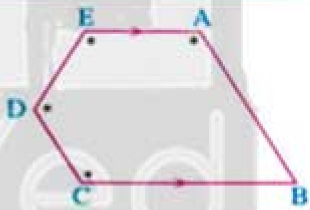


$\overrightarrow{AM}$ bisects $\angle BAD$, $\overrightarrow{DM}$ bisects $\angle ADC$ and

$m(\angle AMD) = 71°$

Prove that : $m(\angle B) + m(\angle C) = 142°$

27 **In the opposite figure :**



$\overline{AE} \parallel \overline{BC}$,

$m(\angle A) = m(\angle E) = m(\angle D) = m(\angle C)$

Find : $m(\angle B)$ « 60° »

Exercise **3**

The parallelogram and its properties

From the school book

1 **Complete the following :**

1. In a parallelogram , every two opposite sides are .... , ...
2. In a parallelogram , every two opposite angles are ....
3. In a parallelogram , every two consecutive angles are ....
4. In a parallelogram , the two diagonals ....
5. The quadrilateral in which two sides only are parallel is called ....
6. A quadrilateral represents a parallelogram if .... (Write only one answer)
7. ABCD is a parallelogram in which m (∠ A) = 50° , then m (∠ B) = ... °
8. In the parallelogram XYZL , if m (∠ X) = $\frac{1}{2}$ m (∠ Y) , then m (∠ Y) = ... °

2 **In the opposite figure :**

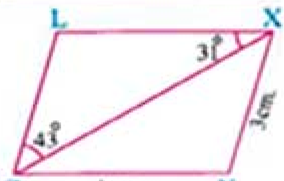
ABCD is a parallelogram in which AB = 2 cm. ,
AD = 6 cm. and m (∠ B) = 105°

Complete the following :

1. BC = ... cm. , DC = ... cm.
2. m (∠ D) = ...° , m (∠ A) = ...° and m (∠ C) = ...°
3. The perimeter of the parallelogram ABCD = ... cm.

3 **In the opposite figure :**

XYZL is a parallelogram in which :
XY = 3 cm. , YZ = 4 cm. , m (∠ LXZ) = 31°
and m (∠ LZX) = 43°



Exercise 3

Complete the following proof to find :

1 m (∠ Y) 2 The perimeter of the parallelogram XYZL

Given ……………………………………………

R.T.F. ……………………………………………

Proof ∵ Δ XLZ in which : m (∠ LXZ) = ……….° , m (∠ LZX) = ……….°

∴ m (∠ L) = 180° – (……….° + ……….°) = ……….°

, ∵ The figure XYZL is a parallelogram.

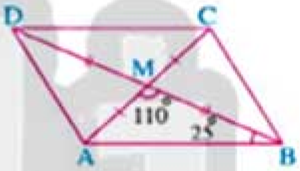
∴ m (∠ Y) = m (∠ ……….) = ……….° (First req.)

, the perimeter of the parallelogram XYZL

= (XY + ……….) × 2 = (3 + ……….) × ………. = ………. × ………. = ………. cm. (Second req.)

4 **In the opposite figure :**

ABCD is a quadrilateral whose diagonals intersect at M , MA = MC , MB = MD , m (∠ AMB) = 110° and m (∠ MBA) = 25°



Complete the following proof :

1 To prove that the figure ABCD is a parallelogram.

2 To find m (∠ ACD)

Given ……………………………………………

R.T.P. ……………………………………………

R.T.F. ……………………………………………

Proof In the figure ABCD :

∵ MA = ………. (given) , MB = ………. (given)

∴ Its diagonals ………. each other.

∴ The figure ABCD is ………. (First req.)

In Δ MBA :

∵ m (∠ AMB) = ……….° , m (∠ MBA) = ……….°

∴ m (∠ MAB) = 180° – (……….° + ……….°) = ……….°

, ∵ The figure ABCD is ……….

∴ $\overline{AB}$ // $\overline{……….}$

, ∵ $\overleftrightarrow{……….}$ is a transversal to them.

∴ m (∠ ACD) = m (∠ ……….) = ……….° (………. angles) (Second req.)

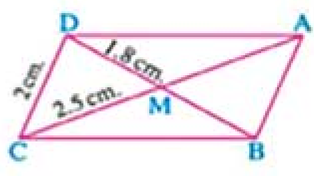
5 **In the opposite figure :**

ABCD is a parallelogram such that :

$\overline{AC} \cap \overline{BD} = \{M\}$ If CD = 2 cm. ,

MC = 2.5 cm. and MD = 1.8 cm.

Calculate the perimeter of Δ AMB



« 6.3 cm. »

Unit 3

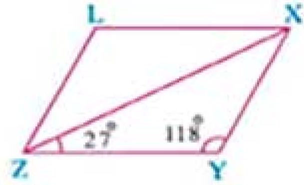
6 **In the opposite figure :**

XYZL is a parallelogram in which :

m (∠ Y) = 118° , m (∠ XZY) = 27°

Find :

1 m (∠ YXZ)
2 m (∠ LZX)
3 m (∠ LXZ)
4 m (∠ L)



« 35° , 35° , 27° , 118° »

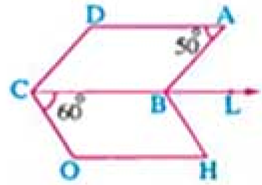
7 **In the opposite figure :**

ABCD is a parallelogram in which :

m (∠ BAD) = 50° and BHOC is a parallelogram

in which m (∠ BCO) = 60° , L ∈ $\overrightarrow{CB}$

Find with proof : m (∠ ABH)



« 110° »

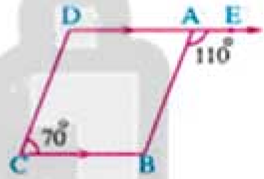
8 **In the opposite figure :**

ABCD is a quadrilateral in which :

$\overline{AD}$ // $\overline{BC}$, E ∈ $\overrightarrow{DA}$, m (∠ BAE) = 110°

and m (∠ DCB) = 70°

Prove that : ABCD is a parallelogram.



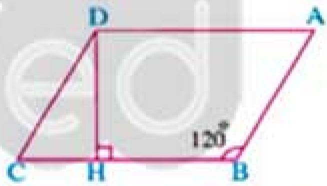
9 **In the opposite figure :**

ABCD is a parallelogram in which :

m (∠ B) = 120° and $\overline{DH}$ ⊥ $\overline{BC}$

where $\overline{DH}$ ∩ $\overline{BC}$ = {H}

Find : m (∠ HDC)



« 30° »

10 **In the opposite figure :**

ABCD is a parallelogram , in which :

m (∠ A) = 60° , m (∠ OBC) = 40° where O ∈ $\overline{AD}$

Find : m (∠ ABO)

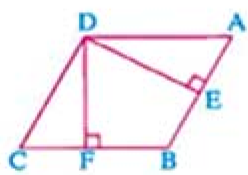
« 80° »

11 **In the opposite figure :**

ABCD is a parallelogram in which :

$\overline{DE}$ ⊥ $\overline{AB}$ and $\overline{DF}$ ⊥ $\overline{BC}$

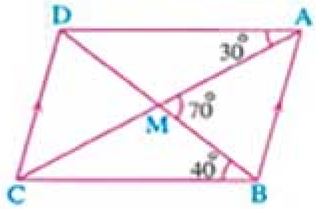
Prove that : m (∠ EDF) = m (∠ A)



12 **In the opposite figure :**

ABCD is a quadrilateral where : $\overline{AC} \cap \overline{BD} = \{M\}$,
$\overline{AB} \parallel \overline{DC}$, m (∠ AMB) = 70° , m (∠ MBC) = 40°
and m (∠ MAD) = 30°

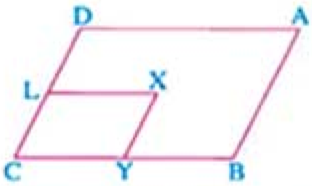
Prove that : ABCD is a parallelogram.



13 **In the opposite figure :**

If ABCD and XYCL are two parallelograms ,

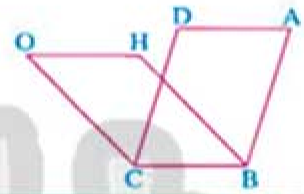
Prove that : m (∠ A) = m (∠ X)



14 **In the opposite figure :**

Each of ABCD
and HBCO is a parallelogram

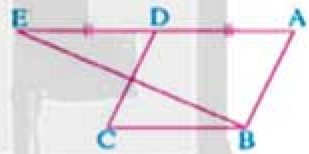
Prove that : AD = HO



15 **In the opposite figure :**

ABCD is a parallelogram ,
E ∈ $\overrightarrow{AD}$ where : AD = DE

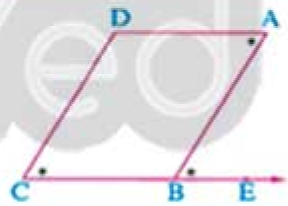
Prove that : $\overline{DC}$ and $\overline{BE}$ bisect each other.



16 **In the opposite figure :**

ABCD is a quadrilateral ,
E ∈ $\overrightarrow{CB}$ and m (∠ BCD) = m (∠ EBA) = m (∠ A)

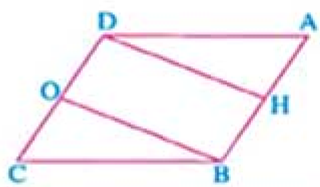
Prove that : ABCD is a parallelogram.



17 **In the opposite figure :**

ABCD is a parallelogram , H is the midpoint of $\overline{AB}$
and O is the midpoint of $\overline{DC}$

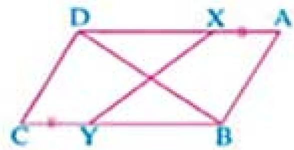
Prove that : HBOD is a parallelogram.



18 **In the opposite figure :**

ABCD is a parallelogram ,
X ∈ $\overline{AD}$, Y ∈ $\overline{BC}$, if AX = CY

Prove that : $\overline{XY}$ and $\overline{BD}$ bisect each other.



Unit 3

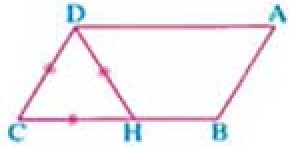
19 **In the opposite figure :**

ABCD is a parallelogram , $H \in \overline{BC}$ where :

Δ DHC is an equilateral triangle.

1 **Prove that :** HC = AB

2 **Find :** m ($\angle$ B) and m ($\angle$ HDA)



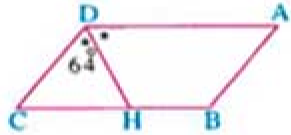
« 120° , 60° »

20 **In the opposite figure :**

ABCD is a parallelogram , $H \in \overline{BC}$

, $\overrightarrow{DH}$ bisects $\angle$ ADC and m ($\angle$ HDC) = 64°

Find : 1 m ($\angle$ DHB) 2 m ($\angle$ ABC)



« 116° , 128° »

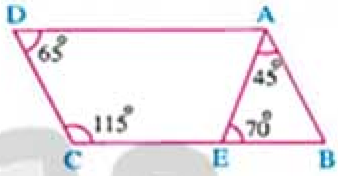
21 **In the opposite figure :**

$E \in \overline{BC}$, m ($\angle$ BAE) = 45°,

m ($\angle$ AEB) = 70° , m ($\angle$ D) = 65°

and m ($\angle$ C) = 115°

Prove that : ABCD is a parallelogram.



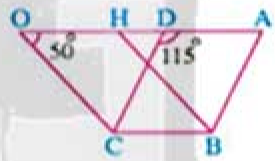
22 **In the opposite figure :**

ABCD and HBCO are two parallelograms

such that m ($\angle$ O) = 50°

and m ($\angle$ ADC) = 115°

Find : m ($\angle$ ABH)



« 65° »

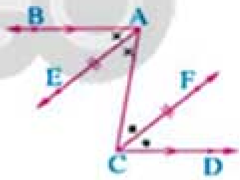
23 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{AE}$ bisects $\angle$ BAC ,

$\overrightarrow{CF}$ bisects $\angle$ ACD

If AE = CF ,

Prove that : AECF is a parallelogram.



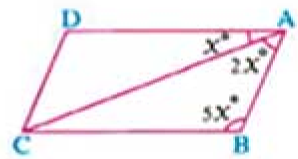
24 **In the opposite figure :**

ABCD is a parallelogram in which :

m ($\angle$ DAC) = x° , m ($\angle$ BAC) = 2 x°

and m ($\angle$ ABC) = 5 x°

Find : m ($\angle$ BCD) and m ($\angle$ ADC) in degrees.



« 67.5° , 112.5° »

25 **Choose the correct answer from the given ones :**

1 ABCD is a parallelogram in which : m ($\angle$ A) = 50° , then m ($\angle$ C) = ...

(a) 50° (b) 60° (c) 130° (d) 150°

2 ABCD is a parallelogram in which : m (∠ A) + m (∠ C) = 140°
, then m (∠ B) = ...

(a) 70° (b) 40° (c) 110° (d) 220°

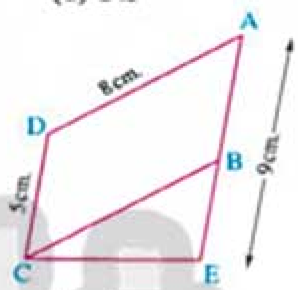
3 If the lengths of two consecutive sides of a parallelogram are 3 cm. and 5 cm. , then its perimeter equals .... cm.

(a) 12 (b) 14 (c) 16 (d) 18

4 If the perimeter of a parallelogram is 25 cm. and if one of its sides is of length 7 cm. , then the consecutive side is of length .... cm.

(a) 7 (b) 18 (c) 12.5 (d) 5.5

5 **In the opposite figure :**



If ABCD is a parallelogram

, $E \in \overrightarrow{AB}$, CD = 5 cm. , AE = 9 cm.

, AD = 8 cm. , the perimeter of Δ BEC = 18 cm.

, then the length of $\overline{EC}$ = ... cm.

(a) 8 (b) 6 (c) 5 (d) 4

Life Application

26 In one of the new cities , each four buildings are put like A , B , C and D according to the following :

- The distance between the two buildings A and B equals the distance between the two buildings C and D
- $\overline{AB} \parallel \overline{CD}$

Explain how we can decide the place where the building D will be built.

For excellent pupils

27 ABCD is a parallelogram in which : E is the midpoint of $\overline{AB}$,
F is the midpoint of $\overline{CD}$, if $\overline{AF} \cap \overline{DE} = \{M\}$, $\overline{BF} \cap \overline{CE} = \{N\}$

Prove that : 1 $\overline{ED} \parallel \overline{FB}$ 2 FMEN is a parallelogram.

28 XYZL is a parallelogram in which : m (∠ Y) = 3 m (∠ X)

Find the measures of the interior angles of XYZL

« m (∠ Y) = m (∠ L) = 135° and m (∠ X) = m (∠ Z) = 45° »

Exercise 4 The special cases of the parallelogram

From the school book

1 Complete the following :

1. The parallelogram whose two diagonals are perpendicular is called ....
2. The parallelogram whose two diagonals are .... is called a rectangle.
3. The parallelogram whose two diagonals are equal in length and perpendicular is called ....
4. The quadrilateral whose sides are equal in length is called ....
5. The quadrilateral whose diagonals bisect each other is called ....
6. The rectangle is a .... with a right angle.
7. The rhombus is a .... in which its diagonals are perpendicular.
8. The square is a .... with a right angle.
9. The rhombus whose two diagonals are equal in length is called ....
10. The rectangle whose two diagonals are perpendicular is called ....
11. The rectangle whose two adjacent sides have the same length is called ....
12. If $\overline{XY}$ // $\overline{ZL}$, XY = ZL, then the quadrilateral XYZL is called ....
13. If ABCD is a rhombus, then .... $\perp$ ....
14. The perimeter of the square = ...
    , the perimeter of the rectangle = ...
    and the perimeter of the rhombus = ...
15. The rhombus whose perimeter is 42 cm., its side length = ... cm.

2 **Choose the correct answer from the given ones :**

1. The two diagonals of a rectangle ....
   (a) are perpendicular.
   (b) are equal in length.
   (c) are perpendicular and equal in length.
   (d) bisect its interior angles.

2. The two diagonals of a rhombus are ....
   (a) perpendicular and not equal in length.
   (b) equal in length and not perpendicular.
   (c) perpendicular and equal in length.
   (d) not equal in length and not perpendicular.

3. The two diagonals of the square are ....
   (a) just perpendicular.
   (b) just equal in length.
   (c) perpendicular and equal in length.
   (d) not equal in length and not perpendicular.

4. The adjacent sides are equal in length in a parallelogram, then the figure is a ....
   (a) square. (b) rhombus. (c) rectangle. (d) trapezium.

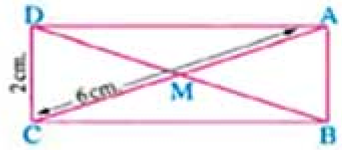
5. If ABCD is a rectangle in which AC = 5 cm., then : BD = ... cm.
   (a) 2.5 (b) 5 (c) 10 (d) 20

6. If ABCD is a square, then : m ($\angle$ CAB) = ...
   (a) 90° (b) 45° (c) 60° (d) 30°

7. If ABCD is a parallelogram in which m ($\angle$ A) = m ($\angle$ B), then : ABCD is a ....
   (a) rectangle. (b) rhombus. (c) square. (d) trapezium.

8. If ABCD is a rhombus in which m ($\angle$ ACB) = 32°, then : m ($\angle$ D) = ...
   (a) 32° (b) 64° (c) 116° (d) 26°

3 **In the opposite figure :**



ABCD is a rectangle in which :

AC = 6 cm. , CD = 2 cm.

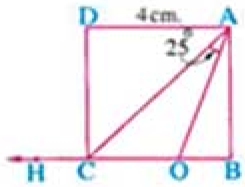
and $\overline{AC} \cap \overline{BD} = \{M\}$

Complete : 1. AB = ... cm.

2. DM = ... cm.

3. The perimeter of Δ ABM = ... cm.

4 **In the opposite figure :**



ABCD is a square in which AD = 4 cm. ,

$O \in \overline{BC}$ such that : m (∠ OAC) = 25°

and $H \in \overline{BC}$

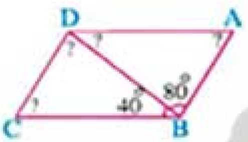
Complete the following :

1 The perimeter of the square = ... cm.

2 m (∠ ACH) = ...°

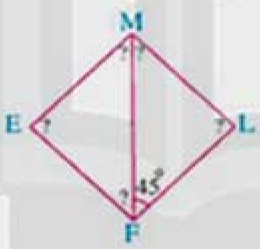
3 m (∠ AOC) = ...°

5 **Find the measures of the angles marked by (?) in each of the following figures :**



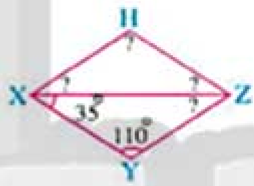
a parallelogram

Fig. (1)



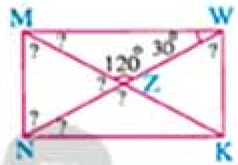
a square

Fig. (2)



a rhombus

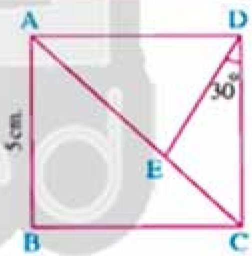
Fig. (3)



a rectangle

Fig. (4)

6 **In the opposite figure :**



ABCD is a square whose side length = 5 cm.,

$E \in \overline{AC}$ where m (∠ EDC) = 30°

Complete the following proof to find :

1 The perimeter of the square ABCD

2 m (∠ AED)

Given ....

R.T.F. ....

Proof ∵ The perimeter of the square = side length × ....

∴ The perimeter of the square ABCD

= .... × .... = ... cm. (First req.)

, ∵ ABCD is a square , $\overline{AC}$ is a diagonal.

∴ m (∠ ACD) = ...°

In Δ DEC :

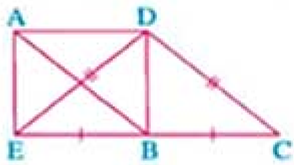
m (∠ DEC) = 180° – (..........° + ...°) = ...°

, ∵ $E \in \overline{AC}$

∴ m (∠ AED) = ...° – ....° = ...° (Second req.)

Exercise 4

7 **In the opposite figure :**



ABCD is a parallelogram,

$E \in \overrightarrow{CB}$ where BC = BE , if DE = DC

Complete the following proof to prove that :

The figure AEBD is a rectangle.

Given ....

R.T.P. ....

Proof $\because$ ABCD is a parallelogram.

$\therefore$ AD = ... , ... // $\overline{BC}$

, $\because$ EB = ... , $E \in \overrightarrow{CB}$ (given)

$\therefore$ AD = ... , ... // $\overline{EB}$

$\therefore$ The figure AEBD is a parallelogram.

, $\because$ DE = ... (given)

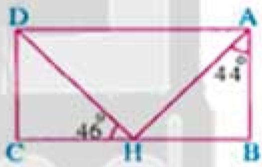
, AB = ... (properties of parallelogram)

$\therefore$ DE = ...

$\therefore$ The two diagonals of the parallelogram AEBD are ....

$\therefore$ The figure AEBD is a .... (Q.E.D.)

8 **In the opposite figure :**



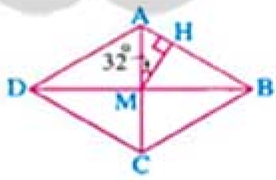
ABCD is a rectangle , $H \in \overline{BC}$ such that :

m ($\angle$ DHC) = 46° and m ($\angle$ BAH) = 44°

Calculate : m ($\angle$ AHD)

« 88° »

9 **In the opposite figure :**



ABCD is a rhombus in which :

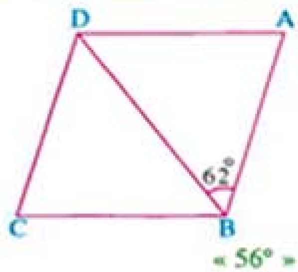
$\overline{AC} \cap \overline{BD} = \{M\}$,

$H \in \overline{AB}$ where $\overline{MH} \perp \overline{AB}$

If m ($\angle$ AMH) = 32° ,

then calculate the measures of the angles of the rhombus ABCD « 116° , 64° , 116° , 64° »

10 **In the opposite figure :**



ABCD is a rhombus,

$\overline{BD}$ is a diagonal in it ,

m ($\angle$ ABD) = 62°

Find with proof : m ($\angle$ A)

« 56° »

Unit 3

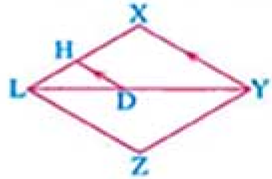
11 **In the opposite figure :**

XYZL is a rhombus , $D \in \overline{YL}$

Draw $\overrightarrow{DH} \parallel \overline{YX}$ such that :

$\overrightarrow{DH} \cap \overline{XL} = \{H\}$

Prove that : m (∠ HDL) = m (∠ HLD)

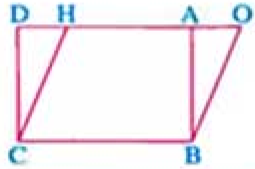


12 **In the opposite figure :**

ABCD is a rectangle

and OBCH is a parallelogram.

Prove that : DH = AO

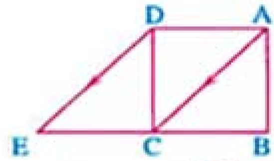


13 **In the opposite figure :**

ABCD is a square, $E \in \overline{BC}$, $\overline{AC} \parallel \overline{DE}$

1 **Prove that :** ACED is a parallelogram.

2 **Find :** m (∠ ACE)



« 135° »

14 ABCD is a rhombus in which m (∠ BAC) = 45°

Prove that : ABCD is a square.

15 **In the opposite figure :**

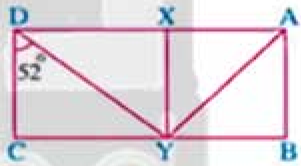
ABCD is a rectangle , $X \in \overline{AD}$

and $Y \in \overline{BC}$ such that :

AXYB is a square.

If m (∠ YDC) = 52° ,

then find with proof : m (∠ AYD)



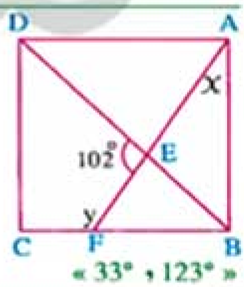
« 97° »

16 **In the opposite figure :**

ABCD is a square.

Find in degrees the value

of each of x and y



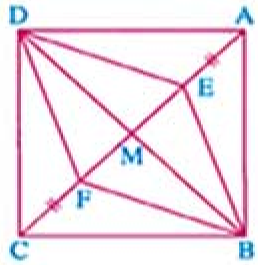
« 33° , 123° »

17 **In the opposite figure :**

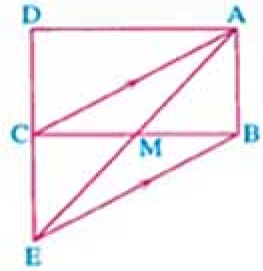
ABCD is a square, its diagonals intersect at M,

$E \in \overline{AC}$, $F \in \overline{AC}$ such that : AE = CF

Prove that : EBFD is a rhombus.



18 **In the opposite figure :**



ABCD is a rectangle,

$\overrightarrow{BE} \parallel \overline{AC}$, $\overrightarrow{BE} \cap \overrightarrow{DC} = \{E\}$,

$\overline{BC} \cap \overline{AE} = \{M\}$

Prove that :

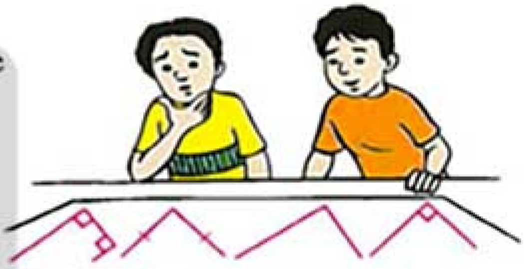
1 $DC = CE$

2 $MC = \frac{1}{2} AD$

Life Application

19 Eslam drew a parallelogram , a rhombus , a rectangle and a square , then he hid parts of them as in the opposite figure and he asked his friend Bassem to recognize each figure.

Help Bassem to name each drawn figure.



1 2 3 4

For excellent pupils

20 **Use "some" or "all" to get a correct statement :**

1 .... squares are rectangles.

2 .... quadrilaterals are parallelograms.

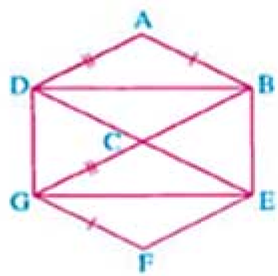
3 .... squares are rhombuses.

4 .... parallelograms are rectangles.

5 .... rectangles are parallelograms.

6 .... rhombuses are squares.

21 **In the opposite figure :**



ABCD is a parallelogram,

CEFG is a rhombus,

if AB = GF , AD = CG

Prove that : BEGD is a rectangle.

Summary of the first part of unit 3
"From lesson 1 to lesson 4"

- If two straight lines intersect , then the measures of each two vertically opposite angles are equal.
- The sum of the measures of the accumulative angles at a point is equal to $360°$
- The polygon is a simple closed line that consists of three or more line segments , and the polygon is named according to the number of its sides.
- The polygon is convex if the measure of any of its interior angles is less than $180°$
- The polygon is concave if the measure of one of its interior angles at least is greater than $180°$ (reflex angle)
- The sum of measures of the interior angles of a polygon of n sides equals $(n - 2) \times 180°$
- The sum of measures of the exterior angles of a convex polygon of n sides $= 360°$
- **The polygon is regular if :**
  1. All its sides are equal in length.
  2. All its angles are equal in measure.
- The measure of each interior angle of the regular polygon of n sides $= \frac{(n-2) \times 180°}{n}$
- The number of sides of the regular polygon in which the measure of one of its interior angles is $x° = \frac{360°}{180° - x°}$
- The number of diagonals of a polygon of n sides $= \frac{n(n-3)}{2}$
- Parallelogram is a quadrilateral , in which each two opposite sides are parallel.
- **Properties of parallelogram :**
  1. Each two opposite sides are equal in length.
  2. Each two opposite angles are equal in measure.
  3. The sum of measures of each two consecutive angles is $180°$
  4. The two diagonals bisect each other.

- A quadrilateral represents a parallelogram if one of the following conditions take place :
  1. Each two opposite sides are parallel.
  2. Each two opposite sides are equal in length.
  3. Two opposite sides are parallel and equal in length.
  4. Each two opposite angles are equal in measure.
  5. The two diagonals bisect each other.
- The rectangle is a parallelogram with a right angle.
- **Properties of the rectangle :**

  The rectangle has the same properties as the parallelogram and more additional properties as the following :
  1. Its four angles are all equal in measure and the measure of each is 90°
  2. Its two diagonals are equal in length.
- The rhombus is a parallelogram in which two adjacent sides are equal in length.
- **Properties of the rhombus :**

  The rhombus has the same properties as the parallelogram and more additional properties as the following :
  1. Its four sides are all equal in length.
  2. Its two diagonals are perpendicular and bisect its interior angles.
- The square is a parallelogram with a right angle and two adjacent sides are equal in length.
- **Properties of the square :**

  The square has the same properties as the parallelogram and more additional properties as the following :
  1. Its four sides are all equal in length.
  2. Its four angles are all equal in measure and each of them is of measure 90°
  3. Its two diagonals are equal in length, perpendicular and each diagonal bisects the vertices angles which this diagonal joins and the measure of each is 45°

Exams on the first part of unit three
from lesson (1) to lesson (4)

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

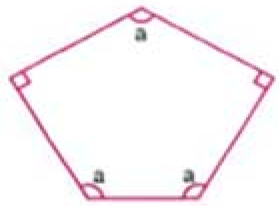
1. The measure of the interior angle of the regular hexagon is ....
   (a) 720° (b) 120° (c) 150° (d) 108°
2. If two adjacent sides are equal in length in a parallelogram , then the figure is a ....
   (a) square. (b) rhombus. (c) rectangle. (d) trapezium.
3. ABCD is a parallelogram in which : m (∠ A) = 70° , then m (∠ C) = ...
   (a) 110° (b) 180° (c) 100° (d) 70°
4. The sum of measures of the exterior angles of a triangle = ...
   (a) 180° (b) 100° (c) 360° (d) 90°
5. The parallelogram which has a right angle is called ....
   (a) rhombus. (b) rectangle. (c) square. (d) trapezium.
6. If the measure of an interior angle of a regular polygon is 150° , then the number of its sides is ....
   (a) 6 (b) 8 (c) 10 (d) 12

2 **Complete the following :**

1. The sum of measures of the interior angles of a heptagon is ....°
2. The sum of measures of the accumulative angles at a point is ....°
3. ABCD is a parallelogram in which m (∠ A) = $\frac{1}{2}$ m (∠ B) , then m (∠ B) = ...°
4. The square is a .... with a right angle.
5. The quadrilateral in which only two sides are parallel is called ....

3 [a] **In the opposite figure :**

Find : The value of a



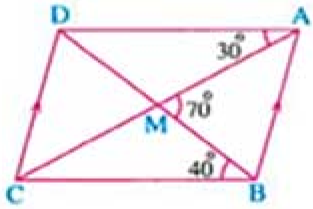
[b] **In the opposite figure :**

$\overline{AC} \cap \overline{BD} = \{M\}$, $\overline{AB} \mathbin{/\!/} \overline{DC}$

, m ($\angle$ AMB) = 70° , m ($\angle$ MBC) = 40°

and m ($\angle$ MAD) = 30°

Prove that : ABCD is a parallelogram.



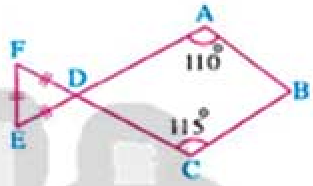
4 [a] **In the opposite figure :**

$\overline{AE} \cap \overline{CF} = \{D\}$

, Δ DEF is an equilateral triangle

, m ($\angle$ A) = 110° and m ($\angle$ C) = 115°

Find : m ($\angle$ B)



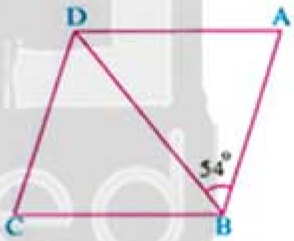
[b] **In the opposite figure :**

ABCD is a rhombus

, $\overline{BD}$ is a diagonal in it

, m ($\angle$ ABD) = 54°

Find with proof : m ($\angle$ A)



5 [a] **In the opposite figure :**

E $\in \overline{BC}$, m ($\angle$ BAE) = 45°

, m ($\angle$ AEB) = 70° , m ($\angle$ D) = 65°

and m ($\angle$ C) = 115°

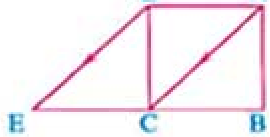
Prove that : ABCD is a parallelogram.

[b] **In the opposite figure :**

ABCD is a square , E $\in \overrightarrow{BC}$, $\overline{AC} \mathbin{/\!/} \overline{DE}$

1 **Prove that :** ACED is a parallelogram.

2 **Find :** m ($\angle$ ACE)



Unit 3

Model 2

Answer the following questions :

1 Choose the correct answer from the given ones :

1. A rhombus of perimeter 60 cm. , then its side length = ... cm.

   (a) 20 (b) 18 (c) 15 (d) 10

2. The sum of measures of the interior angles of a polygon of n sides = ...

   (a) $n \times 180°$ (b) $(n-2) \times 180°$

   (c) $\frac{(n-2) \times 180°}{n}$ (d) $\frac{(n-2) \times 180°}{2n}$

3. If ABCD is a square , then m (∠CAB) = ...

   (a) 90° (b) 45° (c) 60° (d) 30°

4. The sum of measures of the exterior angles of a regular pentagon = ...

   (a) 108° (b) 120° (c) 360° (d) 540°

5. In the parallelogram , the sum of measures of each two consecutive angles equals ....

   (a) 90° (b) 360° (c) 120° (d) 180°

6. The parallelogram in which the two diagonals are equal in length and perpendicular is called a ....

   (a) rectangle. (b) square. (c) rhombus. (d) trapezium.

2 Complete the following :

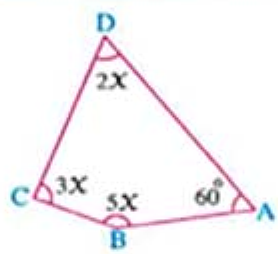
1. The measure of an interior angle of a regular polygon of 10 sides = ...°
2. In the parallelogram , the two diagonals ....
3. The rectangle is a .... with a right angle.
4. ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 110° , then m (∠ B) = ...°
5. If ABCD is a rhombus , m (∠ ACB) = 31° , then m (∠ D) = ...°

3 [a] In the opposite figure :

ABCD is a quadrilateral in which

m (∠ A) = 60°

Find : The value of X

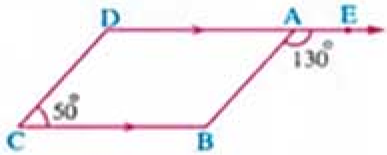


[b] In the opposite figure :

$E \in \overrightarrow{DA}$, m ($\angle$ EAB) = 130°

, m ($\angle$ C) = 50° , $\overrightarrow{DA}$ // $\overrightarrow{CB}$

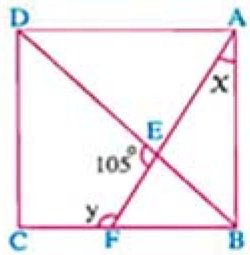
Prove that : The figure ABCD is a parallelogram.



4 **[a] In the opposite figure :**

ABCD is a square

Find in degrees the value of : x , y

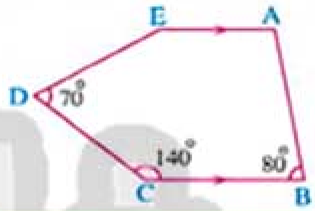


[b] In the opposite figure :

$\overline{AE}$ // $\overline{BC}$, m ($\angle$ B) = 80°

, m ($\angle$ C) = 140° , m ($\angle$ D) = 70°

Find with proof : m ($\angle$ E)



5 **[a] In the opposite figure :**

ABCD is a rectangle

, AC = 8 cm. , CD = 3 cm.

, its two diagonals intersect at M

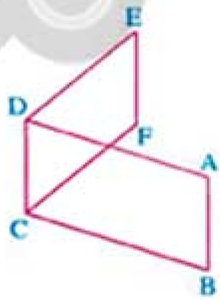
Find :

1. MD , AB
2. The perimeter of Δ ABM

[b] In the opposite figure :

ABCD , EFCD are two parallelograms.

Prove that : AB = EF



Exercise 5

The triangle

From the school book

1 Complete the following :

1. The sum of measures of the interior angles of a triangle = ...$^\circ$
2. The measure of the exterior angle of a triangle is equal to the sum of ....
3. If the measure of an angle in a triangle equals the sum of measures of the other two angles in the triangle , then the triangle is ....
4. If the measure of an angle in a triangle is greater than the sum of measures of the other two angles , then the triangle is ....
5. In Δ ABC : If m ($\angle$ A) + m ($\angle$ C) = m ($\angle$ B) , then m ($\angle$ B) = ...$^\circ$
6. In Δ ABC : If m ($\angle$ B) > m ($\angle$ A) + m ($\angle$ C) , then $\angle$ B is ....
7. It is possible to find a triangle each of its interior angles is of measure ....$^\circ$

2 Choose the correct answer from the given ones :

1. The triangle contains two .... angles at least.
   (a) acute (b) obtuse (c) right (d) reflex
2. The sum of measures of the interior angles of a triangle equals the measure of .... angle.
   (a) a right (b) a straight (c) an acute (d) a reflex
3. In Δ XYZ : If m ($\angle$ X) = 50° , m ($\angle$ Y) = 100°, then m ($\angle$ Z) = ...
   (a) 30° (b) 50° (c) 80° (d) 100°
4. In Δ ABC : If m ($\angle$ A) + m ($\angle$ B) = 110°, then m ($\angle$ C) = ...
   (a) 110° (b) 90° (c) 70° (d) 55°
5. If the measures of two angles in a triangle are 35° and 45°, then the triangle is ....
   (a) acute-angled. (b) right-angled. (c) obtuse-angled. (d) equilateral.

6 The measure of the exterior angle of the equilateral triangle at any one of its vertices equals ....

(a) 60° (b) 120° (c) 150° (d) 30°

3 **In each of the following figures , find the measure of the angle marked by (?) :**

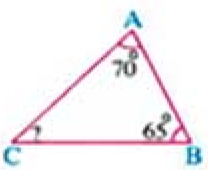


Fig. (1)

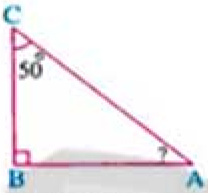


Fig. (2)

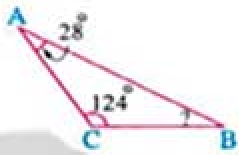


Fig. (3)

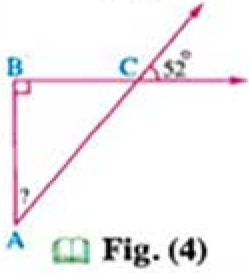


Fig. (4)

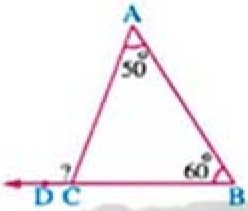


Fig. (5)

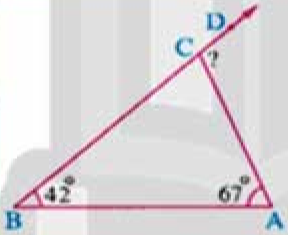


Fig. (6)

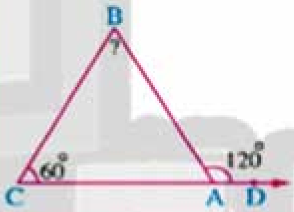


Fig. (7)

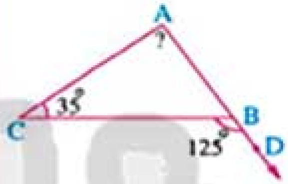


Fig. (8)

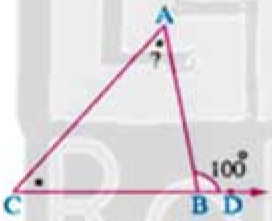


Fig. (9)

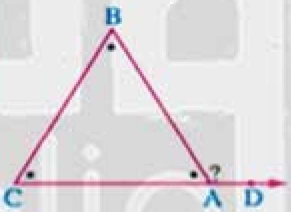


Fig. (10)

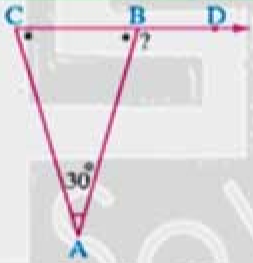


Fig. (11)

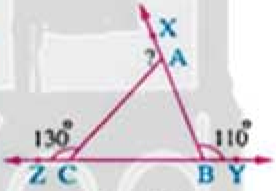


Fig. (12)

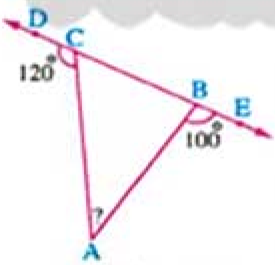


Fig. (13)

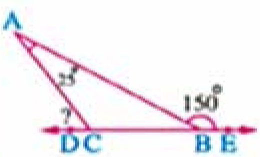


Fig. (14)

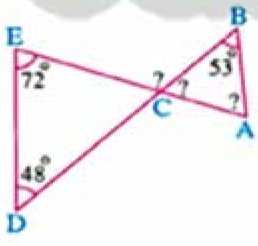


Fig. (15)

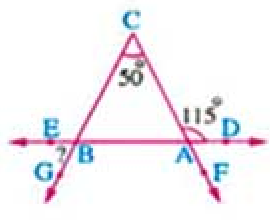


Fig. (16)

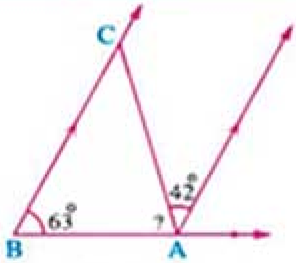


Fig. (17)

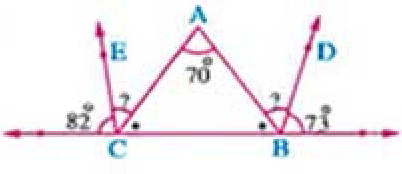
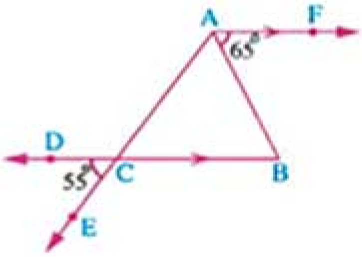


Fig. (18)

4 **In the opposite figure :**



$\overrightarrow{BD} \cap \overrightarrow{AE} = \{C\}$, $\overrightarrow{AF} \mathbin{//} \overrightarrow{BC}$

, m (∠ BAF) = 65° and m (∠ DCE) = 55°

Complete the following proof to find :

The measures of the interior angles of Δ ABC

Given ………………………………………………………………

R.T.F. ………………………………………………………………

Proof $\because \overrightarrow{BD} \cap \overrightarrow{AE} = \{C\}$ (given)

$\therefore$ m (∠ ACB) = m (∠ ………) = ………° (………)

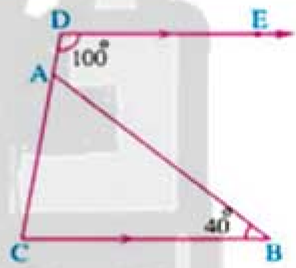
, $\because \overrightarrow{AF} \mathbin{//} \overrightarrow{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore$ m (∠ FAB) = m (∠ ………) = ………° (……… angles)

, $\because$ the sum of measures of the interior anlges of the triangle = ………°

$\therefore$ m (∠ BAC) = ………° − (………° + ………°) = ………° (The req.)

5 **In the opposite figure :**



A ∈ $\overline{DC}$, $\overrightarrow{DE} \mathbin{//} \overrightarrow{CB}$,

m (∠ D) = 100° and m (∠ B) = 40°

Complete the following proof to find : m (∠ BAD)

Given ………………………………………………………………

R.T.F. ………………………………………………………………

Proof $\because \overrightarrow{DE} \mathbin{//} \overline{\cdots\cdots}$, $\overleftrightarrow{\cdots\cdots}$ is a transversal.

$\therefore$ m (∠ D) + m (∠ C) = ………°

(Two interior angles in the same side of the transversal)

$\because$ m (∠ D) = ………°

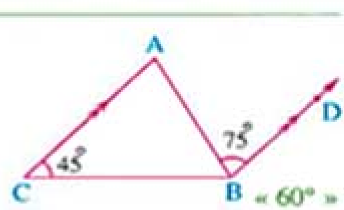
$\therefore$ m (∠ C) = ……… − ……… = ………°

$\because$ ∠ BAD is an exterior angle of Δ ………

$\therefore$ m (∠ BAD) = m (∠ ………) + m (∠ ………)

= ………° + ………° = ………° (The req.)

6 **In the opposite figure :**



« 60° »

$\overrightarrow{BD} \mathbin{//} \overrightarrow{CA}$, m (∠ C) = 45°

and m (∠ ABD) = 75°

Find : m (∠ ABC)

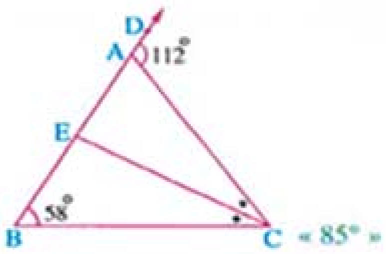
7 **In the opposite figure :**

ABC is a triangle in which : m (∠ B) = 58° ,

E ∈ $\overline{AB}$ such that $\overrightarrow{CE}$ bisects ∠ ACB ,

D ∈ $\overrightarrow{BA}$ and m (∠ CAD) = 112°

Find : m (∠ AEC)



« 85° »

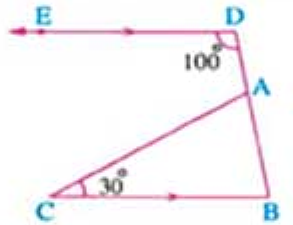
8 **In the opposite figure :**

$\overrightarrow{DE}$ // $\overline{BC}$, m (∠ D) = 100° ,

m (∠ C) = 30°

and A ∈ $\overline{DB}$

Find : m (∠ BAC)



« 70° »

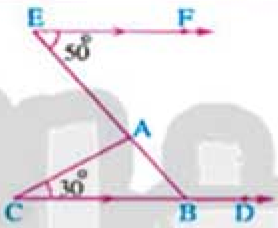
9 **In the opposite figure :**

$\overrightarrow{EF}$ // $\overrightarrow{CD}$,

m (∠ E) = 50° and m (∠ C) = 30°

Find the measures of the angles

of Δ ABC and m (∠ ABD)



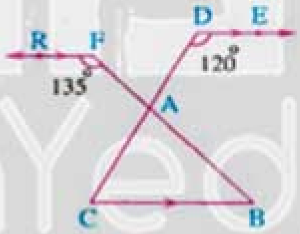
« m (∠ ABC) = 50° , m (∠ BAC) = 100° , m (∠ ABD) = 130° »

10 **In the opposite figure :**

$\overrightarrow{DE}$ // $\overrightarrow{FR}$ // $\overline{BC}$,

m (∠ CDE) = 120° and m (∠ RFB) = 135°

Calculate the measures of the angles of Δ ABC



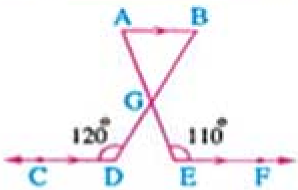
« m (∠ B) = 45° , m (∠ C) = 60° , m (∠ A) = 75° »

11 **In the opposite figure :**

$\overline{AB}$ // $\overrightarrow{DC}$ // $\overrightarrow{EF}$,

m (∠ E) = 110° and m (∠ D) = 120°

Find : m (∠ EGD)



« 50° »

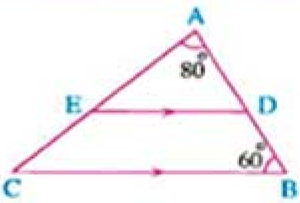
12 **In the opposite figure :**

ABC is a triangle in which :

m (∠ A) = 80° and m (∠ B) = 60°

$\overline{DE}$ // $\overline{BC}$ where : D ∈ $\overline{AB}$ and E ∈ $\overline{AC}$

Find : m (∠ AED) and m (∠ DEC)



« 40° , 140° »

Unit 3

13 **In the opposite figure :**

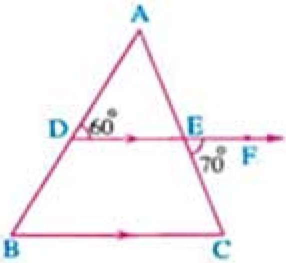
ABC is a triangle , m (∠ ADE) = 60° ,

m (∠ FEC) = 70° ,

$D \in \overline{AB}$, $\overrightarrow{DF} \parallel \overline{BC}$

and $\overline{AC} \cap \overrightarrow{DF} = \{E\}$

Find the measures of the interior angles of Δ ABC



« m (∠ C) = 70° , m (∠ B) = 60° , m (∠ A) = 50° »

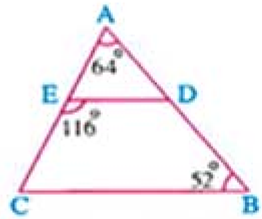
14 **In the opposite figure :**

ABC is a triangle in which m (∠ A) = 64° ,

m (∠ B) = 52° , m (∠ DEC) = 116° ,

$E \in \overline{AC}$ and $D \in \overline{AB}$

Prove that : $\overline{DE} \parallel \overline{BC}$



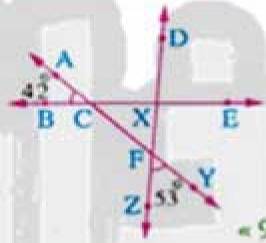
15 **In the opposite figure :**

Prove that :

m (∠ DXE) = 85° ,

then calculate m (∠ DXC)

and m (∠ EXF)



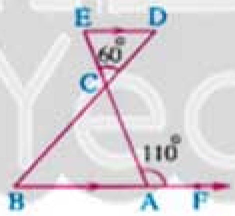
« 95° , 95° »

16 **In the opposite figure :**

$\overline{ED} \parallel \overrightarrow{BA}$, m (∠ CAF) = 110° ,

$\overline{DB} \cap \overline{AE} = \{C\}$,

m (∠ DCE) = 60° and $F \in \overrightarrow{BA}$



Find the measures of each of the angles of the two triangles DCE and ABC

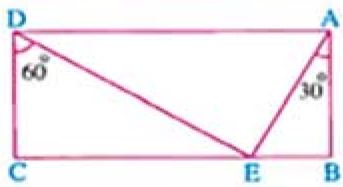
« m (∠ E) = 70° , m (∠ D) = 50° , m (∠ B) = 50° , m (∠ ACB) = 60° , m (∠ BAC) = 70° »

17 **In the opposite figure :**

ABCD is a rectangle , $E \in \overline{BC}$ where :

m (∠ BAE) = 30° and m (∠ EDC) = 60°

Find : m (∠ AED)



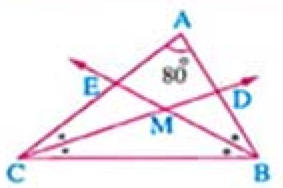
« 90° »

18 **In the opposite figure :**

$\overrightarrow{BM}$ bisects ∠ ABC and $\overrightarrow{CM}$ bisects ∠ ACB

If m (∠ A) = 80° ,

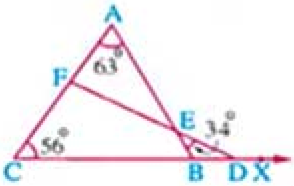
Find : m (∠ EMD)



« 130° »

19 **In the opposite figure :**

ABC is a triangle , $D \in \overrightarrow{CB}$, $X \in \overrightarrow{CB}$,
m (∠ A) = 63° , m (∠ C) = 56°
and m (∠ DEB) = 34°
Find : m (∠ EDX)

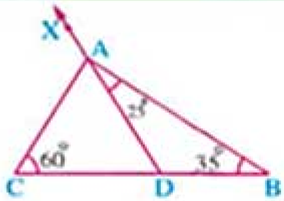


« 153° »

20 **In the opposite figure :**

ABC is a triangle , m (∠ B) = 35° , m (∠ C) = 60° ,
m (∠ BAD) = 25° , $D \in \overline{BC}$ and $X \in \overrightarrow{DA}$

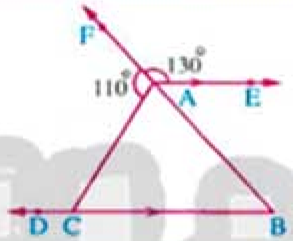
Find : m (∠ XAC)



« 120° »

21 **In the opposite figure :**

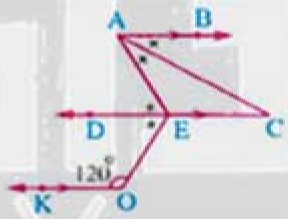
ABC is a triangle , $\overrightarrow{AE} \parallel \overline{BC}$, $D \in \overrightarrow{BC}$, $F \in \overrightarrow{BA}$,
m (∠ FAE) = 130° and m (∠ FAC) = 110°
Find : m (∠ ACD)



« 120° »

22 **In the opposite figure :**

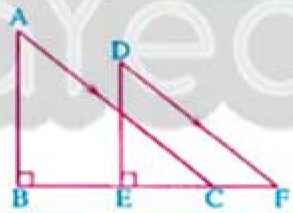
$\overrightarrow{AB} \parallel \overrightarrow{CD} \parallel \overrightarrow{OK}$, $E \in \overline{CD}$, $\overrightarrow{AC}$ bisects ∠ BAE
, $\overrightarrow{ED}$ bisects ∠ AEO and m (∠ O) = 120°
Find : m (∠ C)



« 30° »

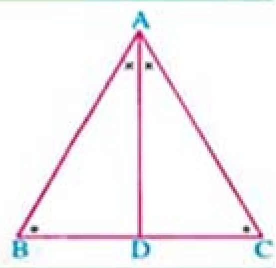
23 **In the opposite figure :**

The points F , C , E and B are collinear ,
m (∠ B) = m (∠ DEC) = 90° and $\overline{AC} \parallel \overline{DF}$
Prove that : m (∠ A) = m (∠ D)



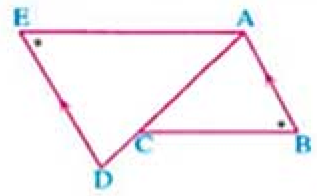
24 **In the opposite figure :**

ABC is a triangle ,
m (∠ B) = m (∠ C)
and $\overrightarrow{AD}$ is the bisector of ∠ A
Prove that : AB = AC



25 **In the opposite figure :**

$\overline{AB} \parallel \overline{ED}$
and m (∠ ABC) = m (∠ AED)
Prove that : $\overline{BC} \parallel \overline{AE}$



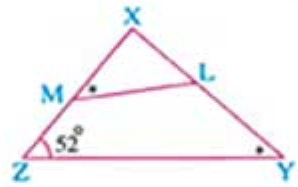
Unit 3

26 **In the opposite figure :**

XYZ is a triangle in which m (∠ Z) = 52° , L ∈ $\overline{XY}$

and M ∈ $\overline{XZ}$ such that : m (∠ Y) = m (∠ XML)

Find : m (∠ XLM)



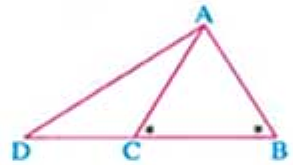
« 52° »

27 **In the opposite figure :**

ABC is a triangle in which :

m (∠ B) = m (∠ ACB) and D ∈ $\overrightarrow{BC}$

Prove that : m (∠ B) > m (∠ D)

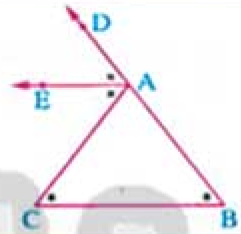


28 **In the opposite figure :**

ABC is a triangle in which : m (∠ B) = m (∠ C) ,

D ∈ $\overrightarrow{BA}$ and $\overrightarrow{AE}$ bisects ∠ DAC

Prove that : $\overrightarrow{AE}$ // $\overline{BC}$

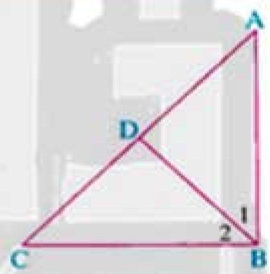


29 **In the opposite figure :**

ABC is a triangle in which : D ∈ $\overline{AC}$,

m (∠ 1) = m (∠ A) and m (∠ 2) = m (∠ C)

Prove that : ∠ ABC is a right angle.



For excellent pupils

30 ABC is a triangle in which : m (∠ A) = 2 m (∠ C) and m (∠ B) = 4 m (∠ C)

Prove that : ∠ B is an obtuse angle.

31 ABC is a triangle in which : m (∠ C) = 28° , m (∠ A) = $4X$°

, m (∠ B) = $(2X + 2)$°

Find : m (∠ A) and m (∠ B)

« 100° , 52° »

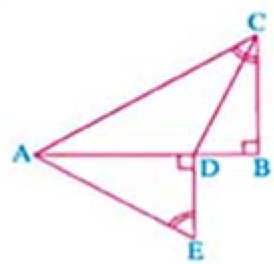
32 **In the opposite figure :**

ABC is a triangle in which : D ∈ $\overline{AB}$

m (∠ B) = 90° , m (∠ ADE) = 90°

and m (∠ ACB) = m (∠ E)

Prove that : m (∠ BDC) > m (∠ DAE)



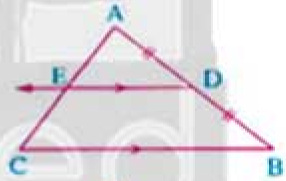
Exercise 6

Theorem 2 and its corollary, and theorem 3

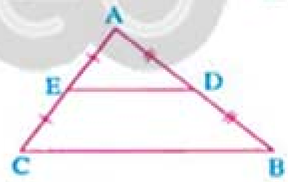
From the school book

1 Complete the following :

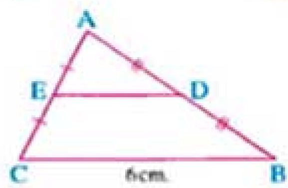
1. The ray drawn from the midpoint of a side of a triangle parallel to another side ....
2. The line segment joining the midpoints of two sides of a triangle is .... the third side.
3. The length of the line segment joining the midpoints of two sides of a triangle equals ....
4. **In the opposite figure :**
   If D is the midpoint of $\overline{AB}$, $\overrightarrow{DE} \parallel \overline{BC}$
   , then .... is the midpoint of ....



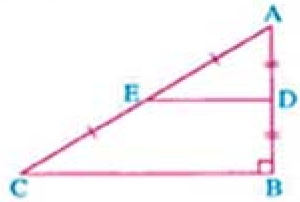
5. **In the opposite figure :**
   If D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively
   , then .... // ....



6. **In the opposite figure :**
   $\because$ D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively
   , BC = 6 cm.
   $\therefore$ DE = ... cm.



7. **In the opposite figure :**
   If m ($\angle$ B) = 90° , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively
   , then m ($\angle$ ADE) = ...°



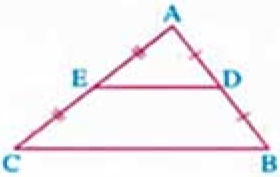
Unit 3

8 **In the opposite figure :**

If D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, and the perimeter of the triangle ABC = 24 cm.

, then the perimeter of the triangle ADE = ... cm.

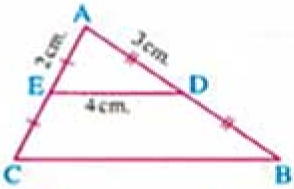


9 **In the opposite figure :**

$\because$ D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, AD = 3 cm., AE = 2 cm. and DE = 4 cm.

$\therefore$ The perimeter of the figure DBCE = ... cm.

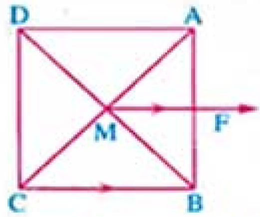


10 **In the opposite figure :**

If the perimeter of the square ABCD = 20 cm.

, $\overrightarrow{MF}$ // $\overline{CB}$ where F $\in \overline{AB}$

, then AF = ... cm.

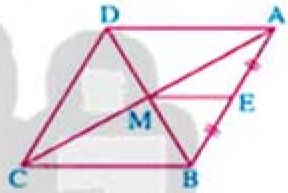


11 **In the opposite figure :**

$\because$ The perimeter of the rhombus ABCD = 24 cm.,

E is the midpoint of $\overline{AB}$

$\therefore$ ME = ... cm.

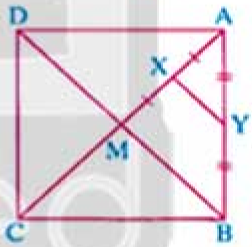


12 **In the opposite figure :**

$\because$ ABCD is a square, X and Y are the midpoints

of $\overline{AM}$ and $\overline{AB}$ respectively and AC = 12 cm.

$\therefore$ XY = ... cm., m ($\angle$ AYX) = ...°



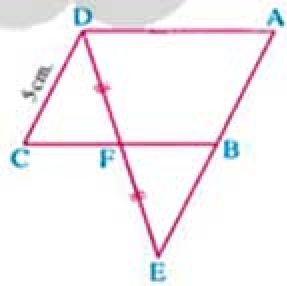
2 **In the opposite figure :**

ABCD is a parallelogram, DC = 5 cm.

, F $\in \overline{BC}$, $\overrightarrow{DF} \cap \overrightarrow{AB} = \{E\}$,

if DF = FE

Complete the following proof to find the length of $\overline{BE}$



Given ....

R.T.F. ....

Proof $\because$ ABCD is a ....

$\therefore$ $\overline{AD}$ // .... $\therefore$ $\overline{BF}$ // ....

, In Δ AED :

$\because$ F is the midpoint of .... , $\overline{BF}$ // ....

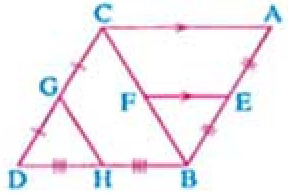
$\therefore$ B is the midpoint of ....

$\therefore$ AB = ...

, $\because$ AB = ... = ... cm. (Properties of ....)

$\therefore$ BE = ... cm. (The req.)

3 **In the opposite figure :**



ABC is a triangle in which CA = CB,

E is the midpoint of $\overline{AB}$, $\overline{EF} \parallel \overline{AC}$,

H and G are the two midpoints of $\overline{BD}$ and $\overline{CD}$ respectively.

Complete the following proof to prove that : EF = GH

Given ....

R.T.P. ....

Proof In Δ ABC :

$\because$ E is the midpoint of .... , $\overline{EF} \parallel$ ....

$\therefore$ F is the midpoint of ....

$\therefore$ EF = $\frac{1}{2}$ ....

, In Δ BDC :

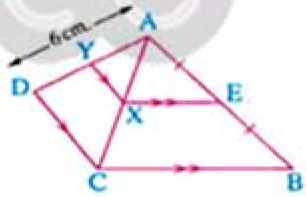
$\because$ H is the midpoint of ....

, ... is the midpoint of $\overline{CD}$

$\therefore$ GH = $\frac{1}{2}$ ....

, $\because$ CA = ... $\therefore$ EF = ... (Q.E.D.)

4 **In the opposite figure :**



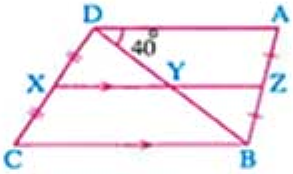
AE = EB , $\overline{EX} \parallel \overline{BC}$,

$\overline{XY} \parallel \overline{CD}$

and AD = 6 cm.

Find the length of : $\overline{AY}$ « 3 cm. »

5 **In the opposite figure :**



X is the midpoint of $\overline{CD}$

, Z is the midpoint of $\overline{AB}$

, $\overline{XY} \parallel \overline{CB}$, m ($\angle$ ADB) = 40°

Find : m ($\angle$ ZYB) « 40° »

Unit 3

6 **In the opposite figure :**

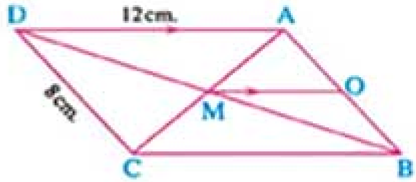
ABCD is a parallelogram , $\overline{AC} \cap \overline{BD} = \{M\}$

Draw $\overline{MO}$ // $\overline{AD}$ to cut $\overline{AB}$ at O

If AD = 12 cm. and DC = 8 cm.

, then find : 1 The perimeter of ABCD

2 The length of $\overline{AO}$



« 40 cm. , 4 cm. »

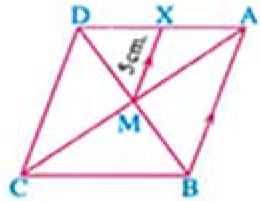
7 **In the opposite figure :**

ABCD is a parallelogram , its diagonals intersect at M

Draw $\overrightarrow{MX}$ // $\overline{BA}$ to intersect $\overline{AD}$ at X

1 **Prove that :** X is the midpoint of $\overline{AD}$

2 If MX = 5 cm. , then find the length of $\overline{CD}$



« 10 cm. »

8 ABCD is a parallelogram , its diagonals intersect at M , $\overrightarrow{ME}$ // $\overline{BA}$ and cuts $\overline{AD}$ at E

Prove that : ME = $\frac{1}{2}$ DC

9 **In the opposite figure :**

ABCD is a parallelogram ,

BC = CH , H $\in \overrightarrow{BC}$

Draw $\overline{AH}$ to cut $\overline{DC}$ at O

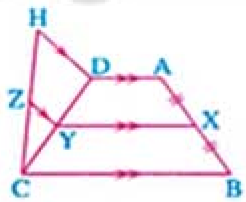
Prove that : AO = OH

10 **In the opposite figure :**

ABCD is a trapezium , $\overline{AD}$ // $\overline{BC}$, X is the midpoint of $\overline{AB}$

If $\overline{AD}$ // $\overline{XY}$ where Y $\in \overline{DC}$, $\overline{YZ}$ // $\overline{DH}$

Prove that : CZ = ZH

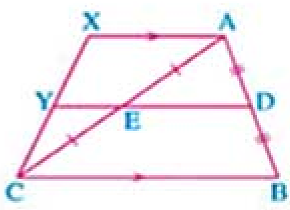


11 **In the opposite figure :**

AD = DB , AE = EC ,

$\overline{AX}$ // $\overline{BC}$, $\overrightarrow{DE} \cap \overline{XC} = \{Y\}$

Prove that : Y is the midpoint of $\overline{XC}$



12 **In the opposite figure :**

ABCD is a quadrilateral in which :

X and Z are the midpoints of $\overline{AB}$

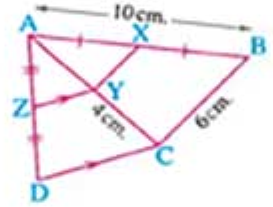
and $\overline{AD}$ respectively and $Y \in \overline{AC}$ such that :

$\overline{YZ} \parallel \overline{CD}$ and YC = 4 cm.

If BC = 6 cm. and AB = 10 cm.

, then find : 1 The length of $\overline{AY}$

2 The perimeter of Δ AXY



« 4 cm. , 12 cm. »

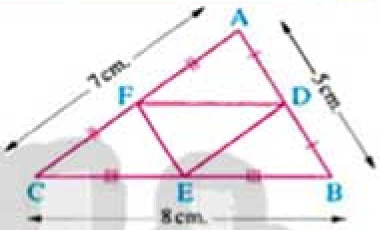
13 **In the opposite figure :**

AB = 5 cm., BC = 8 cm.,

AC = 7 cm., D , E and F are the midpoints of

$\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

Calculate the perimeter of : Δ DEF



« 10 cm. »

14 **In the opposite figure :**

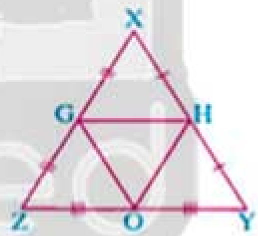
XYZ is a triangle in which :

H , O and G are the midpoints of $\overline{XY}$, $\overline{YZ}$

and $\overline{ZX}$ respectively.

If the perimeter of Δ HOG is 18 cm. ,

then find the perimeter of : Δ XYZ



« 36 cm. »

15 ABC is a triangle , if X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively.

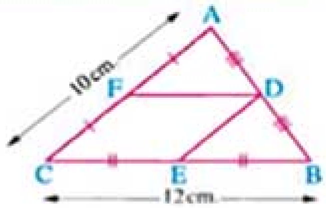
Prove that : The perimeter of $\Delta XYZ = \frac{1}{2}$ that of Δ ABC

16 **In the opposite figure :**

ABC is a triangle in which D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively,

BC = 12 cm., AC = 10 cm.

Find the perimeter of the figure DECF



« 22 cm. »

Unit 3

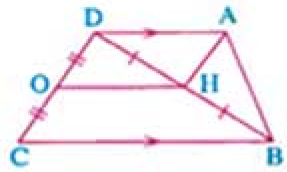
17 **In the opposite figure :**

$\overline{AD} \parallel \overline{BC}$, $AD = \frac{1}{2}$ BC ,

H is the midpoint of $\overline{BD}$,

O is the midpoint of $\overline{CD}$

Prove that : AHOD is a parallelogram.

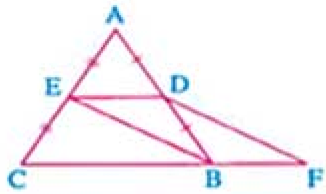


18 **In the opposite figure :**

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively,

$F \in \overrightarrow{CB}$ where $BF = \frac{1}{2} BC$

Prove that : BEDF is a parallelogram.



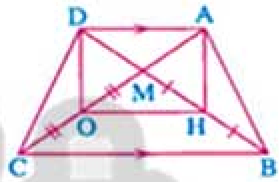
19 **In the opposite figure :**

ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$

and $AD = \frac{1}{2}$ BC and $\overline{AC} \cap \overline{DB} = \{M\}$

Let H and O be the midpoints of $\overline{MB}$ and $\overline{MC}$ respectively.

Prove that : AHOD is a parallelogram.

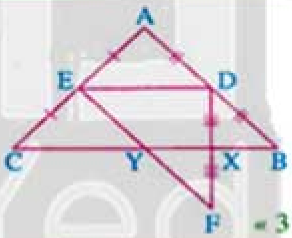


20 **In the opposite figure :**

D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$,

$\overline{DF} \cap \overline{BC} = \{X\}$, DX = XF , BC = 12 cm.

Find the length of : $\overline{XY}$



« 3 cm. »

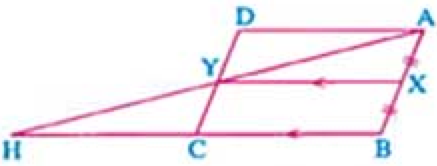
21 **In the opposite figure :**

ABCD is a parallelogram, X is the midpoint of $\overline{AB}$

Draw $\overrightarrow{XY} \parallel \overline{BC}$ to cut $\overline{DC}$ at Y

Draw $\overrightarrow{AY}$ to cut $\overrightarrow{BC}$ at H

Prove that : C is the midpoint of $\overline{BH}$



22 **In the opposite figure :**

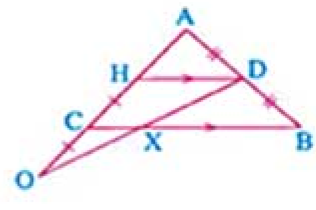
ABC is a triangle , D is the midpoint of $\overline{AB}$,

$\overrightarrow{DH} \parallel \overline{BC}$, $O \in \overrightarrow{AC}$ such that HC = CO

Prove that : $CO = \frac{1}{3}$ AO

If we draw $\overline{DO}$ to cut $\overline{BC}$ at X ,

then prove that : OX = XD



Exercise 6

23 **In the opposite figure :**

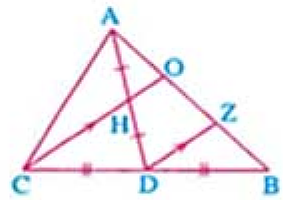
ABC is a triangle , D is the midpoint of $\overline{BC}$

and H is the midpoint of $\overline{AD}$

Draw $\overrightarrow{CH}$ to cut $\overline{AB}$ at O ,

then draw $\overrightarrow{DZ}$ // $\overline{CO}$ to cut $\overline{AB}$ at Z

Prove that : AO = OZ = ZB



24 ABCD is a parallelogram. M is the intersection point of its diagonals ,

Draw $\overleftrightarrow{CE}$ // $\overline{BD}$ to cut $\overrightarrow{AB}$ at E and $\overrightarrow{AD}$ at F

Prove that : 1 AB = BE

2 AD = DF

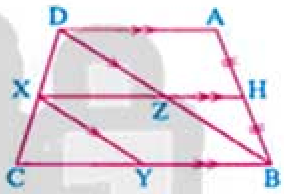
25 **In the opposite figure :**

ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$

Let H be the midpoint of $\overline{AB}$,

$\overline{HX}$ // $\overline{BC}$ and $\overline{XY}$ // $\overline{DB}$

Prove that : Y is the midpoint of $\overline{BC}$



26 ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$, E is the midpoint of $\overline{AB}$, draw $\overrightarrow{EX}$ // $\overline{BC}$ to cut $\overline{DB}$ at X , $\overline{DC}$ at Y , and draw $\overrightarrow{YZ}$ // $\overline{DB}$ to cut $\overline{BC}$ at Z

Prove that : XD = YZ

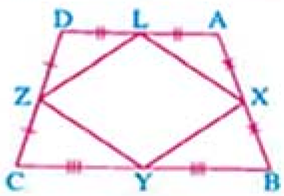
27 ABC is a triangle in which AB = 9 cm. , AC = 8 cm. , D $\in \overline{AB}$, E $\in \overline{AB}$ such that AD = DE = EB and $\overrightarrow{DX}$, $\overrightarrow{EY}$ are drawn parallel to $\overline{BC}$ and cutting $\overline{AC}$ at X and Y respectively , where DX = 4 cm.

Calculate : The perimeter of the shape DEYX « $17\frac{2}{3}$ cm. »

28 **In the opposite figure :**

ABCD is a quadrilateral in which X , Y , Z and L are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

Prove that : XYZL is a parallelogram.

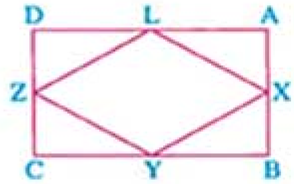


29 ABC is a triangle in which AB = AC , X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively. **Prove that :** AXYZ is a rhombus.

Unit 3

30 **In the opposite figure :**

ABCD is a rectangle and X , Y , Z and L are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.



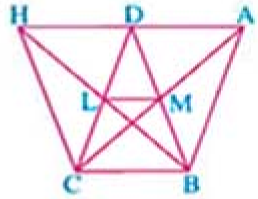
Prove that :

1. XYZL is a rhombus.
2. The perimeter of the rhombus = 2 BD

31 **In the opposite figure :**

ABCD and DBCH are two parallelograms having a common base $\overline{BC}$ and on one side of $\overline{BC}$

Let $\overline{AC} \cap \overline{BD} = \{M\}$ and $\overline{DC} \cap \overline{BH} = \{L\}$

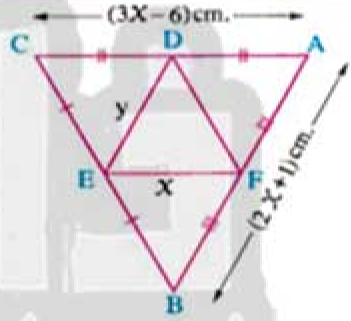


Prove that : 1 $\overline{ML} \parallel \overline{BC}$ 2 $ML = \frac{1}{4}$ AH

32 **Connecting with algebra :**

In the opposite figure :

Find the value of each of : x and y



« 6 cm. , 6.5 cm. »

Life Application

33 Sara wants to design a kite whose two diagonals are of length 64 cm. and 90 cm.
She wants to put a stripe to decorate the kite such that the stripe joins the midpoints of the sides of the kite.
How long is the stripe ?

« 154 cm. »

For excellent pupils

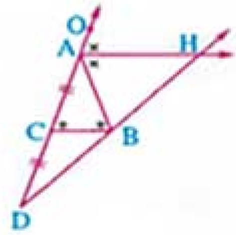
34 **In the opposite figure :**

ABC is a triangle in which : m (∠ ABC) = m (∠ ACB)

, D $\in \overrightarrow{AC}$ such that AC = CD and O $\in \overrightarrow{CA}$

Let $\overrightarrow{AH}$ bisect ∠ BAO such that : $\overrightarrow{AH} \cap \overrightarrow{DB} = \{H\}$

Prove that : DB = BH



35 ABCD is a quadrilateral in which $\overline{AC} \perp \overline{BD}$ and X , Y , Z and L are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

Prove that : XYZL is a rectangle whose area equals $\frac{1}{4}$ AC × BD

Exercise 7 Pythagoras' theorem

From the school book

1 **In each of the following figures , find the length of the unknown side :**

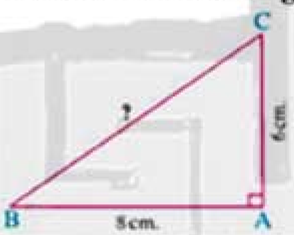


Fig. (1)

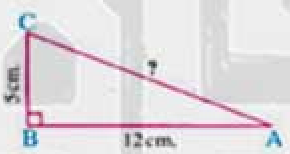


Fig. (2)

Fig. (3)

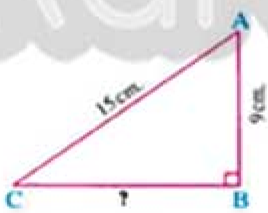


Fig. (4)

Fig. (5)

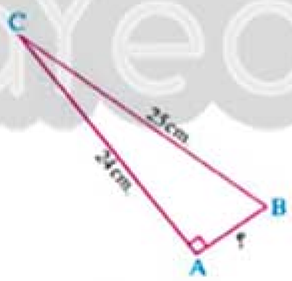


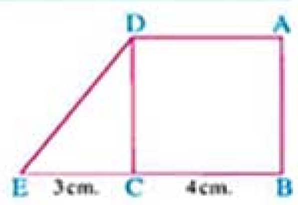
Fig. (6)

2 **In the opposite figure :**

ABCD is a square whose side length = 4 cm.

and $E \in \overrightarrow{BC}$ where CE = 3 cm.

Complete the following proof to find the length of $\overline{DE}$



$\because$ ABCD is a square. $\therefore$ DC = ... cm. , m ($\angle$ DCB) = ...°

$\therefore$ In Δ DCE : $(DE)^2 = (\text{..........})^2 + (CE)^2 = (\text{..........})^2 + (3)^2 = \text{..........} + \text{..........} = \text{..........}$

$\therefore$ DE = ... cm. (The req.)

3 **In the opposite figure :**

$\overline{AD} \perp \overline{BC}$, BD = 9 cm. ,

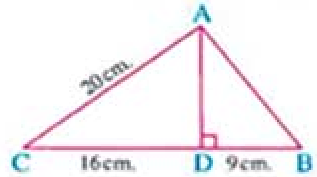
DC = 16 cm. and AC = 20 cm.

Find :

1 AD

2 AB

3 The area of Δ ABC



« 12 cm. , 15 cm. , 150 cm^2. »

4 **In the opposite figure :**

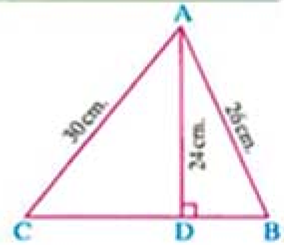
ABC is a triangle and $\overline{AD} \perp \overline{BC}$

If AD = 24 cm.

, AB = 26 cm.

and AC = 30 cm.

Find BC and calculate the area of Δ ABC



« 28 cm. , 336 cm^2. »

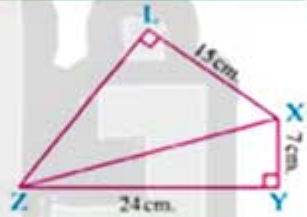
5 **In the opposite figure :**

XYZL is a quadrilateral in which :

m ($\angle$ XYZ) = m ($\angle$ XLZ) = 90° ,

XY = 7 cm. , YZ = 24 cm. and XL = 15 cm.

Find : The length of each of $\overline{XZ}$ and $\overline{ZL}$



« 25 cm. , 20 cm. »

6 **In the opposite figure :**

m ($\angle$ B) = m ($\angle$ ACD) = 90° , AB = 9 cm. ,

BC = 12 cm. and DC = 20 cm.

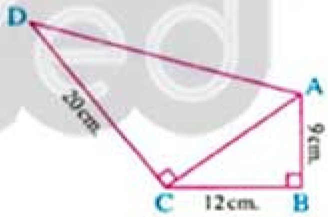
Find :

1 The length of $\overline{AC}$

2 The length of $\overline{AD}$

3 The perimeter of the figure ABCD

4 The area of the figure ABCD



« 15 cm. , 25 cm. , 66 cm. , 204 cm^2. »

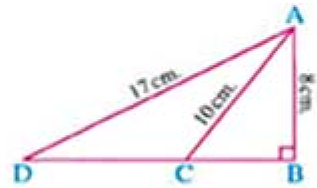
7 **In the opposite figure :**

Δ ABD is a right-angled triangle at B

, AB = 8 cm. , AD = 17 cm.

and C $\in \overline{BD}$ such that AC = 10 cm.

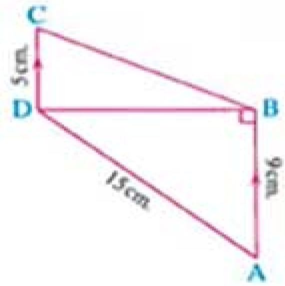
Find : The length of each of $\overline{CB}$, $\overline{BD}$ and $\overline{CD}$



« 6 cm. , 15 cm. , 9 cm. »

Unit 3

8 **In the opposite figure :**



m (∠ ABD) = 90° , $\overline{BA} \parallel \overline{CD}$

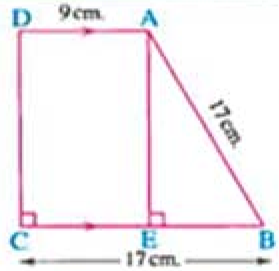
, AB = 9 cm. , AD = 15 cm.

and DC = 5 cm.

Calculate the length of : $\overline{BC}$

« 13 cm. »

9 **In the opposite figure :**



ABCD is a trapezium , $\overline{AD} \parallel \overline{BC}$

, m (∠ DCB) = 90° , $\overline{AE} \perp \overline{BC}$

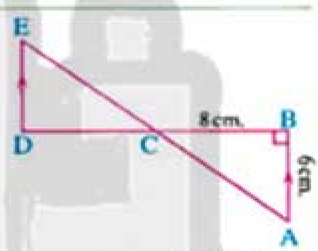
If AB = BC = 17 cm. and AD = 9 cm.

, find the length of $\overline{DC}$ and calculate

the area of the trapezium ABCD

« 15 cm. , 195 cm.^{2} »

10 **In the opposite figure :**



$\overline{BD} \cap \overline{AE} = \{C\}$, $\overline{AB} \parallel \overline{DE}$

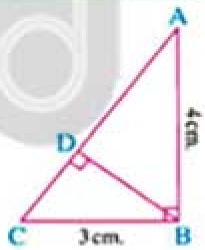
, AB = 6 cm. , BC = 8 cm.

and C is the midpoint of $\overline{BD}$

Calculate the length of : $\overline{CE}$

« 10 cm. »

11 **In the opposite figure :**



ABC is a right-angled triangle at B

, $\overline{BD} \perp \overline{AC}$, AB = 4 cm.

and BC = 3 cm.

Calculate the length of : $\overline{BD}$

« 2.4 cm. »

12 **Complete the following :**

1. In the right-angled triangle , the area of the square on the hypotenuse equals ....
2. If XYZ is a right-angled triangle at X , XY = 12 cm. and XZ = 9 cm.,
   then YZ = ... cm.
3. If ABC is a right-angled triangle at B , AB = 20 cm. and AC = 25 cm. ,
   then BC = ... cm.

4 **In the opposite figure :**

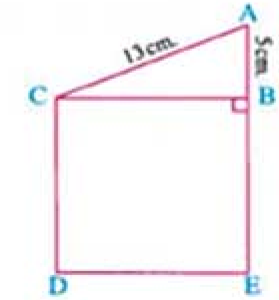
If m (∠ ABC) = 90°

, AB = 5 cm.

and AC = 13 cm.

, then the area of

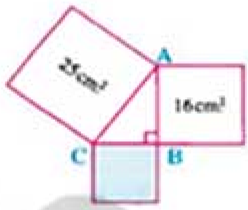
the square BEDC = ·········· cm^2.



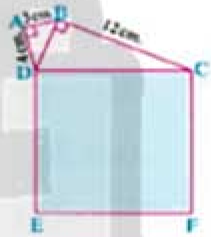
5 A rectangle is of length 8 cm. and width 6 cm. ,
then the length of its diagonal equals ·········· cm.

6 If the area of a rectangle equals 60 cm^2 and its width is 5 cm. ,
then the length of its diagonal = ·········· cm.

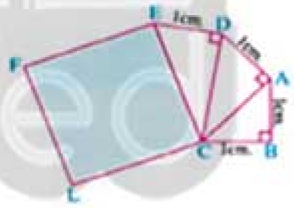
7 If Δ ABC is right-angled at B ,
then the side length of the shaded
square = ·········· cm.



8 If Δ ABD is right-angled at A
and Δ BCD is right-angled at B ,
then the area of the shaded
square = ·········· cm^2.



9 If the triangles ABC , ACD and DCE
are right-angled at B , A and D respectively,
AB = BC = AD = DE = 1 cm.
, then the area of the shaded
square = ·········· cm^2.



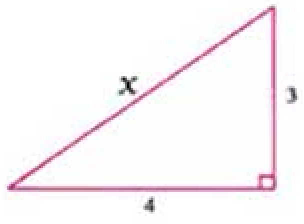
13 **Choose the correct answer from those given :**

1 **In the opposite figure :**

Which of the following relations is true ?

(a) $x = 4^2 + 3^2$ (b) $x^2 = 4^2 - 3^2$

(c) $x^2 + 9 = 16$ (d) $x^2 = 25$

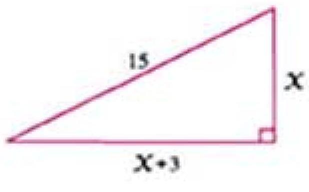


2 **In the opposite figure :**

Which of the following relations is true ?

(a) $x + 3 + x = 15$ (b) $x^2 + 3x = 108$

(c) $(x + 3)^2 = 15 - x^2$ (d) $x^2 + 6x + 9 = 225$

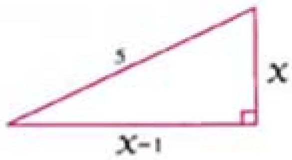


3 **In the opposite figure :**

Which of the following relations is true ?

(a) $x^2 + (x-1)^2 = 5$ (b) $x + (x-1) = 25$

(c) $x^2 - x = 12$ (d) $(x-1)^2 - x^2 = 25$



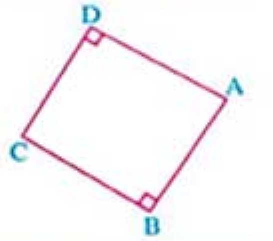
4 If ABCD is a square , then $(AC)^2 = \cdots\cdots$

(a) AB (b) $(AB)^2$ (c) $2\,(AB)^2$ (d) $4\,(AB)^2$

14 **In the opposite figure :**

If m (∠ B) = m (∠ D) = 90°

Prove that : $(AB)^2 + (BC)^2 = (AD)^2 + (DC)^2$



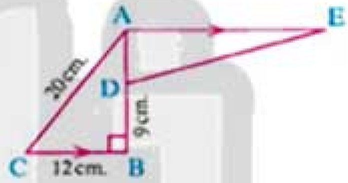
15 **In the opposite figure :**

ABC is a triangle , m (∠ B) = 90°

, $\overline{AE} \parallel \overline{BC}$, If BC = 12 cm. , AC = 20 cm.

, D ∈ $\overline{AB}$ where BD = 9 cm. and AE = 2 BC

Find : The length of each of $\overline{AD}$ and $\overline{ED}$



« 7 cm. , 25 cm. »

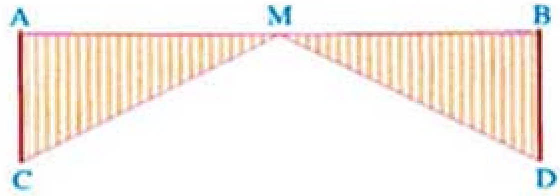
Life Applications

16 A window cleaner has a ladder which is 5 metres long.
He places it so that it reaches a window till 4 metres from the ground.
How far is the wall from the foot of the ladder ?

« 3 m. »

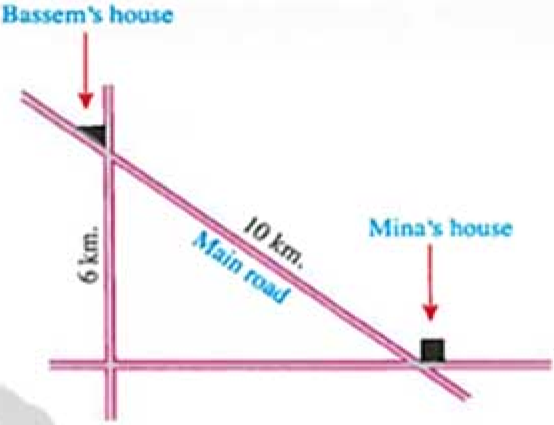
17 A wooden bridge $\overline{AB}$ of length 15 m. is built horizontally on two vertical walls $\overline{AC}$ and $\overline{BD}$ resting on two supports $\overline{MC}$ and $\overline{MD}$ if AC = 4 m. and AM = MB

Calculate the length of the support $\overline{MC}$



« 8.5 m. »

18 If Mina wants to go to the house of his friend Bassem.
What is the distance saved if he takes the main road instead of the other two roads ?



« 4 km. »

For excellent pupils

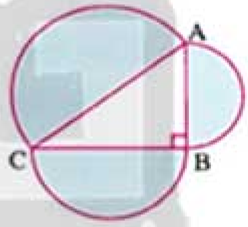
19 If Δ ABC is right-angled at B , D is the midpoint of $\overline{BC}$,

prove that : $(AC)^2 - (AD)^2 = 3\,(BD)^2$

20 **In the opposite figure :**

Prove that the sum of areas of the two semicircles drawn on the two sides of the right angle in a right-angled triangle equals the area of the semicircle drawn on the hypotenuse.

[Given the area of the circle = $\pi\, r^2$]



Summary of the second part of unit 3
"From lesson 5 to lesson 7"

- The sum of the measures of the interior angles of a triangle is 180°
- The measure of an exterior angle of a triangle is equal to the sum of the measures of its non adjacent interior angles.
- If two angles of one triangle equal two angles of another triangle in measure , then the third angle of the first triangle is equal in measure to the third angle of the other triangle.
- If the sum of measures of two angles in a triangle equals 90° , then the third angle is right.
- If the sum of measures of two angles in a triangle is less than 90° , then the third angle is obtuse.
- If the sum of measures of two angles in a triangle is more than 90° , then the third angle is acute.
- If the measure of an angle in a triangle equals the sum of measures of the other two angles , then the triangle is right-angled.
- The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.
- The line segment joining the midpoints of two sides of a triangle is parallel to the third side.
- The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side.

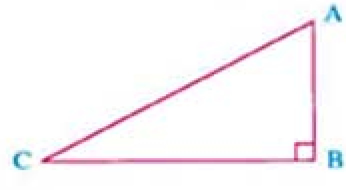
Pythagoras' theorem :

The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.

In the opposite figure :

If Δ ABC is right-angled at B , then

- $(AC)^2 = (AB)^2 + (BC)^2$
- $(AB)^2 = (AC)^2 - (BC)^2$
- $(BC)^2 = (AC)^2 - (AB)^2$



Exams on the second part of unit three
from lesson (5) to lesson (7)

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. The measure of the exterior angle at any vertex of the equilateral triangle = ...
   (a) 60° (b) 120° (c) 150° (d) 30°
2. A rectangle of length 4 cm. and width 3 cm. , then the length of its diagonal = ...
   (a) 25 cm. (b) 3 cm. (c) 4 cm. (d) 5 cm.
3. Any triangle has at least two .... angles.
   (a) acute (b) obtuse (c) right (d) reflex
4. If the measures of two angles in a triangle are 30° and 50° , then the triangle is ....
   (a) acute-angled. (b) right-angled.
   (c) obtuse-angled. (d) equilateral.
5. If Δ ABC is right at B , AB = 20 cm. and AC = 25 cm. , then BC = ... cm.
   (a) 225 (b) 400 (c) 15 (d) 10
6. In Δ ABC : if m ($\angle$ B) = m ($\angle$ A) + m ($\angle$ C) , then $\angle$ B is ....
   (a) acute. (b) right. (c) obtuse. (d) reflex.

2 Complete the following :

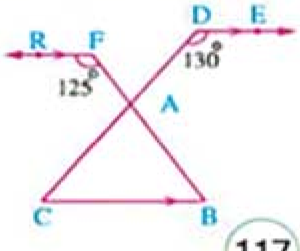
1. The measure of the exterior angle of a triangle equals ....
2. In Δ ABC : if m ($\angle$ B) > m ($\angle$ A) + m ($\angle$ C) , then $\angle$ A is ....
3. The line segment joining the midpoints of two sides of a triangle is .... to the third side.
4. In the right-angled triangle , the area of the square on the hypotenuse equals ....
5. The ray drawn from the midpoint of a side in a triangle parallel to another side ....

3 [a] In the opposite figure :

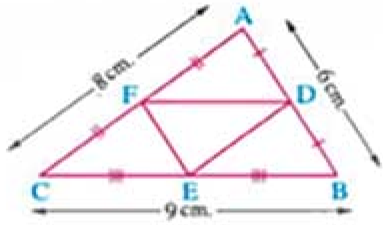
$\overrightarrow{DE}$ // $\overrightarrow{FR}$ // $\overline{BC}$

, m ($\angle$ CDE) = 130° and m ($\angle$ RFB) = 125°

Calculate the measures of the angles of : Δ ABC



[b] In the opposite figure :

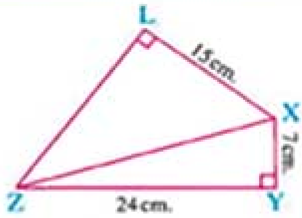


AB = 6 cm. , BC = 9 cm.

, AC = 8 cm. , D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

Calculate the perimeter of : Δ DEF

4 **[a] In the opposite figure :**



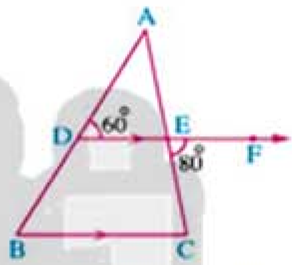
XYZL is quadrilateral in which :

m ($\angle$ XYZ) = m ($\angle$ XLZ) = 90° , XY = 7 cm.

, YZ = 24 cm. and XL = 15 cm.

Find the length of each of : $\overline{XZ}$ and $\overline{LZ}$

[b] In the opposite figure :



ABC is a triangle , m ($\angle$ ADE) = 60°

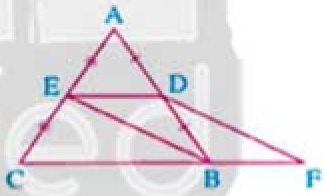
, m ($\angle$ FEC) = 80°

, D $\in \overline{AB}$, $\overrightarrow{DF}$ // $\overline{BC}$

and $\overline{AC} \cap \overrightarrow{DF}$ = {E}

Find : The measures of the interior angles of Δ ABC

5 **[a] In the opposite figure :**

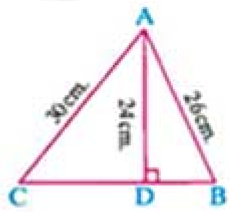


D and E are midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, F $\in \overrightarrow{CB}$ where BF = $\frac{1}{2}$ BC

Prove that : BEDF is a parallelogram.

[b] In the opposite figure :



ABC is a triangle in which : $\overline{AD} \perp \overline{BC}$

If AD = 24 cm. , AB = 26 cm. and AC = 30 cm.

1 **Find :** BC

2 **Calculate the area of :** Δ ABC

Model 2

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1 In Δ ABC , m ($\angle$ A) + m ($\angle$ B) = 110°

, then m ($\angle$ C) = ...

(a) 110° (b) 90° (c) 70° (d) 55°

2 Δ XYZ is right-angled at Y , XY = 12 cm. , YZ = 5 cm.
, then XZ = ... cm.

(a) 169 (b) 25 (c) 17 (d) 13

3 Δ ABC is right-angled at B , if the measure of the exterior angle at A is 120°
, then m ($\angle$ C) = ...

(a) 60° (b) 90° (c) 120° (d) 30°

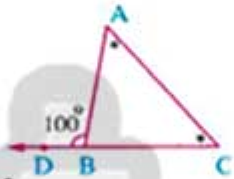
4 The length of the line segment joining the midpoints of two sides of a triangle is equal to .... the length of the third side.

(a) twice (b) half (c) third (d) quarter

5 In Δ ABC , m ($\angle$ A) = m ($\angle$ C) – m ($\angle$ B) , m ($\angle$ B) = 50°
, then m ($\angle$ A) = ...

(a) 40° (b) 90° (c) 50° (d) 45°

6 **In the opposite figure :**



D $\in \overrightarrow{CB}$, m ($\angle$ ABD) = 100°
, m ($\angle$ A) = m ($\angle$ C) , then m ($\angle$ C) = ...

(a) 40° (b) 80° (c) 50° (d) 100°

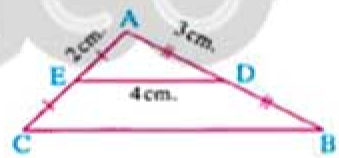
2 Complete the following :

1 If the measure of an angle of a triangle is greater than the sum of measures of the other two angles , then the triangle is ....

2 In Δ ABC , m ($\angle$ B) = 90° , then $(AB)^2$ = ...

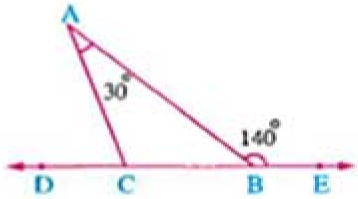
3 **In the opposite figure :**

The perimeter of the figure DBCE = ... cm.



4 **In the opposite figure :**

m ($\angle$ ACD) = ...°



5 If the area of a rectangle is 12 cm^2 and its length is 4 cm. , then its diagonal length = ... cm.

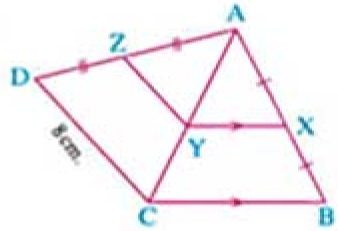
3 **[a] In the opposite figure :**

X is the midpoint of $\overline{AB}$

, $\overline{XY} \parallel \overline{BC}$, Z is the midpoint of $\overline{AD}$

, CD = 8 cm

Find : The length of $\overline{YZ}$

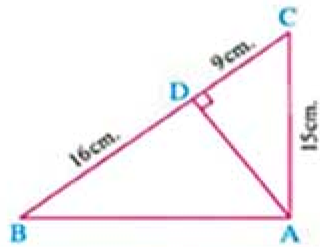


[b] In the opposite figure :

AC = 15 cm. , DC = 9 cm.

, DB = 16 cm. , $\overline{AD} \perp \overline{BC}$

Find with proof the length of each of : $\overline{AB}$, $\overline{AD}$



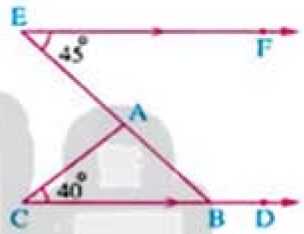
4 **[a] In the opposite figure :**

$\overrightarrow{EF} \parallel \overrightarrow{CD}$, m (∠ E) = 45°

, m (∠ C) = 40°

Find :

The measures of angles of Δ ABC , m (∠ ABD)



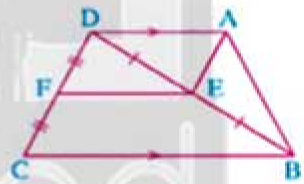
[b] In the opposite figure :

$\overline{AD} \parallel \overline{BC}$, AD = $\frac{1}{2}$ BC

, E is the midpoint of $\overline{BD}$

, F is the midpoint $\overline{CD}$

Prove that : AEFD is a parallelogram.



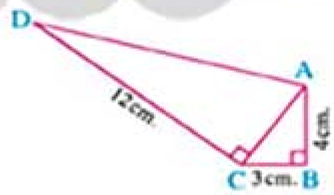
5 **[a] In the opposite figure :**

m (∠ B) = m (∠ ACD) = 90°

, AB = 4 cm. , BC = 3 cm. , CD = 12 cm.

Find : 1 The perimeter of the figure ABCD

2 The area of the figure ABCD



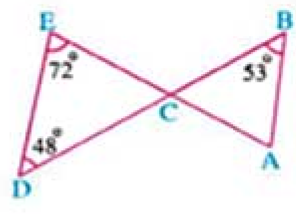
[b] In the opposite figure :

m (∠ B) = 53° , m (∠ E) = 72°

, m (∠ D) = 48°

Find : 1 m (∠ A)

2 m (∠ BCE)



Exercise 8

Geometric transformations

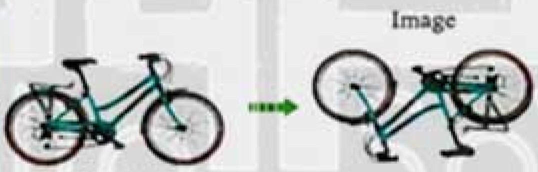
From the school book

1 **Describe the type of the geometric transformation (reflection , translation or rotation) in each of the following :**

1 Original

Image

2 Image



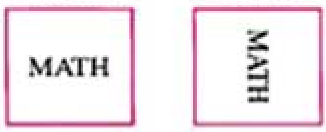
Original

3 Original

Image

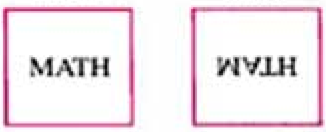
2 **Write below each shape the type of the geometric transformation (reflection , translation or rotation) :**

1 Original Image



....

2 Original Image



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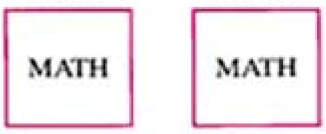
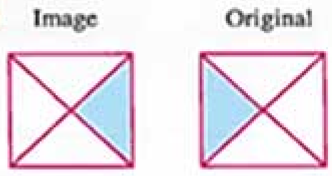
3 Original Image



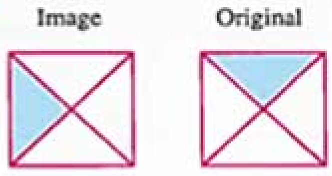
....

Unit 3

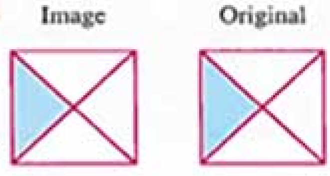
3 **Write below each shape the type of the geometric transformation (reflection , translation or rotation) :**

1


……………………………

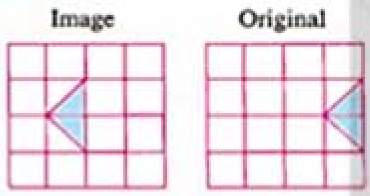
2


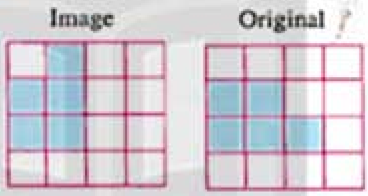
……………………………

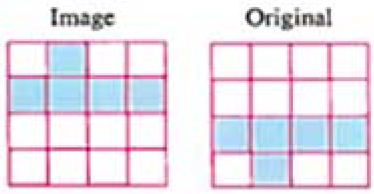
3


……………………………

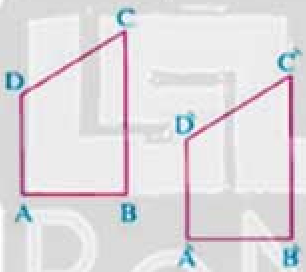
4 **Write the type of the geometric transformation in each of the following shapes :**

1


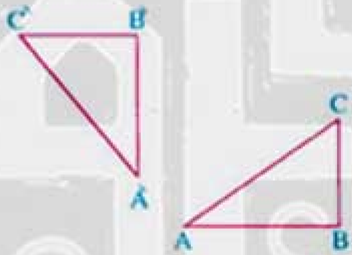
2


3


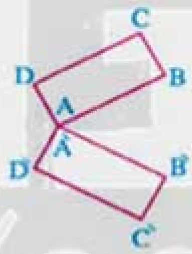
5 **Describe the type of transformation in each of the following figures (reflection , translation , rotation) :**



1 ……………………

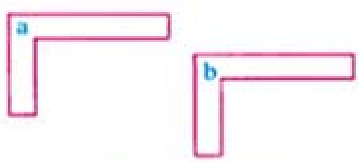


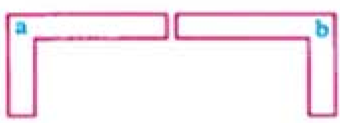
2 ……………………

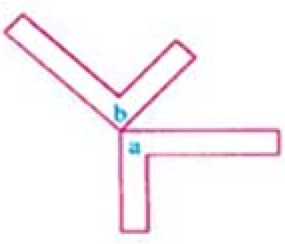


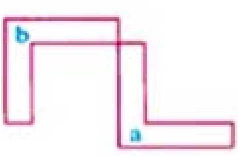
3 ……………………

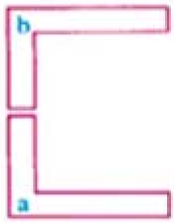
6 **Figure b is the image of figure a by a geometric transformation. Identify each transformation as (a translation , a reflection or a rotation) :**

1


2


3


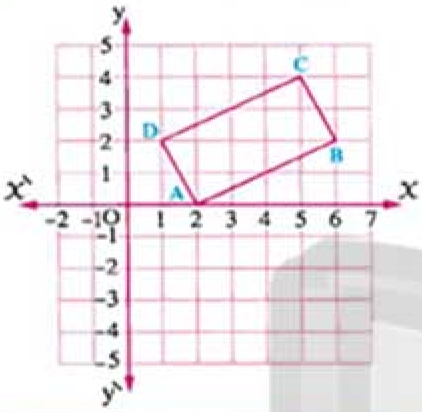
4


5


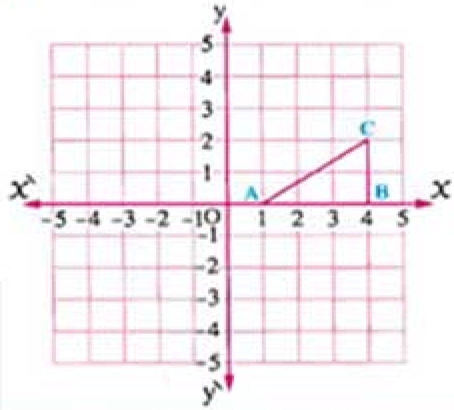
Exercise 8

7 **Draw the image of each figure according to the shown transformation , then describe each type :**

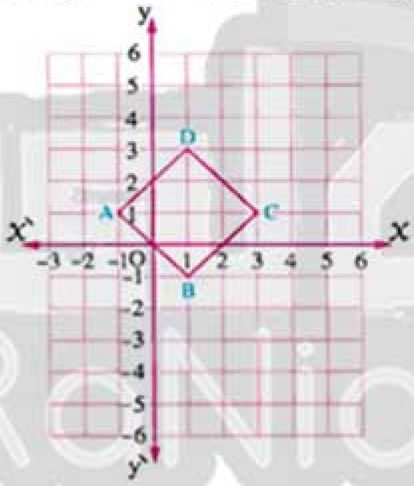
1 $(x , y) \longrightarrow (-x , y)$



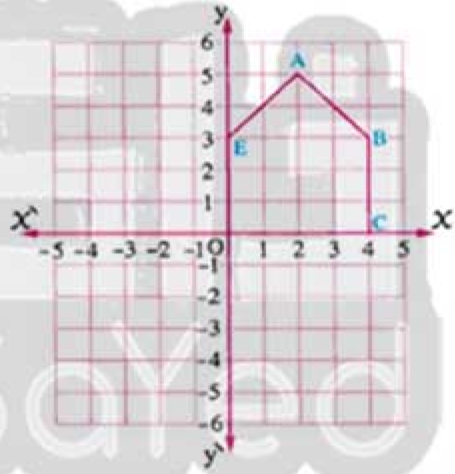
2 $(x , y) \longrightarrow (-x , -y)$



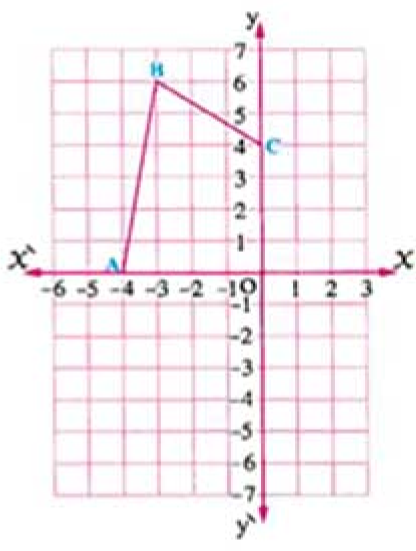
3 $(x , y) \longrightarrow (x + 2 , y + 3)$



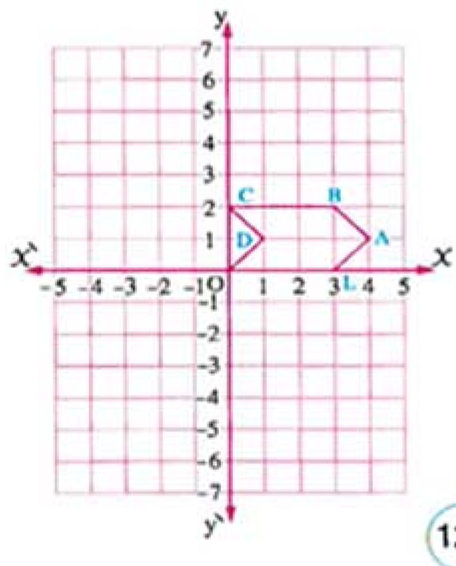
4 $(x , y) \longrightarrow (x , y - 3)$



5 $(x , y) \longrightarrow (y , -x)$



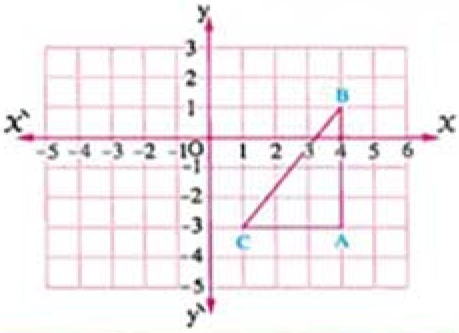
6 $(x , y) \longrightarrow (-x , y)$



Unit 3

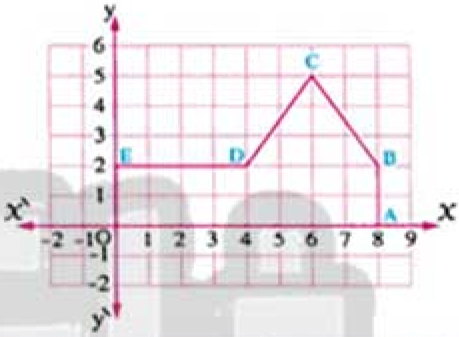
8 **Map the image of ΔABC where $A(4, -3)$, $B(4, 1)$, $C(1, -3)$ according to the following transformations then describe its type :**

1. $(x, y) \longrightarrow (-x, y)$
2. $(x, y) \longrightarrow (-x, -y)$
3. $(x, y) \longrightarrow (x, y-2)$
4. $(x, y) \longrightarrow (-y, x)$



9 **Draw the image of the polygon ABCDEO according to each transformation and describe the type :**

1. $(x, y) \longrightarrow (-x, y)$
2. $(x, y) \longrightarrow (x, y+5)$
3. $(x, y) \longrightarrow (-x, -y)$
4. $(x, y) \longrightarrow (x-5, y)$
5. $(x, y) \longrightarrow (x, -y)$



10 **Draw the image of ΔABC where $A(1, 2)$, $B(3, 2)$ and $C(3, 5)$ by the following transformations :**

1. $(x, y) \longrightarrow (x, -y)$
2. $(x, y) \longrightarrow (x+1, y-3)$
3. $(x, y) \longrightarrow (-y, x)$

11 **On a square lattice, draw ΔABO where $A(3, 1)$, $B(1, 3)$ and O is the origin point, and draw its images by the following transformations then discribe its type :**

1. $(x, y) \longrightarrow (x+1, y-2)$
2. $(x, y) \longrightarrow (x, -y)$
3. $(x, y) \longrightarrow (-y, x)$
4. $(x, y) \longrightarrow (-x, -y)$

12 **On a square lattice, draw the quadrilateral ABCD where $A(1, 1)$, $B(4, 2)$, $C(3, 4)$ and $D(1, 4)$, and draw its image by the following transformation, then discribe its type :**

1. $(x, y) \longrightarrow (y, -x)$
2. $(x, y) \longrightarrow (-x, y)$
3. $(x, y) \longrightarrow (x-1, y+1)$

For excellent pupils

13 Draw ΔABC whose image $\Delta \grave{A}\grave{B}\grave{C}$ by the transformation $(x, y) \longrightarrow (-y, x)$ where $\grave{A}(1, -1)$, $\grave{B}(3, 1)$ and $\grave{C}(4, -1)$, then discribe the transformation type.

Exercise 9

Reflection in a straight line

From the school book

First Problems on reflection in the plane :

1 Find the image of each of A , $\overline{AB}$ and Δ ABC by reflection in the straight line L :

Fig. (1)

Fig. (2)

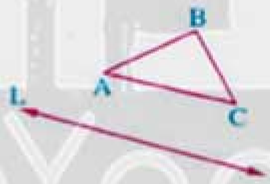


Fig. (3)

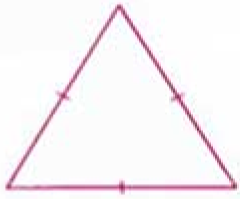
2 Copy the figures below in your notebook , then draw the images of Δ DEF and the circle M by reflection in L :

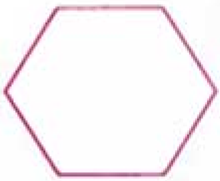
1

2

Unit 3

3 **Draw the axes of symmetry of each of the following figures if they are existed :**

1


2


3


4


5


6


4 **Draw the triangle ABC in which :** AB = 6 cm. , m (∠ A) = 90° and m (∠ B) = 30° , then draw its image by reflection in $\overleftrightarrow{AB}$

5 **Draw the image of Δ ABC in which :** AB = 3 cm. , BC = 4 cm. and AC = 5 cm. by reflection in the straight line containing the shortest side.

6 **Draw the image of Δ XYZ in which :** XY = 3 cm. , YZ = 5 cm. and ZX = 7 cm. by reflection in the straight line containing the longest side.

7 **Draw Δ ABC in which :** AB = 3 cm. , BC = 6 cm. and m (∠ ABC) = 90° , then find its image by reflection in the straight line L which is perpendicular to $\overline{BC}$ at C

8 **Draw the rectangle ABCD in which :** AB = 6 cm. and CB = 4 cm. , then draw its image by reflection in $\overleftrightarrow{AD}$. Say the name of the resulting figure which consists of the rectangle and its image , then find its perimeter. « 32 cm. »

9 Draw the image of the circle M whose radius length is 2 cm. , by reflection in the straight line which is far from the centre by 1 cm.

10 Draw the circle N with radius length 2.5 cm. , then draw its image by reflection in the straight line which is far from its centre by 2.5 cm.

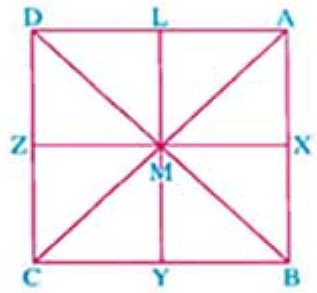
11 Draw Δ ABC where BC = 3 cm. , AB = 4 cm. and AC = 5 cm. If the point D is the image of the point C by reflection in $\overleftrightarrow{AB}$ **Find :**

1. The perimeter of Δ ACD
2. The area of Δ ACD « 16 cm. , 12 cm^2 »

12 **In the opposite figure :**

ABCD is a square. M is the point of intersection of its diagonals X , Y , Z and L are the midpoints of its sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

Complete the following :

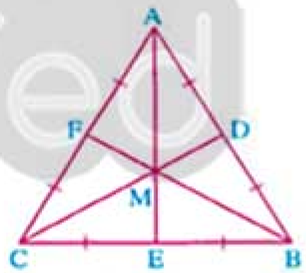


1. The image of the point A by reflection in $\overleftrightarrow{LY}$ is ....
2. The image of the $\overline{AM}$ by reflection in $\overleftrightarrow{XM}$ is ....
3. The image of the Δ ALM by reflection in $\overleftrightarrow{LY}$ is ....
4. The image of the Δ ALM by reflection in $\overleftrightarrow{XZ}$ is ....
5. The image of the Δ ALM by reflection in $\overleftrightarrow{AM}$ is ....
6. The image of the Δ AMB by reflection in $\overleftrightarrow{LY}$ is ....
7. The image of the Δ AMB by reflection in $\overleftrightarrow{XZ}$ is ....
8. The image of the square AXML by reflection in $\overleftrightarrow{LY}$ is .... and by reflection in $\overleftrightarrow{AM}$ is ....
9. The image of the square ABCD by reflection in $\overleftrightarrow{LY}$ is ....
10. Δ MZD is the image of Δ MZC by reflection in ....
11. Δ AXM is the image of Δ CYM by reflection in ....

13 **In the opposite figure :**

Δ ABC is an equilateral triangle , where D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively , and $\overline{AE} \cap \overline{BF} \cap \overline{CD} = \{M\}$:

Complete :



1. Axes of symmetry of Δ ABC are ....
2. $\overline{AB}$ is the reflected image of $\overline{AC}$ by reflection in ....
3. The reflected image of $\overline{AF}$ by reflection in $\overleftrightarrow{BF}$ is .... , and the reflected image of $\overline{CF}$ in $\overleftrightarrow{AE}$ is ....
4. The reflected image of Δ AMD by reflection in $\overleftrightarrow{AE}$ is ....
   $\therefore$ m ($\angle$ AMD) = m ($\angle$ ....) , because reflection in a line reserves ....
5. The reflected image of Δ AMB by reflection in $\overleftrightarrow{AE}$ is ....
6. Δ BMC is the reflected image of .... by reflection in $\overleftrightarrow{CD}$, and the reflected image of .... by reflection in $\overleftrightarrow{BF}$
   $\therefore$ BM = AM , and CM = AM , because the reflection reserves ....

14 **Complete the following :**

1. The reflection in a plane reserves :
   (a) .... (b) .... (c) .... (d) ....

Unit 3

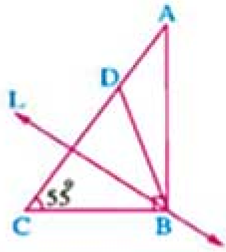
2 If the reflection in a straight line transforms the figure to itself then this straight line is called ....

3 **The number of axes of symmetry of :**

(a) The equilateral triangle is ....
(b) The isosceles triangle is ....
(c) The scalene triangle is ....
(d) The parallelogram is ....
(e) The rectangle is ....
(f) The rhombus is ....
(g) The square is ....
(h) The trapezium which is not isosceles is ....
(i) The isosceles trapezium is ....
(j) The circle is ....

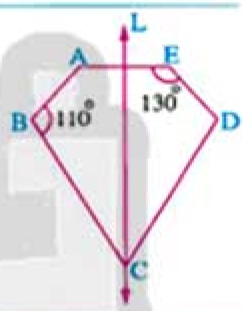
4 **In the opposite figure :**

If m (∠ ABC) = 90°
and m (∠ C) = 55° , the straight line L is the axis of symmetry of Δ DBC
, then m (∠ ABD) = ... °



15 **In the opposite figure :**

If the straight line L is the axis of symmetry of the figure ABCDE ,
calculate : m (∠ BCD)



« 60° »

16 By using geometric instruments , draw the rectangle ABCD , where AB = 3 cm. and BC = 4 cm. locate À as the reflected image of A by reflection in $\overleftrightarrow{CD}$ and locate C̀ as the reflected image of C by reflection in $\overleftrightarrow{AB}$

Prove that :

1 m (∠ C̀AC) = 2 m (∠ CAB)

2 $\overleftrightarrow{AC'}$ // $\overleftrightarrow{A'C}$

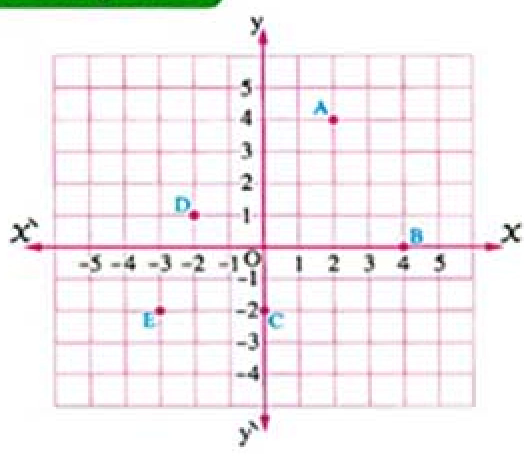
Second Problems on reflection in the Cartesian plane :

1 **In the opposite figure :**

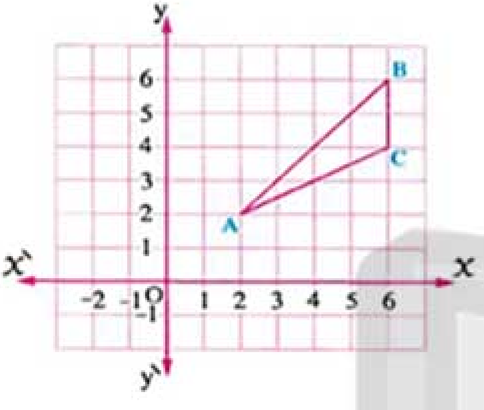
Write the coordinates of the image of each point by reflection in :

1 The x-axis

2 The y-axis

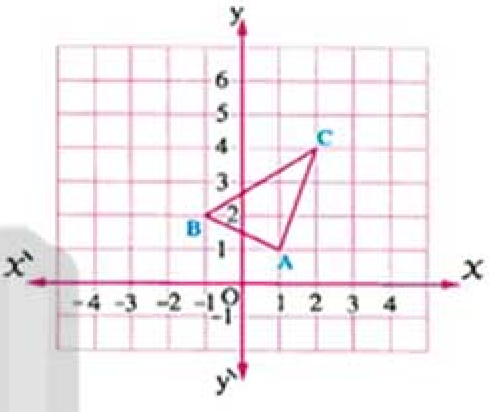


2 Copy each of the following figures on a lattice and draw the image of the figure by a geometric transformation as shown below each figure , then write the coordinates of each vertex of the figure.



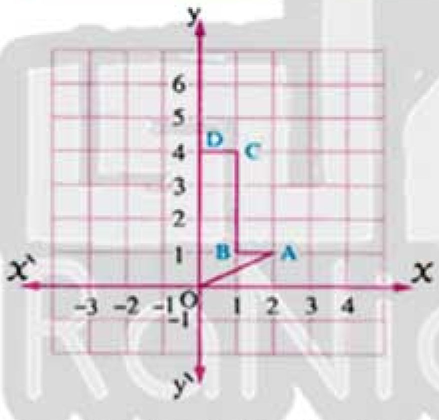
reflection in the x-axis

Fig. (1)



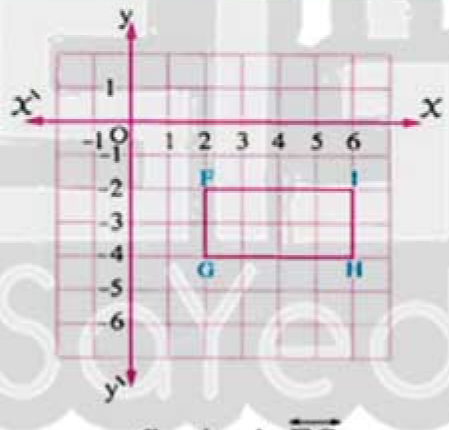
reflection in the y-axis

Fig. (2)



reflection in the y-axis

Fig. (3)



reflection in $\overleftrightarrow{FG}$

Fig. (4)

3 Draw $\overline{AB}$ where A (4 , 3) and B (1 , – 2) , then draw its image by reflection in :

1. The x-axis.
2. The y-axis.

4 If A (3 , 1) and B (3 , – 2) , find $\overline{DC}$ which is the image of $\overline{AB}$ by reflection in the y-axis and name the figure ABCD and calculate its perimeter. « 18 length units »

5 Find the image of Δ ABC where A (– 6 , – 1) , B (– 2 , – 1) and C (– 5 , – 6) by reflection in the x-axis.

Unit 3

6 Draw the image of Δ OBC where O (0 , 0) , B (3 , 0) and C (– 1 , 2) by reflection in the y-axis.

7 On a square lattice , draw Δ ABC where A (2 , – 2) , B (3 , 4) and C (– 3 , 2) , then draw $\Delta A'B'C'$ which is the image of Δ ABC by reflection in the y-axis , then draw $\Delta A''B''C''$ which is the image of $\Delta A'B'C'$ by reflection in the X-axis.

8 On a square lattice , draw the rectangle whose vertices are A (3 , 2) , B (8 , 2) , C (8 , 6) and D (3 , 6) , then draw its image by reflection in the y-axis.

9 Graph the square ABCD and its image by reflection in the X-axis , then compare the lengths of the sides and the area where A (0 , 2) , B (– 5 , 0) , C (– 3 , – 5) and D (2 , – 3)

10 **ABCD is a rectangle in which :** A (1 , 1) , B (1 , 3) and C (– 3 , 3) Determine the coordinates of D from the graph , then find the image of the rectangle ABCD by reflection in the X-axis.

11 Draw the image of the square ABCD where A (2 , 3) and B (2 , – 1) by reflection in the y-axis. What do you notice ?

12 Draw the image of the rectangle XYZL where X (2 , 2) and Y (– 3 , 2) with width 3 units by reflection in the X-axis.

13 **Complete the following table :**

No.	The point	Its image by reflection in the X-axis	Its image by reflection in the y-axis
1	(3 , – 2)	....	....
2	....	(1 , 2)	....
3	....	....	(– 2 , 4)
4	(0 , 5)	....	....
5	....	(3 , 0)	....
6	....	....	(0 , 0)

14 **Complete the following :**

1. The image of the point (1 , 3) by reflection in the x-axis is ....
2. The image of the point (– 2 , 5) by reflection in the y-axis ....
3. The image of the point (2 , – 3) by reflection in the .... is (2 , 3)
4. The image of the point (– 1 , – 4) by reflection in the .... is (1 , – 4)
5. The image of the point (0 , 3) by reflection in the .... is itself.
6. The image of the point (– 5 , 0) by reflection in the .... is itself.
7. The image of the point (2 , 1) by reflection in the x-axis followed by reflection in the y-axis is ....
8. The image of the point (2 , – 3) by reflection in the y-axis followed by reflection in the x-axis is ....
9. If $\grave{A}$ (– 2 , 3) is the image of the point A (2 , 3) by reflection in y-axis , then the image of the point $\grave{A}$ by reflection in the y-axis is ....

For excellent pupils

15 Determine on a square lattice the points A (5 , 4) , B (5 , 1) , C (2 , 1) , $\grave{A}$ (4 , 5) , $\grave{B}$ (1 , 5) and $\grave{C}$ (1 , 2)

1. If $\Delta\, \grave{A}\grave{B}\grave{C}$ is the image of Δ ABC by reflection in the straight line L , draw this straight line.
2. If the figure AB$\grave{B}\grave{A}$ is the image of the figure CB$\grave{B}\grave{C}$ by reflection in the straight line M , draw this straight line.

16 If the geometric transformation $(x , y) \longrightarrow (y , x)$ is a reflection in a straight line L , draw on a square lattice the straight line L

Exercise 10 Reflection in a point

From the school book

First Problems on reflection in the plane :

1 Choose the correct answer from the given ones :

1 If $\overline{\grave{A}\grave{B}}$ is the image of $\overline{AB}$ by reflection in M , then $\grave{A}\grave{B}$ .... AB

(a) > (b) < (c) = (d) ≠

2 **In the opposite figure :**

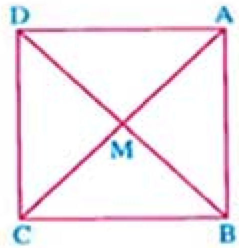
The image of $\overline{AB}$

by reflection in the point M is ....

(a) $\overline{AM}$ (b) $\overline{BA}$ (c) $\overline{BM}$

3 **In the opposite figure :**

ABCD is a square whose diagonals intersect at M The image of Δ ABM by reflection in M is Δ ....



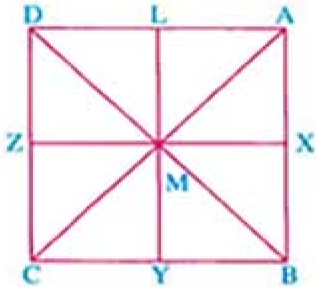
(a) ADM (b) BCM

(c) DCM (d) CDM

4 If $\grave{A}$ is the image of A by reflection in M and if MA = 5 cm. , then $A\grave{A}$ = ...

(a) 5 cm. (b) 7 cm. (c) 10 cm. (d) 15 cm.

2 **In the opposite figure :**



ABCD is a square whose diagonals intersect at M
X , Y , Z and L are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively

Complete the following :

1. The image of the point A by reflection in M is ....
2. The image of the point X by reflection in M is ....
3. The image of $\overline{AL}$ by reflection in M is ....
4. The image of $\overline{MZ}$ by reflection in M is ....
5. The image of $\overline{BM}$ by reflection in M is ....
6. The image of $\overline{AX}$ by reflection in X is ....
7. The image of Δ ALM by reflection in M is ....
8. The image of Δ BXM by reflection in M is ....
9. The image of Δ AMB by reflection in M is ....
10. The image of the square AXML by reflection in M is ....

3 **Using the geometric tools , draw the image of each of the following by reflection in A (Answer in the same page of the book) :**

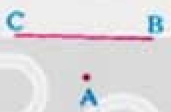


Fig. (1)

Fig. (2)

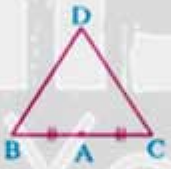


Fig. (3)

4 Draw Δ ABC in which AB = BC = 4 cm. and AC = 5 cm. , then find its image by reflection in the point B

5 In each of the following figures, draw $\Delta\ \grave{A}B\grave{C}$ as the image of Δ ABC by reflection in the point B and mention the name of the figure $A\grave{C}\grave{A}C$ giving reason.

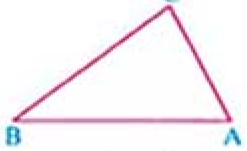


Fig. (1)

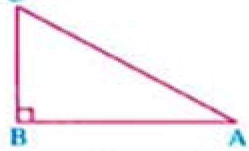


Fig. (2)

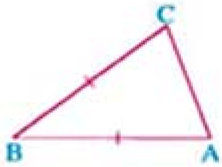


Fig. (3)

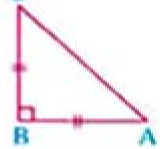


Fig. (4)

Unit 3

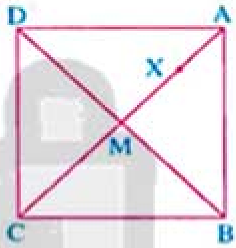
6 Draw Δ ABC in which BC = 3 cm. , AB = 4 cm. and m ($\angle$ B) = 90° , then draw $\Delta \grave{A}\grave{B}C$ as the image of Δ ABC by reflection in C Prove that the quadrilateral $AB\grave{A}\grave{B}$ is a parallelogram.

7 Draw the square ABCD whose side length is 5 cm. then draw its image by reflection in the point M where M is the point of intersection of its diagonals. What do you observe ?

8 ABC is a triangle , F is the midpoint of $\overline{AC}$ Draw D as the image of B by reflection in F What is the type of the figure ABCD and what is the type of the triangle ABC required to transfer the figure ABCD to :

1 Rectangle. 2 Rhombus.

9 In the opposite figure :

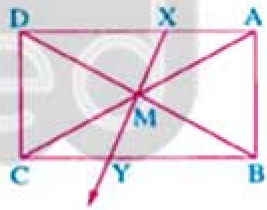


ABCD is a square , M is the point of intersection of its diagonals and X $\in \overline{AM}$ Find Y as the image of X by reflection in M then,

Prove that : 1 Δ DAX $\equiv$ Δ BCY

2 The figure DXBY is a parallelogram.

10 In the opposite figure :



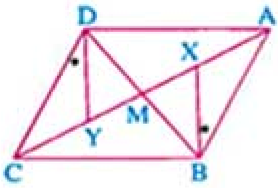
ABCD is a rectangle , M is the point of intersection of its diagonals , X $\in \overline{AD}$ and $\overrightarrow{XM} \cap \overline{BC} = \{Y\}$

Prove that :

1 Y is the reflected image of X in M

2 The figure AXCY is a parallelogram.

11 In the opposite figure :



ABCD is a parallelogram , M is the point of intersection of its diagonals and X $\in \overline{AC}$, Y $\in \overline{AC}$ such that m ($\angle$ ABX) = m ($\angle$ CDY)

Prove that :

1 Δ ABX is the image of Δ CDY by reflection in M

2 The figure XBYD is a parallelogram.

Second Problems on reflection in the Cartesian plane :

1 **Choose the correct answer from those given :**

1 The image of the point $(-3, 2)$ by reflection in the origin point is ....

(a) $(3, 2)$ (b) $(-3, -2)$ (c) $(3, -2)$ (d) $(-3, 2)$

2 The point $(5, -2)$ is the image of the point .... by reflection in the origin point.

(a) $(5, -2)$ (b) $(-5, -2)$ (c) $(-5, 2)$ (d) $(5, 2)$

3 The point whose image by reflection in the origin point is itself is ....

(a) $(0, 1)$ (b) $(1, 0)$ (c) $(0, 0)$ (d) $(-1, 0)$

4 The image of the point $(3, -2)$ by reflection in the origin point followed by reflection in X-axis is ....

(a) $(3, -2)$ (b) $(-3, -2)$ (c) $(-3, 2)$ (d) $(3, 2)$

2 On a square lattice, draw ΔABC where $A(3, 1)$, $B(1, 4)$ and $C(0, 0)$, then find its image by reflection in the point C

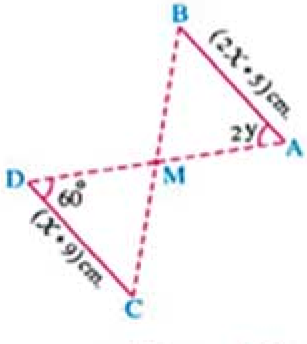
3 In Xy - coordinate plane, draw ΔABC, where $A(-2, 4)$, $B(5, 0)$ and $C(3, -3)$, then find the reflected image of ΔABC in the origin point.

4 On a square lattice, draw ΔABC where $A(2, -2)$, $B(3, 4)$ and $C(-3, 2)$, then map $\Delta \grave{A}\grave{B}\grave{C}$ as the image of ΔABC by reflection in y-axis then map $\Delta \ddot{A}\ddot{B}\ddot{C}$ as the image of $\Delta \grave{A}\grave{B}\grave{C}$ by reflection in X-axis. What is the image of ΔABC by reflection in the origin point ? What do you deduce ?

5 ABCD is a rectangle where $A(2, 5)$, $B(6, 5)$, $C(6, 8)$ and $D(2, 8)$, then find the image of the rectangle ABCD by reflection in the origin point.

For excellent pupils

6 **In the opposite figure :**



If $\overline{CD}$ is the image of $\overline{BA}$ by reflection in the point M and $BA = (2X + 5)$ cm., $CD = (X + 9)$ cm., $m(\angle A) = 2y$ and $m(\angle D) = 60°$

Find : 1 The length of $\overline{CD}$

2 The value of y

« 13 cm. , 30° »

Exercise 11

Translation

From the school book

First Problems on translation in the plane :

1 **Using the geometric tools , draw the image of each of the following :**
By translation MN in the direction of $\overrightarrow{MN}$ as shown in each case.

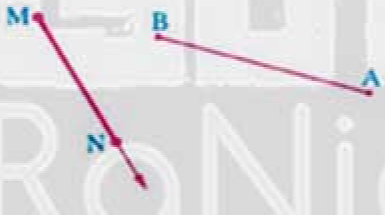


Fig. (1)

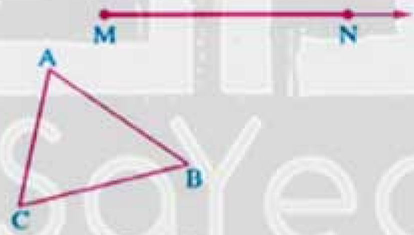
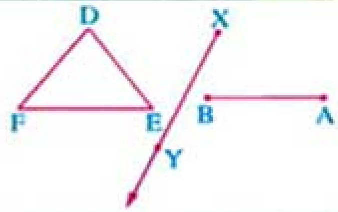
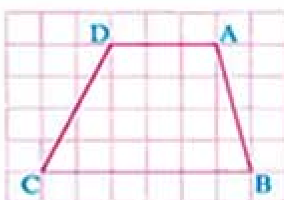


Fig. (2)

2 **In the opposite figure :**
Using the geometric tools , find the image of each of the opposite figures by the translation of displacement XY in the direction of $\overrightarrow{XY}$



3 Using the grid , draw the image of the figure ABCD by the translation of 4 units in the direction of $\overrightarrow{BC}$



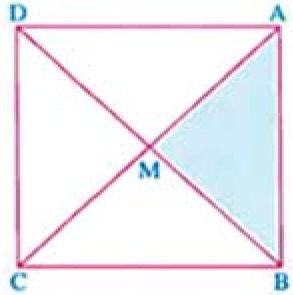
4 Draw a line segment $\overline{AB}$ where AB = 5 cm. , then draw the image of $\overline{AB}$ by a translation of magnitude of 8 cm. in the direction of $\overrightarrow{AB}$

Exercise 11

5 Using the geometric instruments , draw the square ABCD whose side length is 4 cm. , then draw its image by translation of magnitude of 4 cm. in the direction of $\overrightarrow{AB}$

6 Draw Δ ABC in which AB = 4 cm. , BC = 6 cm. and CA = 5 cm. , then draw the image of Δ ABC by a translation of magnitude of 3 cm. in the direction of $\overrightarrow{CB}$

7 **In the opposite figure :**



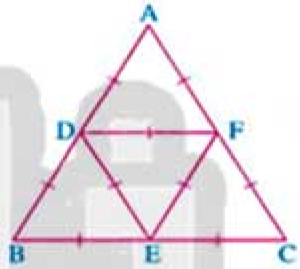
ABCD is a square whose side length is 4 cm.

M is the point of intersection of its diagonals. Draw :

1 The image of Δ MAB by the translation of distance 2 cm. in the direction of $\overrightarrow{AD}$

2 The image of Δ AMB by the translation AM in the direction of $\overrightarrow{AM}$

8 **In the opposite figure :**



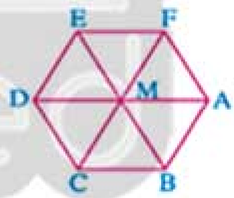
The triangles ADF , BDE , DEF and EFC are congruent.

Complete :

1 The image of Δ ADF by a translation of magnitude of AD in the direction of $\overrightarrow{AD}$ is ....

2 Δ FEC is the image of Δ DBE by a translation of magnitude .... in the direction of ....

9 **In the opposite figure :**



ABCDEF is a regular hexagon. **Complete the following :**

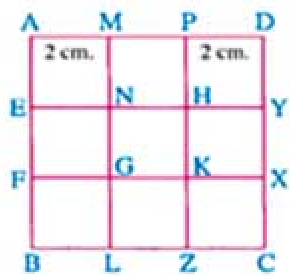
1 The image of the point D by translation DM in the direction of $\overrightarrow{DM}$ is ....

2 The image of $\overline{AF}$ by translation ED in the direction of $\overrightarrow{ED}$ is ....

3 The image of Δ MCD by translation EF in the direction of $\overrightarrow{EF}$ is ....

4 The translation which makes Δ DME the image of Δ MAF is ....

10 **In the opposite figure :**



ABCD is a square and all the interior squares are congruent.

Complete :

1 The image of $\overline{AE}$ by a translation of magnitude of 2 cm. in the direction of $\overrightarrow{GK}$ is ....

2 The image of the square AENM by a translation of magnitude of 4 cm. in the direction of $\overrightarrow{PK}$ is ....

3 The square MNHP is the image of the square GLZK by a translation of magnitude .... in the direction of ....

11 Δ ABC is right-angled at B where AB = 3 cm. and BC = 4 cm. If Δ $\grave{A}\grave{B}\grave{C}$ is the image of Δ ABC by translation of a distance 3 cm. in the direction of $\overrightarrow{CB}$

Prove that : The figure $A\grave{A}\grave{C}C$ is a parallelogram.

12 Draw Δ ABC which is right-angled at B , in which AB = BC = 3 cm. , then find the image of Δ ABC by translation of a distance 3 cm. in the direction of $\overrightarrow{AB}$, then

Prove that : The figure $B\grave{B}\grave{C}C$ is a square.

13 ABCD is a rectangle , where $E \in \overline{AD}$ Find the translated image of Δ ABE by translation of a magnitude DA in the direction of $\overrightarrow{AD}$ If $\grave{E}$ is the image of E by the same translation.

Prove that : The figure $BC\grave{E}E$ is a parallelogram.

14 ABCD is a parallelogram , $\overline{BE} \perp \overline{AD}$ cutting it at E Find Δ $\grave{A}\grave{B}D$ as the image of Δ ABE by translation of a distance ED in the direction of $\overrightarrow{AD}$, then

Prove that : The figure $EB\grave{B}D$ is a rectangle.

Second Problems on translation in the Cartesian plane :

1 **Complete the following :**

1. The image of the point (2 , 5) by translation $(x , y) \longrightarrow (x + 2 , y + 1)$ is ....
2. The image of the point (3 , 2) by translation $(x , y) \longrightarrow (x + 3 , y - 2)$ is ....
3. The image of the point (– 5 , 4) by translation $(x , y) \longrightarrow (x + 4 , y - 5)$ is ....
4. The image of the point (– 2 , – 5) by translation $(x , y) \longrightarrow (x - 2 , y)$ is ....
5. The image of the point (3 , – 2) by translation $(x , y) \longrightarrow (x , y + 3)$ is ....

2 **Choose the correct answer from those given :**

1. The image of the point (– 1 , 2) by translation of magnitude of 3 units in the positive direction of the x-axis is ....
   (a) (– 1 , 5) (b) (2 , 2) (c) (– 2 , 2) (d) (– 1 , 3)
2. The image of the point (– 3 , 4) by translation of magnitude of 4 units in the negative direction of the y-axis is ....
   (a) (– 3 , 0) (b) (– 7 , 4) (c) (– 3 , 8) (d) (– 1 , 4)
3. If $\grave{A}$ (3 , – 3) is the image of A by translation $(x , y) \longrightarrow (x - 1 , y - 4)$, then the point A is ....
   (a) (2 , – 7) (b) (4 , 1) (c) (– 4 , – 1) (d) (2 , 1)
4. The image of the point (– 1 , 4) by the translation (3 , – 2) followed by reflection in the x-axis is ....
   (a) (2 , 2) (b) (– 2 , 2) (c) (– 2 , – 2) (d) (2 , – 2)

5 If the point (a , – 1) is the image of (2 , 4) by the translation $(x , y) \longrightarrow (x + 1 , y - b)$, then (a , b) is ....

(a) (3 , 3) (b) (1 , 3) (c) (3 , 5) (d) (1 , – 5)

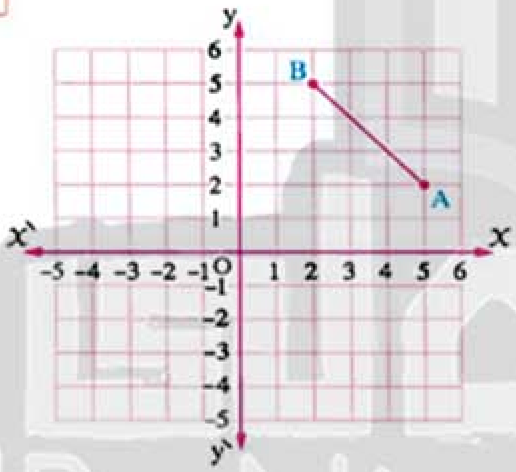
6 If : $\grave{A}$ is the image of the point A (2 , 3) by reflection in the y-axis , then A is the image of $\grave{A}$ by the translation ....

(a) $(x , y) \longrightarrow (x + 4 , y)$ (b) $(x , y) \longrightarrow (x , y + 6)$

(c) $(x , y) \longrightarrow (x - 4 , y)$ (d) $(x , y) \longrightarrow (x , y - 6)$

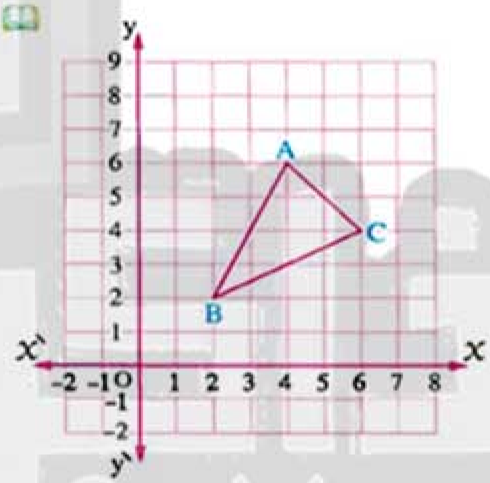
3 Find the image of each of the following figures by the translation shown under each figure :

1



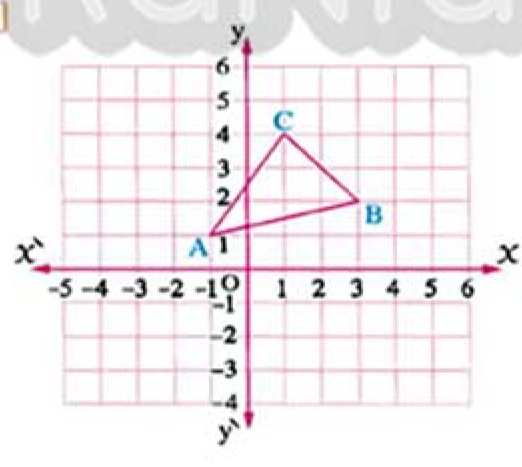
$(x , y) \longrightarrow (x - 3 , y - 4)$

2



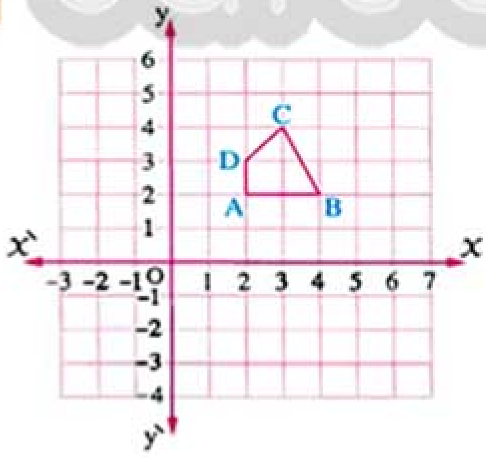
$(x , y) \longrightarrow (x + 2 , y + 3)$

3



$(x , y) \longrightarrow (x + 2 , y)$

4

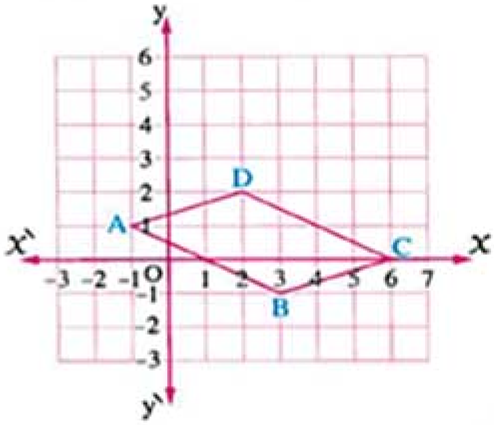


$(x , y) \longrightarrow (x + 3 , y - 2)$

Unit 3

4 **Copy the graph , then draw the image of the parallelogram ABCD under each of the following translations :**

1. $(x , y) \longrightarrow (x + 5 , y + 2)$
2. $(x , y) \longrightarrow (x - 8 , y - 1)$
3. $(x , y) \longrightarrow (x + 2 , y - 4)$
4. $(x , y) \longrightarrow (x - 4 , y + 2)$



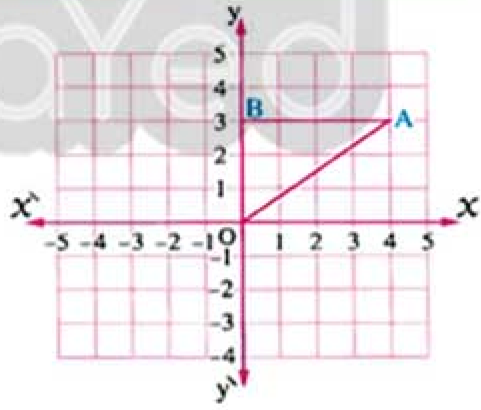
5 Find the image of each of the following points by the translation $(x , y) \longrightarrow (x + 2 , y - 3)$ followed by the translation $(x , y) \longrightarrow (x - 3 , y + 1)$

1. A (4 , – 2)
2. B (– 1 , 3)
3. C (0 , 2)

6 On a square lattice , draw $\overline{AB}$ where A (2 , 3) and B (4 , 1) , then draw the image of $\overline{AB}$ by the translation $(x , y) \longrightarrow (x + 3 , y + 2)$

7 Using the square lattice , draw Δ OBC where O is the origin point , B (3 , 0) , C (0 , 2) , then draw its image by the translation $(x , y) \longrightarrow (x - 4 , y + 1)$

8 Find the image of Δ AOB by the translation of magnitude = AO and in the direction of $\overrightarrow{AO}$



9 Using the lattice , find the image of each of the following points by the translation of LM in the direction of $\overrightarrow{LM}$ where : L (1 , 3) and M (4 , 5)

1. B (– 2 , 3)
2. C (5 , 4)
3. D (3 , 0)

Exercise 11

10 On a square lattice , draw ΔABC where A (2 , 1) , B (1 , – 1) and C (0 , 1) , then draw its image by a translation of 2 AB in the direction of $\overrightarrow{AB}$

11 A square has vertices A (1 , 1) , B (4 , 2) , C (3 , 5) and D (0 , 4)

1. Graph the square and its image under the translation which maps vertex A onto vertex B
2. Write the mapping rule for the translation.

12 Use the translation : $(x , y) \longrightarrow (x + 2 , y + 3)$ to locate the point whose image is (2 , 3)

13 If the image of the point A (1 , 1) by translation in the Cartesian plane is $\grave{A}$ (2 , 2) , find the images of the points O (0 , 0) , B (– 1 , 3) and C (– 3 , 5) by the same translation.

14 If A (– 3 , 1) and B (1 , – 2) , write the mapping rule of the translation that makes B the image of A

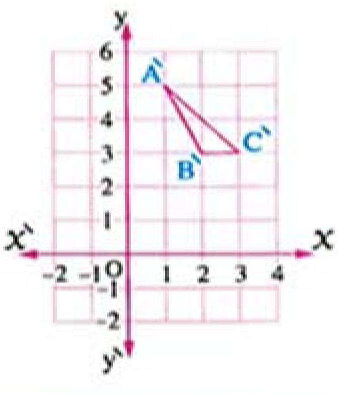
15 If A (3 , 2) , B (5 , 1) , **find :**

1. $\grave{C}$ which is the image of C (1 , – 1) under translation of AB in the direction of $\overrightarrow{AB}$
2. D whose image is $\grave{D}$ (2 , 1) under translation of AB in the direction of $\overrightarrow{AB}$

16 The point $\grave{A}$ (3 , – 3) is the image of the point A by the translation $(x , y) \longrightarrow (x – 1 , y – 4)$ Locate A then by the same translation , draw the image of ΔABC where B (5 , 0) and C (– 1 , – 2)

17 **In the opposite figure :**

Copy the graph , then draw the triangle ABC whose image is $\Delta \grave{A}\grave{B}\grave{C}$ by the translation $(x , y) \longrightarrow (x + 2 , y + 3)$



18 If A (– 2 , 1) and B (1 , 3) , find on a square lattice the image of $\overline{AB}$ by reflection in x-axis followed by the translation $(x , y) \longrightarrow (x + 4 , y)$

Unit 3

19 State whether the graph shows a reflection or a translation :

1. Name the line of reflection.
2. Describe the translation.

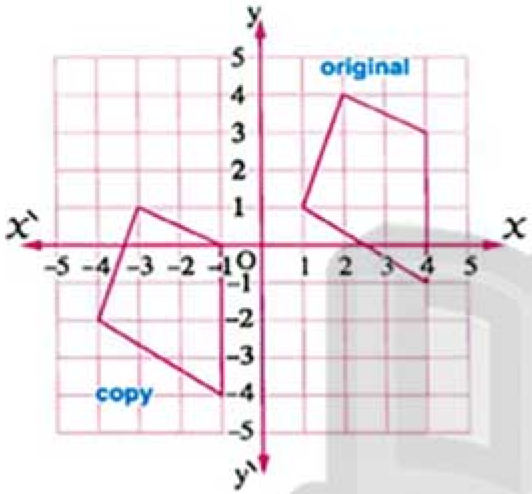


Fig. (1)

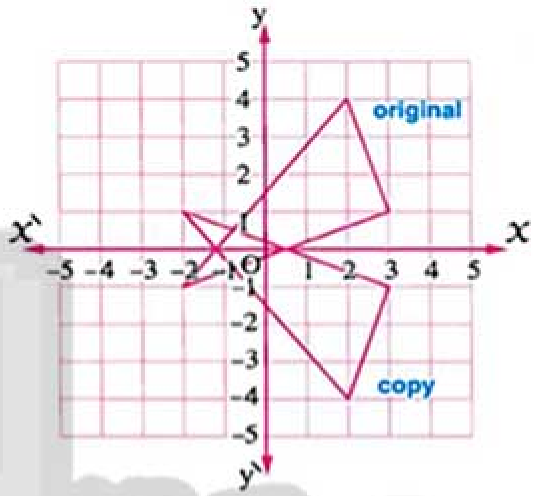


Fig. (2)

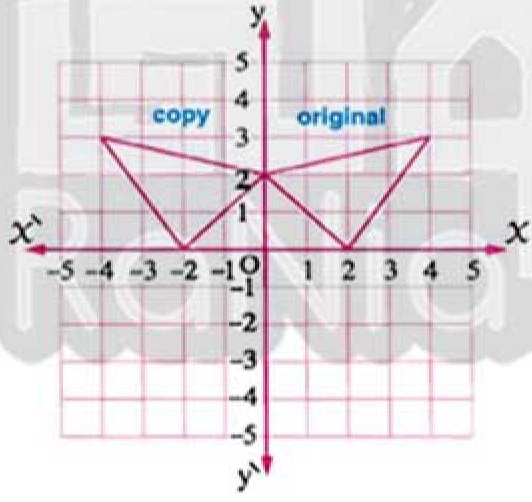


Fig. (3)

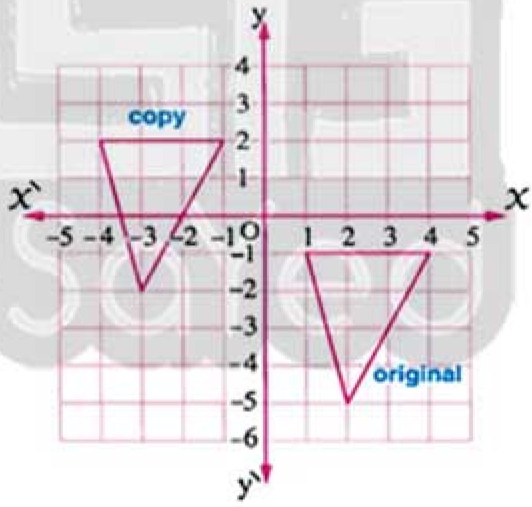


Fig. (4)

For excellent pupils

20 Draw Δ ABC on a square lattice where A (4 , 4) , B (4 , 2) and C (1 , 2) , then draw its image by the translation of magnitude 3 AB in the direction of $\overrightarrow{AB}$

21 If A (2 , 1) is the image of B by reflection in x-axis followed by reflection in y-axis , state the translation which makes A the image of the point B

Exercise 12

Rotation

From the school book

First Problems on rotation in the plane :

1 Draw the image of the point A , $\overline{AB}$ and Δ ABC by the required rotation :

Fig. (1)
R (M , 30°)

Fig. (2)
R (M , – 60°)

Fig. (3)
R (M , 60°)

2 Use the geometric tools to draw $\overline{AB}$ with length 3 cm. , then draw its image by rotation R (B , 135°)

3 Draw the equilateral triangle ABC with side length 6 cm. Draw the image of the triangle ABC by rotation R (A , 60°)

4 Draw the triangle ABC in which AB = 5 cm. , BC = 6 cm. and CA = 7 cm. , then draw the image of Δ ABC by rotation :

1 R (A , 180°) 2 R (A , 360°)

5 Draw the triangle XYZ in which XY = XZ = 3 cm. and YZ = 4 cm. , then draw the image of Δ XYZ in each of the two cases :

1. By rotation about X with an angle of measure 90°
2. By rotation about X with an angle of measure 270°

6 Draw Δ ABC in which AB = 5 cm. , AC = 3 cm. , m (∠ A) = 40° , then draw $\grave{C}$ the image of C by rotation R (A , 40°) , $\grave{B}$ the image of B by rotation R (A , – 40°)

7 Draw the square ABCD with side length 5 cm. Draw the image of the square ABCD :

1. By rotation R (B , 90°)
2. By rotation R (A , 180°)

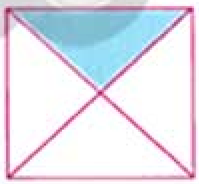
8 Using the geometric tools , draw the square ABCD with side length 4 cm. , then draw its image by rotation about its centre (The point of diagonals intersection) with an angle of measure 90°

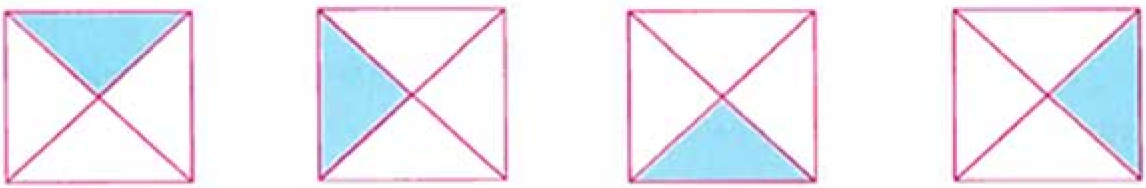
9 Draw the rectangle ABCD in which BC = 6 cm., AB = 4 cm. Draw the image of the rectangle ABCD :

1. By rotation R (A , 90°)
2. By rotation R (M , 180°) where M is the point of intersection of its diagonals.

10 **Choose the correct answer from those given :**

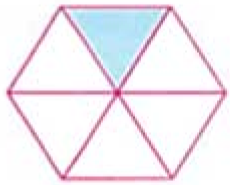
1. Which of these figures represents the rotation of the opposite square about its centre with an angle of measure 270° ?

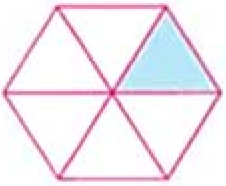




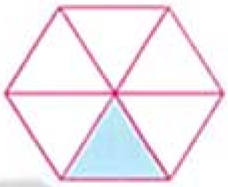
(a) (b) (c) (d)

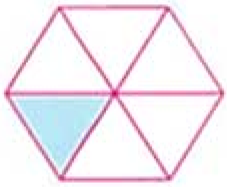
2 Which of these figures represents the rotation of the opposite regular hexagon about its centre with an angle of measure (– 120°) ?




(a)


(b)


(c)


(d)

3 **In the opposite figure :**

If B is the midpoint of $\overline{AC}$, then the image of $\overline{AC}$ by rotation about B with an angle of measure 180° is ....

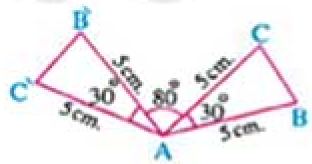
(a) $\overline{AC}$ (b) $\overline{AB}$ (c) $\overline{CA}$ (d) $\overline{CB}$

4 **In the opposite figure :**

$\overline{CD}$ is the image of $\overline{AB}$ under a rotation about M and the measure of its angle is ....

(a) 75° (b) 30° (c) – 30° (d) – 150°

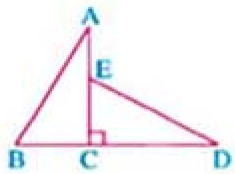
5 **In the opposite figure :**



Δ AB̀C̀ is the image of Δ ABC by a rotation about A with an angle of measure ....

(a) – 110° (b) 80° (c) 110° (d) 140°

6 **In the opposite figure :**



Δ ABC is the image of Δ DEC which is right-angled at C by rotation about C with an angle of measure ....

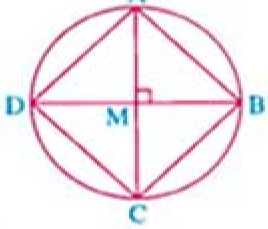
(a) 90° (b) – 90° (c) 180° (d) 360°

Unit 3

11 **In the opposite figure :**

The radius length of circle M is 3 cm. ,

$\overline{AC}$ and $\overline{BD}$ are two perpendicular diameters in it.



Complete :

1 By the rotation R (M , 90°) , then the image of the point A is ....

and the image of the point B is ....

$\therefore$ The image of $\overline{AB}$ is .... and the image of $\overrightarrow{AB}$ is ....

2 By rotation R (M , – 90°) , the image of $\overline{AB}$ is .... and the image of $\overrightarrow{AB}$ is .... and the image of $\overleftrightarrow{AB}$ is ....

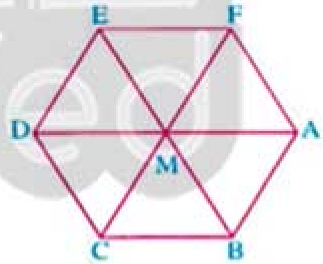
3 By rotation R (M , 180°) , the image of the point A is .... , the image of the point B is ....

$\therefore$ The image of $\overline{AB}$ is ....

4 By rotation R (M , – 180°) , the image of $\overline{AB}$ is ....

12 **In the opposite figure :**

ABCDEF is a regular hexagon whose centre is M



Complete the following :

1 The image of the point E by rotation about M with an angle of measure 120° is ....

2 The image of $\overline{AF}$ by rotation about M with an angle of measure 180° is ....

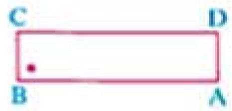
3 The image of $\overline{DE}$ by rotation about M with an angle of measure – 60° is ....

4 The image of Δ MCD by rotation about M with an angle of measure 300° is ....

5 Δ ABM is the image of Δ CDM by rotation about .... with an angle of measure ....

6 Δ BMC is the image of .... by rotation about M with an angle of measure (– 120°)

13 **Referring to the opposite figure ,**
choose the correct answer from those given :



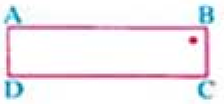


Fig. (1)

Fig. (2)

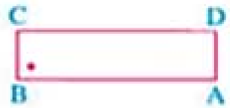


Fig. (3)

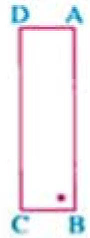
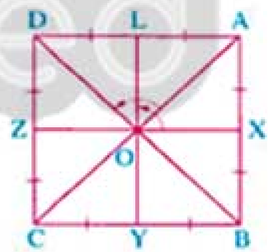


Fig. (4)

1. The image of the figure by reflection in $\overleftrightarrow{AD}$ is ....
   (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)
2. The image of the figure by rotation about A with an angle of measure 90° is ....
   (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)
3. The image of the figure by translation to the right is ....
   (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)
4. The image of the figure by rotation about A with an angle of measure 180° is ....
   (a) fig. (1) (b) fig. (2) (c) fig. (3) (d) fig. (4)

14 **In the opposite figure :**

ABCD is a square , O is the point of intersection of its diagonals , X , Y , Z and L are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.



Find :

1. The image of Δ AXO by reflection in $\overleftrightarrow{AO}$ followed by another reflection in $\overleftrightarrow{LO}$
2. The image of Δ AXO by rotation R (O , 90°)

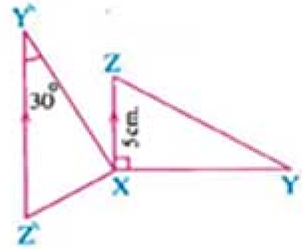
15 ABC is a right-angled triangle at B with AB = 5 cm. and BC = 12 cm. **Find :**

1. X as the image of B by translation 9 cm. in the direction of $\overrightarrow{BA}$
2. Y as the image of B by rotation R (A , – 90°)
3. The length of $\overline{XY}$

16 **In the opposite figure :**

If the point X is the centre of rotation such that $\grave{Y}$ is the image of Y and $\grave{Z}$ is the image of Z

if $\overline{XZ} \parallel \overline{\grave{Y}\grave{Z}}$ **Find :**



1. The measure of the angle of rotation.
2. The length of $\overline{X\grave{Z}}$

« 120° , 5 cm. »

Second Problems on rotation in the Cartesian plane :

1 **Complete the following :**

1. The image of the point (2 , – 3) by rotation about the origin point with an angle of measure 90° is .... and with an angle of measure 180° is ....
2. The image of the point (– 1 , 0) by rotation about the origin point with an angle of measure 90° is .... and with an angle of measure 360° is ....
3. The point (3 , – 2) is the image of the point (2 , 3) by rotation about the origin point with an angle of measure ....°
4. The image of the point .... by rotation about the origin point with an angle of measure 90° is (– 1 , 4)
5. The image of the point .... by rotation about the origin point with an angle of measure (– 180°) is (5 , – 2)
6. The image of the point (– 3 , 7) by rotation with an angle of measure 90° about the origin point followed by reflection in y-axis is ....
7. The image of the point (– 2 , 0) by translation $(x , y) \longrightarrow (x + 3 , y - 1)$ followed by rotation about the origin point with an angle of measure 90° is ....
8. The rotation with an angle of measure 90° about the origin point maps the point $(x , - y)$ onto the point ....
9. The image of (a , b) is the same point by rotation about the origin point with an angle of measure ....°
10. If the image of the point (x , y) by rotation about the origin point with an angle of measure 90° is (a , b) , then a + y = ...

2 **Complete the following diagram :**

The image of the point $(2, -1)$		
	by reflection in the x-axis	The point (……… , ………)
	by reflection in the y-axis	The point (……… , ………)
	by reflection in the origin point	The point (……… , ………)
	by translation $(x, y) \Rightarrow (x-3, y+4)$	The point (……… , ………)
	by rotation $R(O, 90°)$	The point (……… , ………)
	by rotation $R(O, -90°)$	The point (……… , ………)
	by rotation $R(O, \pm 180°)$	The point (……… , ………)
	by rotation $R(O, \pm 360°)$	The point (……… , ………)

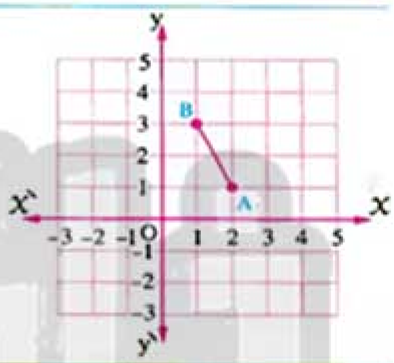
3 **In the opposite figure :**

The point A $(2, 1)$ and B $(1, 3)$

Find the image of $\overline{AB}$

by rotation about the origin point

with an angle of measure $90°$



4 On the lattice , draw the image of Δ OAB

by rotation about the origin with an angle

of measure :

1 $90°$ 2 $180°$

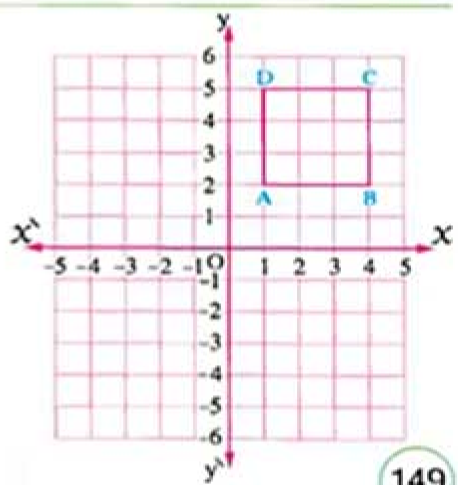
5 **In the opposite figure :**

Draw the image of the square ABCD

by rotation about the origin with an angle

of measure :

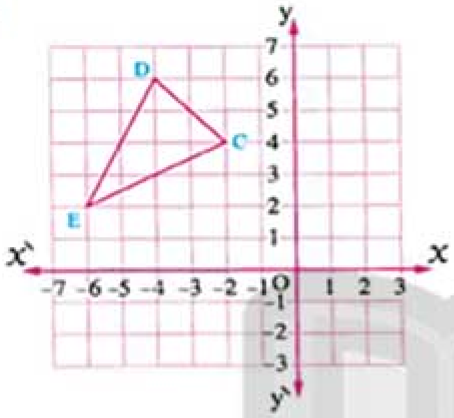
1 $90°$ 2 $180°$



Unit 3

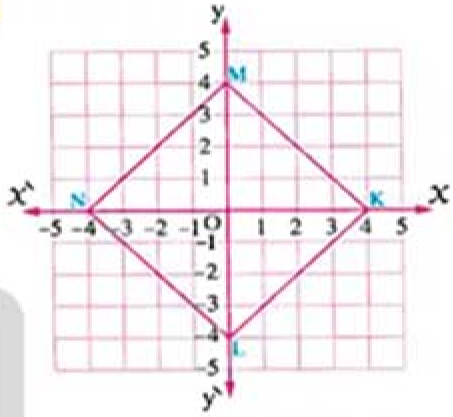
6 Copy each figure on a graph paper. Draw their images under the transformation indicated. Give the coordinates of the images vertices in each case.

1



Rotation of 90°
Clockwise about O

2



Rotation of 90°
anticlockwise about O

7 Draw on graph paper Δ ABC where A (3 , – 1) , B (5 , 2) and C (– 2 , 4) , then draw its image by rotation about the origin point with an angle of measure 180°

8 In an orthogonal Cartesian coordinate system , determine the two points A (3 , 0) and B (0 , 2) , then draw the image of Δ AOB by rotation about «O» with an angle of measure 90° where O is the origin point.

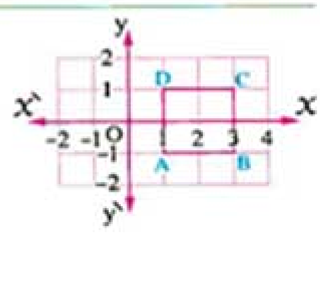
9 A rectangle has vertices A (– 1 , – 2) , B (7 , 2) , C (5 , 6) and D (– 3 , 2)
Graph the rectangle and its image under rotation about the origin where $(x , y) \longrightarrow (-y , x)$

10 On a lattice , draw the quadrilateral ABCD where A (0 , 4) , B (4 , 4) , C (7 , 0) and D (0 , 0) , then draw its image by rotation :

1 R (O , 90°) **2** R (O , – 180°)

11 If the image of the point C by rotation with an angle of measure 90° about the origin is $\grave{C}$ (– 4 , 5) , locate the point C , then draw its image $\grave{\grave{C}}$ by rotation about the origin with an angle of measure 180°

12 Draw the image of the square ABCD by rotation about A with an angle of measure 90°



13 On a lattice , draw ΔABC where A (4 , 4) , B (4 , 2) and C (1 , 2) , then draw its image by rotation about the point B with an angle of measure 180°

14 On graph paper , draw the rectangle ABCD with vertices A (0 , 0) , B (0 , 2) , C (4 , 2) and D (4 , 0)

(a) Draw three images formed by rotating the rectangle about the origin through an angle of measure

1 90° 2 180° 3 270°

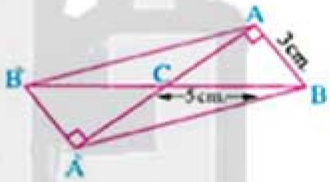
(b) What are the coordinates of the centre of the rectangle?

(c) Draw three images formed by rotating the rectangle about its centre through an angle of measure

1 90° 2 180° 3 270°

For excellent pupils

15 **In the opposite figure :**



ABC is a right-angled triangle at A

, AB = 3 cm. and BC = 5 cm.

If : $\Delta CA'B'$ is the image of ΔCAB by rotation about C with an angle of measure 180°

Find the area of $\Delta AA'B'$ « 12 cm^2 »

Summary of the third part of unit 3
"From lesson 8 to lesson 12"

Properties of reflection in a straight line :

1. Reserves the lengths of the line segments.
2. Reserves the measures of angles.
3. Reserves parallelism.
4. Doesn't reserve the orientation of the vertices of the figure.
5. Reserves the betweenness.

Properties of reflection in a point :

1. Reserves the lengths of the line segments.
2. Reserves the measures of angles.
3. Reserves parallelism.
4. Reserves the orientation of vertices of the figure.
5. Reserves the betweenness.

Properties of translation :

1. Reserves the lengths of the line segments.
2. Reserves the measures of angles.
3. Reserves the parallelism.
4. Reserves the orientation of vertices of the figure.
5. Reserves the betweenness.

To determine the translation, we should know two important elements which are :

1. The magnitude of the translation.
2. The direction of the translation.

Properties of rotation :

1. Reserves the lengths of the line segments.
2. Reserves the measures of angles.
3. Reserves the parallelism.
4. Reserves the orientation of vertices of the figure.
5. Reserves the betweenness.

The rotation is determined completely by the following elements :

1. The centre of rotation.
2. The measure of the angle of rotation.
3. The direction of rotation.

Summary for geometrical transformations (reflection , translation , rotation) in the Cartesian plane :

The image of the point (x , y)	by reflection in the x-axis	The point $(x , -y)$
	by reflection in the y-axis	The point $(-x , y)$
	by reflection in the origin point	The point $(-x , -y)$
	by translation $(x, y) \Rightarrow (x + k , y + l)$	The point $(x + k , y + l)$
	by rotation R (O , 90°) ($\frac{1}{4}$ turn)	The point $(-y , x)$
	by rotation about O with an angle of measure (– 90°) or (270°)	The point $(y , -x)$
	by rotation R (O , ± 180°) ($\frac{1}{2}$ turn)	The point $(-x , -y)$
	by rotation R (O , ± 360°) (identity rotation)	The point (x , y)

Exams on the third part of unit three
from lesson (8) to lesson (12)

Model 1

Answer the following questions :

1 Choose the correct answer from the given ones :

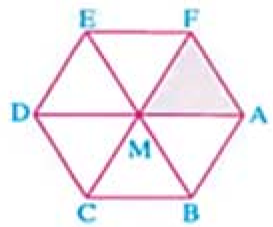
1. The image of the point (– 3 , – 5) by reflection in X-axis is ....
   (a) (3 , – 5) (b) (3 , 5) (c) (– 3 , 5) (d) (– 3 , – 5)
2. If A' is the image of A by reflection in M and MA = 6 cm. , then AA' = ...
   (a) 6 cm. (b) 3 cm. (c) 12 cm. (d) 9 cm.
3. The image of the point (– 2 , 3) by translation of magnitude of 4 units in the negative direction of y-axis is ....
   (a) (2 , 3) (b) (– 2 , 7) (c) (– 6 , 3) (d) (– 2 , – 1)
4. The image of the point (– 4 , 2) by rotation around the origin point with an angle of measure 90° is ....
   (a) (– 2 , 4) (b) (– 2 , – 4) (c) (4 , – 2) (d) (2 , – 4)
5. The image of the point (2 , – 7) by reflection in the origin point is ....
   (a) (2 , 7) (b) (– 2 , 7) (c) (– 2 , – 7) (d) (2 , – 7)
6. If the image of the point (5 , – 2) by rotation around the origin point is itself , then the measure of angle of rotation is ....
   (a) 90° (b) 180° (c) 270° (d) 360°

2 Complete the following :

1. The number of axes of symmetry of the rhombus = ... and the number of axes of symmetry of the parallelogram = ...
2. The reflection in a straight line reserves .... , ... , ...
3. The image of the point (– 3 , – 4) by translation (– 4 , 1) is ....
4. The image of the point (– 2 , 5) by rotation R (O , – 90°) is ....
5. The point (– 2 , 4) is the image of the point .... by reflection in the origin point.

3 **[a] Draw Δ ABC in which :** A (3 , 2) , B (– 1 , 5) and C (0 , – 3) , then draw its image by reflection in the y-axis.

[b] In the opposite figure :



If ABCDEF is a regular hexagon whose centre is M , **find :**

1. The image of Δ AMF by reflection in $\overleftrightarrow{AM}$
2. The image of Δ AMF by reflection in M
3. The image of Δ AMF by translation of distance AM in the direction of $\overrightarrow{AM}$
4. The image of Δ AMF by rotation R (M , 120°)

4 **[a]** ABCD is a rectangle where : A (4 , 3) , B (4 , 1) , C (– 1 , 1) and D (– 1 , 3) , draw the rectangle and its image by rotation R (O , 180°)

[b] Draw the image of Δ ABC where : AB = 3 cm. , BC = 4 cm. and AC = 5 cm. by reflection in the straight line which contains the longest side.

5 **[a]** Draw Δ ABC in which : AB = 4 cm. , BC = 6 cm. and AC = 5 cm. , then draw its image by translation of magnitude 3 cm. in the direction of $\overrightarrow{CB}$

[b] Draw Δ ABC in which : AB = BC = 4 cm. and AC = 5 cm. , then find its image by reflection in point B

Model 2

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1. The image of the point (– 7 , 4) by rotation around the origin point with an angle of measure 180° is ....
   (a) (– 4 , – 7) (b) (4 , 7) (c) (7 , – 4) (d) (– 7 , – 4)
2. The point whose image is itself by reflection in the origin point is ....
   (a) (0 , 1) (b) (1 , 0) (c) (0 , – 1) (d) (0 , 0)
3. The image of the point (– 3 , 2) by translation of magnitude 4 units in the positive direction of the x – axis is ....
   (a) (1 , 2) (b) (– 3 , 6) (c) (– 7 , 2) (d) (– 3 , – 2)
4. The image of the point (– 10 , 0) by reflection in the x – axis is ....
   (a) (10 , 0) (b) (– 10 , 0) (c) (0 , – 10) (d) (0 , 10)
5. The image of the point (5 , – 7) by rotation R (0 , – 90°) is ....
   (a) (– 7 , – 5) (b) (7 , 5) (c) (– 7 , 5) (d) (– 5 , 7)

Unit 3

6 If $\grave{A}$ (– 4 , 5) is the image of A by translation (– 2 , 3) , then the point A is ....

(a) (– 6 , 8) (b) (– 2 , 8) (c) (– 2 , 2) (d) (– 6 , 2)

2 Complete the following :

1 The rotation in the plane reserves .... , ... , ...

2 The number of axes of symmetry of the isosceles trapezium = ...

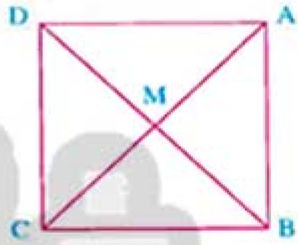
3 The measure of the angle of the identity rotation equals .... °

4 The translation in the plane is determined by .... , ...

5 The point (– 3 , 4) is the image of the point (3 , 4) by reflection in ....

3 [a] Draw on a graph paper Δ ABC where A (2 , 0) , B (3 , 2) , C (4 , 1) , then draw its image by reflection in the x – axis.

[b] In the opposite figure :



ABCD is a square , M is the point of intersection of its diagonals , **find :**

1 The image of Δ AMB by reflection in M

2 The image of $\overline{AD}$ by translation of magnitude DC in direction of $\overrightarrow{DC}$

4 [a] If the image of the point (1 , 2) by a translation is (1 , 4)

Find :

1 The translation.

2 The image of the point (3 , 1) by the same translation.

[b] Draw the equilateral triangle ABC of side length 5 cm. , then draw its image by rotation R (B , 60°)

5 [a] Find the image of A (– 3 , 2) by translation BC in the direction of $\overrightarrow{BC}$ where B (1 , 3) , C (4 , 5)

[b] On the square lattice, draw ΔABC where A (1 , 1) , B (1 , 4) , C (5 , 1) , then draw its image by rotation R (O , – 90°)

Accumulative basic skills

1 Choose the correct answer from the given ones :

1. A square of area 144 cm^2, then its perimeter = ... cm.
   (a) 12 (b) 48 (c) 288 (d) 576

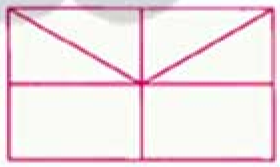
2. A rectangle, its length is 6 cm. and its perimeter is 16 cm., then its area = ... cm^2.
   (a) 10 (b) 8 (c) 12 (d) 16

3. The supplementary of the angle whose measure is 30°, is an angle of measure ....
   (a) 30° (b) 60° (c) 120° (d) 150°

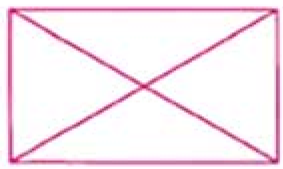
4. Which of the following figures is suitable to form a circle ?

   (a) (b) (c) (d)

5. The area of the shaded part from the area of the figure = ...
   (a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$



6. The great number of triangles in the opposite figure = ...
   (a) 4 (b) 6 (c) 8 (d) 10

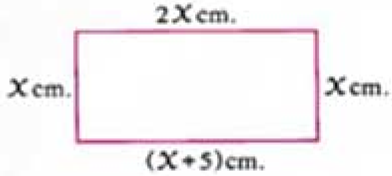


7. If X is an angle, then m (∠ X) + m (reflex ∠ X) = ...
   (a) two right angles. (b) three right angles.
   (c) five right angles. (d) four right angles.

Basic skills

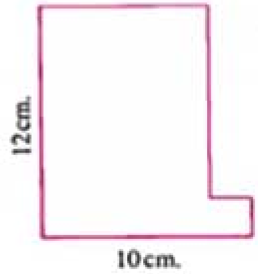
8 The area of the rectangle in the opposite figure = ... cm^2.

(a) 50 (b) 30 (c) 20 (d) 15



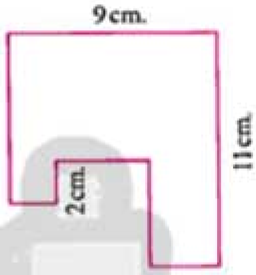
9 The perimeter of the opposite figure = ... cm.

(a) 22 (b) 24 (c) 44 (d) 120



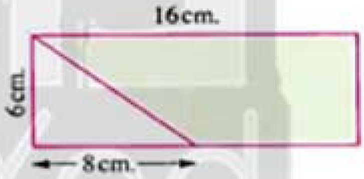
10 The perimeter of the opposite figure = ... cm.

(a) 99 (b) 44 (c) 22 (d) 20



11 The area of the shaded part in the opposite figure = ... cm^2.

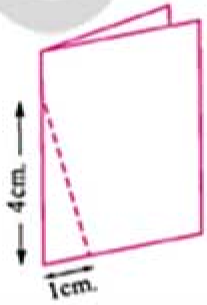
(a) 24 (b) 44 (c) 48 (d) 72



12 A rectangular piece of paper is folded as in the opposite figure, and a part of it is cut off with aligning the dashed line, then at opening the cut small part, it will be in the form of ....

(a) an equilateral triangle. (b) an isosceles triangle.

(c) a right-angled triangle. (d) two isosceles triangles.



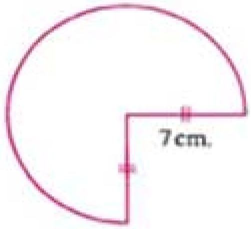
2 Complete the following :

1 A cube, area of one of its faces is 25 cm^2, then its volume = ... cm^3.

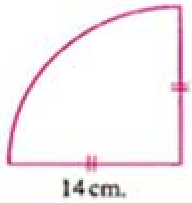
2 A cuboid, its volume = 48 cm^3, if the length of its base equals 6 cm. and its width equals 4 cm., then its height = ... cm.

3 The angle whose measure is 89°, is .... angle.

4 If m (∠ A) = 2 m (∠ B) , ∠ A complements ∠ B , then m (∠ A) = ...°

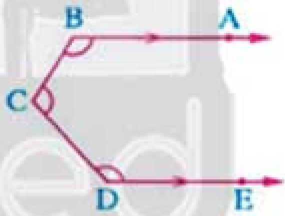


5 The area of the opposite figure = ... cm^2. ($\pi = \frac{22}{7}$)

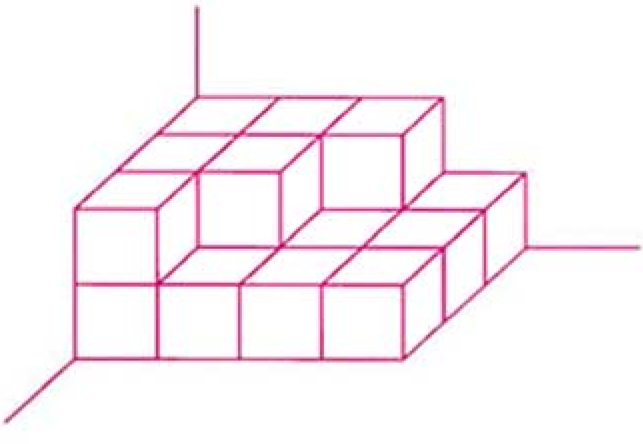


6 The perimeter of the opposite figure = ... cm. ($\pi = \frac{22}{7}$)

7 The number of acute angles in the opposite figure is ....



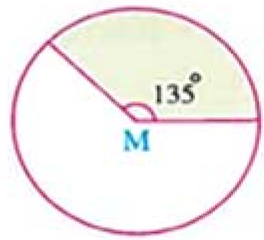
8 **In the opposite figure :**
m (∠ B) + m (∠ C) + m (∠ D) = ... °



9 The volume of the opposite figure = ... volume units.

Basic skills

10 The percentage of the area of the shaded part to the area of the circle is ....



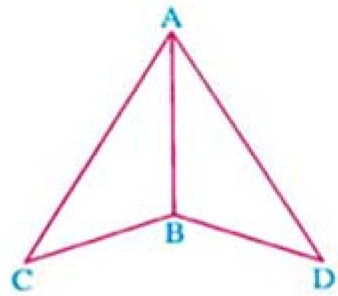
11 **In the opposite figure :**

If $\Delta ABC \equiv \Delta ABD$,

the perimeter of the figure ABCD = 20 cm.

and AB = 6 cm.

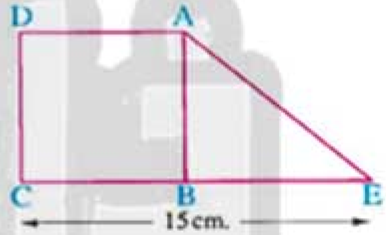
, then the perimeter of ΔABC = ... cm.



12 **In the opposite figure :**

ABCD is a square of area 49 cm^2.

, if EC = 15 cm. , then the area of ΔABE = ... cm^2.



EL-MOASSER

In Mathematics Notebook

For 1st Prep.
Second Term

- Quizzes
- Final Revision
- Final Examinations

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By
A group of supervisors

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- Final revision
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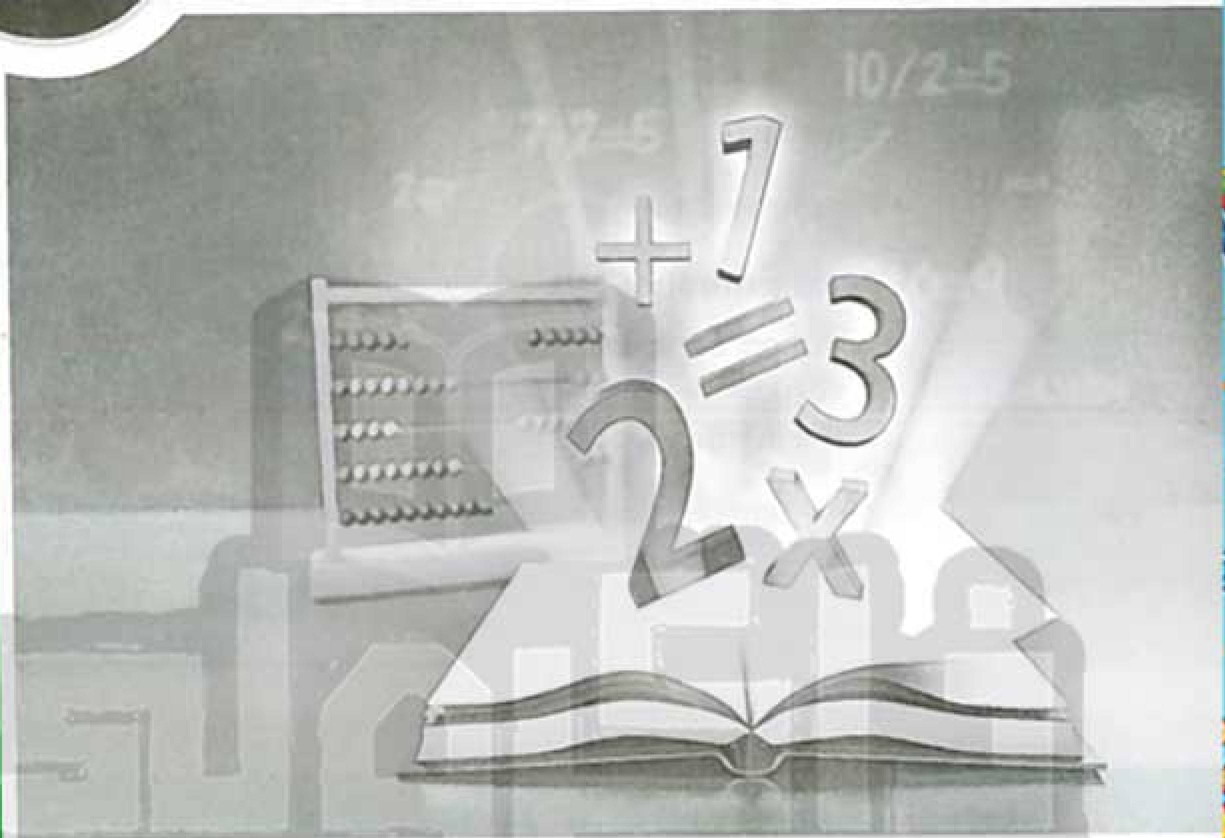
- Final examinations :
  - School book examinations (2 models + model for the merge students)
  - 15 schools examinations

First Algebra and Statistics



Quizzes

on Algebra and Statistics

Quiz 1 on lesson 1 – unit 1

time 15 min.

1 Choose the correct answer from those given :

1 The multiplicative inverse of the number $\left(-\frac{3}{4}\right)^{zero}$ is ....

(a) -1 (b) $-\frac{4}{3}$ (c) $\frac{4}{3}$ (d) 1

2 If $a = b$, then $\left(\frac{5}{7}\right)^{a-b} =$ ....

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$ (c) 1 (d) zero

3 If $x = -\frac{2}{3}$, $y = 2$, then $x^y =$ ....

(a) $\frac{4}{9}$ (b) $-\frac{4}{9}$ (c) $\frac{8}{27}$ (d) $-\frac{8}{27}$

2 [a] Simplify to the simplest form :

$\left(-\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^3 \div \left(-\frac{2}{9}\right)^2$

[b] If $x = -\frac{3}{2}$, $y = \frac{1}{2}$, $z = -\frac{3}{4}$,

find in the simplest form the numerical value of : $\left(\frac{x z}{y}\right)^2$

Quiz 2 till lesson 2 – unit 1

time 15 min.

1 Choose the correct answer from those given :

1 $(a^2)^4 =$ ....

(a) a^6 (b) a^8 (c) a^2 (d) a^4

2 The quarter of the number $4^{20} =$ ....

(a) 4^5 (b) 4^{10} (c) 4^{19} (d) 1^{20}

3 $2^5 + 2^5 =$ ....

(a) 4^5 (b) 2^{10} (c) 2^6 (d) 4^{10}

2 [a] Find each of the following in the simplest form :

1 $\left(\frac{3}{5}\right)^7 \div \left(\frac{3}{5}\right)^5 \times \frac{3}{5}$ 2 $\frac{x^5 \times x^8}{x^3 \times x^2 \times x^4}$ where $x \neq 0$ 3 $\left(\left(-\frac{2}{3}\right)^2\right)^3$

[b] If $a = -\frac{1}{2}$, $b = 2$, $c = \frac{3}{4}$,

find the numerical value of the expression : $a^3 b^2 + b^2 c - 8 abc$

Quiz 3 till lesson 3 – unit 1

time 15 min.

1 Choose the correct answer from those given :

1 If $x^{-1} = \frac{1}{2}$, then $x =$ ....

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) -2

2 $\frac{6 a^2 x^4}{2 a^3 x^3} =$ .... where $a x \neq 0$

(a) $3 a x$ (b) $3 a^5 x^7$ (c) $\frac{3 x}{a}$ (d) $\frac{3}{a x}$

3 $5^6 \times 5^{-6} = 7^{\cdots}$

(a) 5 (b) 1 (c) 12 (d) zero

2 [a] Find each of the following in the simplest form :

1 $\frac{5^{-2} \times 5^5}{5^3}$ 2 $\left(\frac{3^4 \times 7^2}{7^3 \times 3^2}\right)^{-1}$ 3 $\left(-\frac{x^3}{y^2}\right)^{-2}$ such that $x \neq 0$, $y \neq 0$

[b] If $x = \frac{1}{2}$, $y = \frac{1}{8}$, find the value of the expression : $(2x - y)^{-2} \times y$

Quiz 4 till lesson 4 – unit 1

time 15 min.

1 Choose the correct answer from those given :

1 $2.37 \times 10^{-4} =$ ....

(a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237

2 $3 y^{-1} =$ .... , $y \neq 0$

(a) $\frac{1}{3} y$ (b) $\frac{3}{y}$ (c) y^{-3} (d) $\frac{1}{3^y}$

3 Which of the following $= \frac{1}{4}$ million ?

(a) 25×10^5 (b) 0.25×10^5 (c) 0.25×10^6 (d) 0.25×10^4

2 [a] Write the result of each of the following on the standard form :

1 $(5.8 \times 10^7) + (3.2 \times 10^5)$ 2 $(65.5 \times 10^{-2}) \div (5 \times 10^2)$ 3 60000×5000

[b] Calculate the value of each of the following :

1 $\frac{(3^{-2})^4}{3^{-5} \times 3^{-2}}$ 2 $\frac{(-3)^7 \times (-3)^{-2}}{(-3)^5}$

Quiz 5 till lesson 5 – unit 1

time 15 min.

1 Choose the correct answer from those given :

1. If $0.000237 = 2.37 \times 10^n$, then n = ...

(a) 4 (b) 3 (c) -4 (d) -3

2. $-11 + 3 \times 7 =$ ....

(a) 11 (b) 10 (c) -56 (d) -1

3. $3^2 + 3^2 + 3^2 =$ ....

(a) 3^3 (b) 9^6 (c) 9^2 (d) 3^6

2 [a] Calculate the value of each of the following :

1. $8 \times 2^2 - 7 \times (4 + 1)$
2. $2\left((5^2 + 1) - (4^2 - 1)\right)$
3. $\frac{5 + 2 \times 5}{2^2 + 1}$

[b] Find the value of : $\frac{7^{-3} \times 7^4}{7 \times 7^{-2}}$

Quiz 6 till lesson 6 – unit 1

time 15 min.

1 Choose the correct answer from those given :

1. $\sqrt{(-7)^2} =$ ....

(a) 49 (b) 7 (c) -7 (d) ± 7

2. $\sqrt{10^2 - 8^2} =$ ....

(a) 2 (b) 6 (c) ± 2 (d) ± 6

3. $\sqrt{9 + 16} = 3 +$ ....

(a) 4 (b) 5 (c) 2 (d) 22

2 [a] 1 If $(AB)^2 = 144 \text{ cm}^2$, $(BC)^2 = 625 \text{ cm}^2$ and $B \in \overline{AC}$, then find the length of : $\overline{AC}$

2 Reduce to the simplest form : $\left(-\frac{1}{2}\right)^3 \times \sqrt{\frac{25}{9}} \times \sqrt{\left(\frac{8}{5}\right)^2} \times 3^{-1}$

[b] If $a = \frac{1}{3}$, $b = -\frac{2}{3}$, find the value of : $|(a^3 \div b^3)^{-1}|$

Quiz 7 till lesson 7 – unit 1

time 15 min.

1 Choose the correct answer from those given :

1 The S.S. of the equation : $2x = -6$ in $\mathbb{Z}$ is ....

(a) $\{-3\}$ (b) $\varnothing$ (c) $\{-6\}$ (d) $\{3\}$

2 If $3y = 6$, then $6y =$ ....

(a) 2 (b) 3 (c) 6 (d) 12

3 The number $0.0000014 =$ ....

(a) 1.4×10^{-5} (b) 1.4×10^{5} (c) 1.4×10^{-6} (d) 1.4×10^{6}

2 [a] 1 Find the S.S. of the following equation in $\mathbb{Q}$: $3(2x - 1) = 15$

2 Two natural numbers, the difference between them is 5 and their sum is 15 find the two numbers.

[b] Find the value of : $\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^{\text{zero}}$

Quiz 8 till lesson 8 – unit 1

time 15 min.

1 Choose the correct answer from those given :

1 The S.S. of the inequality : $-5x <$ zero in $\mathbb{Q}$ is ....

(a) $\varnothing$ (b) $\mathbb{Q}_+$ (c) $\mathbb{Q}_-$ (d) $\mathbb{Z}_+$

2 The quarter of the number 2^{10} is ....

(a) 2^5 (b) 2^9 (c) 2^8 (d) 1^{10}

3 If $x < y$, $y < z$, then ....

(a) $x > z$ (b) $x < z$ (c) $x = z$ (d) $x \geq z$

2 [a] 1 Find the S.S. of the following inequality in $\mathbb{Q}$: $2 - 3x < -4$

2 **Find the S.S. of the following equation in $\mathbb{Q}$:** $5x + 4 = 6$

[b] If $a = -\frac{1}{2}$, $b = 2$, $c = \frac{3}{2}$, **find the numerical value of the expression :** $a^2 b^3 + (a + c)^5$

Quiz 9 till lesson 2 – unit 2

time 15 min.

1 Choose the correct answer from those given :

1 As flipping a fair coin once , the probability of appearing a head is ....

(a) 5 (b) 5 % (c) 50 % (d) $\frac{1}{5}$

2 If the probability of occuring an event is $\frac{5}{8}$, then the probability of not occuring of the same event = ...

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) 1 (d) $\frac{1}{2}$

3 $x^4 \div x^{-2} =$ ....

(a) x^2 (b) x^{-6} (c) x^{-8} (d) x^6

2 [a] Find the S.S. of the following equation in $\mathbb{Q}$: $3(x+1) - x = 8$

[b] A card is drawn randomly from 8 cards numbered from 1 to 8 Write the sample space , then find the probability of each of the following :

1 Appearing an even number. 2 Appearing a number divisible by 3

Final Revision

of Algebra and Statistics

Revision for the important rules of algebra and statistics

Remember The repeated multiplication and the laws of powers

- **If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , m and n are two integers , then :**

1. $\left(\frac{a}{b}\right)^n \times \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n+m}$ "when multiplying the like base , add their powers (indices)"
2. $\left(\frac{a}{b}\right)^n \div \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n-m}$ where $\frac{a}{b} \neq 0$
   when dividing like bases , we subtract their powers (indices)
3. $\left(\left(\frac{a}{b}\right)^n\right)^m = \left(\frac{a}{b}\right)^{n \times m}$
4. $\left(\frac{a}{b} \times \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \times \left(\frac{c}{d}\right)^n$
5. $\left(\frac{a}{b} \div \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \div \left(\frac{c}{d}\right)^n$ where $\frac{c}{d} \neq 0$

- If $\frac{a}{b}$ is a rational number , then $\left(\frac{a}{b}\right)^0 = 1$ where $a \neq 0$
- If a is a rational number , $a \neq 0$, n is a positive integer , then : $a^{-n} = \frac{1}{a^n}$, $a^n = \frac{1}{a^{-n}}$
- If $\frac{a}{b}$ is a rational number , where $\frac{a}{b} \neq 0$, n is a positive integer , then : $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Example

Simplify each of the following to the simplest form :

1. $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3$
2. $\left(\frac{2}{9}\right)^7 \div \left(\frac{2}{9}\right)^5$
3. $\left(-\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$
4. $\frac{6^{-3} \times 6^5}{6^2}$
5. $\left(\frac{5^3 \times 5^4}{5^5}\right)^{-2}$
6. $\frac{(-4x^3y^4)^2}{(-2xy^2)^4}$

Solution

1. $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$
2. $\left(\frac{2}{9}\right)^7 \div \left(\frac{2}{9}\right)^5 = \left(\frac{2}{9}\right)^2 = \frac{2^2}{9^2} = \frac{4}{81}$
3. $\left(-\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2 = -\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2 = -\left(\frac{1}{2}\right)^5 = -\frac{1}{32}$
4. $\frac{6^{-3} \times 6^5}{6^2} = 6^{-3+5-2} = 6^0 = 1$
5. $\left(\frac{5^3 \times 5^4}{5^5}\right)^{-2} = (5^{3+4-5})^{-2} = (5^2)^{-2} = 5^{-4} = \frac{1}{5^4} = \frac{1}{625}$
6. $\frac{(-4x^3y^4)^2}{(-2xy^2)^4} = \frac{(-4)^2 \times x^6 \times y^8}{(-2)^4 \times x^4 \times y^8} = \frac{\cancel{16}x^6\cancel{y^8}}{\cancel{16}x^4\cancel{y^8}} = x^2$ where $xy \neq 0$

Remember **The standard scientific notation of the rational number**

The number is written in the standard form as : $a \times 10^n$ where $1 \leq |a| < 10$ and $n \in \mathbb{Z}$

For example : Each of the following two numbers is written in its standard form :

- 4.6×10^8
- 5.27×10^{-6}
- -9.6×10^{10}
- 1×10^{-13}

Each of the previous numbers is the product of two numbers :

- The first number could be positive or negative and its absolute value must be greater than or equal 1 and less than 10
- The second number expresses the powers of the number 10 (These powers could be positive on negative)

Example 1

Put each of the following numbers in the standard form :

(1) 8 200 000 000 (2) 0.000 000 135 (3) 45×10^8

(4) 706.4×10^5 (5) -0.0015×10^{-9}

Solution

(1) $8\ 200\ 000\ 000 = 8.2 \times 10^9$

Notice that

To move the decimal point 9 places towards left , we multiplied by 10^9

(2) $0.000\ 000\ 135 = 1.35 \times 10^{-7}$

Notice that

To move the decimal point 7 places towards right , we multiplied by 10^{-7}

(3) $45 \times 10^8 = 4.5 \times 10 \times 10^8 = 4.5 \times 10^9$

(4) $706.4 \times 10^5 = 7.064 \times 10^2 \times 10^5 = 7.064 \times 10^7$

(5) $-0.0015 \times 10^{-9} = -1.5 \times 10^{-3} \times 10^{-9} = -1.5 \times 10^{-12}$

Example 2

Find the value of n in each of the following :

(1) $500\ 000 = 5 \times 10^n$ (2) $0.00\ 052 = 5.2 \times 10^n$ (3) $7\ 293 = n \times 10^3$

Solution

(1) $\because 500\ 000 = 5 \times 10^5$ $\therefore n = 5$

(2) $\because 0.00\ 052 = 5.2 \times 10^{-4}$ $\therefore n = -4$

(3) $\because 7\ 293 = 7.293 \times 10^3$ $\therefore n = 7.293$

Remember **The order of mathematical operations**

The order of performing the mathematical operations is as the following :

1. Perform the operations within parentheses (interior parentheses then exterior ones).
2. Evaluate the powers.
3. Perform multiplications and divisions in order from left to right.
4. Perform additions and subtractions in order from left to right.

Notice that

In the problems containing fractions , we should perform the operations in the numerator and denominator before divisions.

Example

Calculate the value of each of the following :

(1) $4 - 3\,[4 - 2\,(6 - 3)] \div 2$

(2) $8 \times 2^2 - 7 \times (4 + 1)$

(3) $\frac{11 - (5 - 4)}{5^2 - 10 \times 2}$

Solution

(1) $4 - 3\,[4 - 2\,(6 - 3)] \div 2 = 4 - 3\,[4 - 2 \times 3] \div 2$ (the interior parentheses)

$= 4 - 3\,[4 - 6] \div 2$ (multiplication inside parentheses)

$= 4 - 3\,[-2] \div 2$ (subtraction inside parentheses)

$= 4 + 6 \div 2$ (multiplication by parentheses)

$= 4 + 3$ (division)

$= 7$ (addition)

(2) $8 \times 2^2 - 7 \times (4 + 1) = 8 \times 2^2 - 7 \times 5$ (addition inside parentheses)

$= 8 \times 4 - 7 \times 5$ (powers)

$= 32 - 35$ (multiplication)

$= -3$ (subtraction)

(3) $\frac{11 - (5 - 4)}{5^2 - 10 \times 2} = \frac{11 - 1}{25 - 20} = \frac{10}{5} = 2$

Remember **The square root of a perfect square rational number**

- The square root of the perfect square rational number "a" is the number whose square equals "a"

  For example : $\sqrt{16} = 4$, $-\sqrt{16} = -4$, $\pm\sqrt{16} = \pm 4$
- It is meaningless to find $\sqrt{a}$ if a is a negative rational number.
- $\sqrt{a^2} = |a|$ *For example :* $\sqrt{(-3)^2} = |-3| = 3$
- When there is an addition or a subtraction operation under the square root , it must be performed first before finding the square root.

Example 1

Find each of the following in the simplest form :

① $\sqrt{36}$ ② $-\sqrt{\frac{16}{25}}$ ③ $\pm\sqrt{2\frac{1}{4}}$ ④ $\sqrt{\left(-\frac{2}{7}\right)^2}$

⑤ $-\sqrt{0.25}$ ⑥ $\sqrt{16+9}$ ⑦ $\sqrt{100-36}$ ⑧ $\sqrt{\frac{36\,a^8}{49\,d^4}}$

Solution

① $\sqrt{36} = 6$ because $6^2 = 36$

② $-\sqrt{\frac{16}{25}} = -\frac{4}{5}$ because $\left(\frac{4}{5}\right)^2 = \frac{16}{25}$

③ $\pm\sqrt{2\frac{1}{4}} = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$

④ $\sqrt{\left(-\frac{2}{7}\right)^2} = \left|-\frac{2}{7}\right| = \frac{2}{7}$

⑤ $-\sqrt{0.25} = -\sqrt{\frac{25}{100}} = -\frac{5}{10} = -\frac{1}{2}$

⑥ $\sqrt{16+9} = \sqrt{25} = 5$

⑦ $\sqrt{100-36} = \sqrt{64} = 8$

⑧ $\sqrt{\frac{36\,a^8}{49\,d^4}} = \frac{6\,a^4}{7\,d^2}$

Example 2

Simplify each of the following to the simplest form :

① $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^2$ ② $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0$

Solution

① $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^2 = -\frac{2}{7} \times \frac{7}{2} \times \frac{4}{49} = -\frac{4}{49}$

② $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0 = \frac{9}{4} \times \frac{8}{3} \times 1 = 6$

Remember **The properties of the equality relation for solving the equation**

- We can add any rational number to both sides of the equation.
  For example : If $X - 1 = 5$, then $X - 1 + 1 = 5 + 1$ *i.e.* $X = 6$
- We can subtract any rational number from both sides of the equation.
  For example : If $X + 3 = 2$, then $X + 3 - 3 = 2 - 3$ *i.e.* $X = -1$
- We can multiply both sides of the equation by the same rational number.
  For example : If $\frac{1}{5} X = 2$, then $\frac{1}{5} X \times 5 = 2 \times 5$ *i.e.* $X = 10$
- We can divide both sides of the equation by the same rational number not equal to zero.
  For example : If $7 X = 14$, then $\frac{7 X}{7} = \frac{14}{7}$ *i.e.* $X = 2$

Example

Find in $\mathbb{Q}$ the solution set of each of the following equations :

(1) $2 X - 5 = 13$ (2) $3 X + 4 = 2 (X + 1)$

Solution

(1) $\because 2 X - 5 = 13$ "Adding 5 to both sides"

$\therefore 2 X - 5 + 5 = 13 + 5$ $\therefore 2 X = 18$ "Dividing both sides by 2"

$\therefore \frac{2 X}{2} = \frac{18}{2}$ $\therefore X = 9$

$\therefore$ The S.S. $= \{9\}$

(2) Notice that the variable (X) exists in the two sides , then we try to collect it in one side (say the left side)

$\because 3 X + 4 = 2 (X + 1)$

Using the distribution property $\therefore 3 X + 4 = 2 X + 2$

Subtracting $2 X$ from both sides $\therefore 3 X - 2 X + 4 = 2 X - 2 X + 2$

$\therefore X + 4 = 2$

Subtracting 4 from both sides $\therefore X + 4 - 4 = 2 - 4$

$\therefore X = -2$ $\therefore$ The S.S. $= \{-2\}$

Remarks for solving word problems

- If a number $= x$, then its twice $= 2x$ and its three times $= 3x$, ...
- If a number $= x$ and another number exceeds it by 5 , then the other number $= x + 5$
- If a number $= x$ and another number decreases than it by 5 , then the other number $= x - 5$
- If the age of a man now $= x$ years , then :
  * His age after 3 years $= (x + 3)$ years.
  * His age 3 years ago $= (x - 3)$ years.
- Three consecutive integers are : x , $x + 1$ and $x + 2$
- Three consecutive natural (even or odd) numbers are x , $x + 2$ and $x + 4$
- The perimeter of a rectangle = 2 (length + width)
- The perimeter of a square = side length × 4
- The perimeter of the triangle = the sum of its sides lengths.
- The area of the triangle $= \frac{1}{2}$ the base length × the height.
- The sum of measures of the interior angles of the triangle = 180°

Example 1

Two natural numbers , one of them is thrice of the other , if the sum of them is 16
Find the two numbers.

Solution

Let one of the two numbers be x

$\because$ The other number is thrice of this number $\therefore$ The other number $= 3x$

$\because$ The sum of the two numbers = 16 $\therefore x + 3x = 16$

$\therefore 4x = 16$ "Dividing by 4" $\therefore x = 4$

$\therefore$ One of the two numbers = 4 , the other number $= 3 \times 4 = 12$

Example 2

A rectangle with length equals twice its width and its perimeter = 18 cm.
Find the dimensions of the rectangle.

Solution

Let the width of the rectangle be x cm. $\therefore$ Its length $= 2x$ cm.

$\because$ The perimeter of the rectangle = 2 (length + width)

$\therefore 18 = 2(2x + x)$ $\therefore 18 = 2 \times 3x$

$\therefore 18 = 6x$ $\therefore x = 3$

$\therefore$ The width of the rectangle = 3 cm. and its length = 6 cm.

Remember **The properties of inequality for solving the inequalities**

- We can add any rational number to both sides of the inequality without change in the inequality relation.

For example : If $x - 1 > 5$, then $x - 1 + 1 > 5 + 1$ *i.e.* $x > 6$

- We can subtract any rational number from the two sides of the inequality without change in the inequality relation.

For example : If $x + 3 < 2$, then $x + 3 - 3 < 2 - 3$ *i.e.* $x < -1$

- We can multiply any positive rational number by both sides of the inequality without change in the inequality relation.

For example : If $\frac{1}{3} x < 2$, then $\frac{1}{3} x \times 3 < 2 \times 3$ *i.e.* $x < 6$

- We can multiply any negative rational number by both sides of the inequality with change in the inequality relation.

For example : If $-\frac{1}{3} x < 2$, then $-\frac{1}{3} x \times -3 > 2 \times -3$ *i.e.* $x > -6$

- We can divide any positive rational number by both sides of the inequality without change in the inequality.

For example : If $7x > 14$, then $\frac{7x}{7} > \frac{14}{7}$ *i.e.* $x > 2$

- We can divide any negative rational number by both sides of the inequality with change in the inequality.

For example : If $-7x > 14$, then $\frac{-7x}{-7} < \frac{14}{-7}$ *i.e.* $x < -2$

Example

Find in $\mathbb{Q}$ the solution set of each of the following inequalities :

(1) $2x - 5 > 5$ (2) $4 - 2x \le 2$

Solution

(1) $\because 2x - 5 > 5$ "Adding 5 to both sides" $\therefore 2x - 5 + 5 > 5 + 5$

$\therefore 2x > 10$ "Multiplying both sides by $\frac{1}{2}$" $\therefore \frac{1}{2} \times 2x > 10 \times \frac{1}{2}$

$\therefore x > 5$ $\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x > 5\}$

(2) $\because 4 - 2x \le 2$ "Adding -4 to both sides" $\therefore -4 + 4 - 2x \le -4 + 2$

$\therefore -2x \le -2$ "Dividing both sides by (-2)" $\therefore \frac{-2x}{-2} \ge \frac{-2}{-2}$

Notice that : the change of inequality sign

$\therefore x \ge 1$ $\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x \ge 1\}$

Remember **The probability**

- The probability of any event occurrence $A \subset S$ is denoted by P (A) and it is given by using the relation : $P(A) = \frac{\text{The number of elements of the event « A »}}{\text{The number of elements of sample space « S »}} = \frac{n(A)}{n(S)}$
- The probability of the impossible event = 0
- The probability of the certain event = 1
- The value of probability of any event is not less than zero and not more than one
  i.e. $0 \le$ The probability of an event occurrence ≤ 1
- It is meaningless that the probability of the occurrence of an event is 140% or – 0.2

Example 1

A fair die is rolled once and we observe the apparent number on the upper face , what is the probability of getting :

(1) the number 4
(2) an even number.
(3) a number greater than or equal to 5
(4) a prime number.
(5) a number greater than 6
(6) a number smaller than 10

Solution

(1) The probability of getting the number 4 = $\frac{1}{6}$
(2) The probability of getting an even number = $\frac{3}{6} = \frac{1}{2}$
(3) The probability of getting a number greater than or equal 5 = $\frac{2}{6} = \frac{1}{3}$
(4) The probability of getting a prime number = $\frac{3}{6} = \frac{1}{2}$
(5) The probability of getting a number greater than 6 = $\frac{0}{6} = 0$
(6) The probability of getting a number smaller than 10 = $\frac{6}{6} = 1$

Example 2

A bag contains 4 red balls , 6 green balls and 5 black balls , if a ball is drawn randomly from it , **calculate :**

(1) The probability that the drawn ball is green.
(2) The probability that the drawn ball is black.
(3) The probability that the drawn ball is not red.

Solution

(1) The probability that the drawn ball is green = $\frac{\text{The number of green balls}}{\text{The total number of balls}} = \frac{6}{15} = \frac{2}{5}$
(2) The probability that the drawn ball is black = $\frac{\text{The number of black balls}}{\text{The total number of balls}} = \frac{5}{15} = \frac{1}{3}$
(3) The probability that the drawn ball is not red = $\frac{6+5}{15} = \frac{11}{15}$

Final Examinations

on Algebra and Statistics

Model Examinations of the School Book on Algebra and Statistics

Model 1

Answer the following questions :

1 Complete :

1. $\frac{81}{625} = \left(\frac{25}{9}\right)^{\cdots\cdots}$
2. If $7 - 2x = 3$, then $x =$ .... where $x \in \mathbb{N}$
3. $3^{-1} + 4^{-1} =$ ....
4. The standard form of the number $0.7 \times 0.005 =$ ....
5. The probability of the certain event = ...

2 Choose the correct answer :

1. The sum of the probabilities for all possible outcomes of a randomly experiment is ....
   (a) zero (b) 1 (c) > 1 (d) < 1
2. If $3a = \sqrt{4}\, b$, then $\frac{a}{b} =$ ....
   (a) 2 : 3 (b) 3 : 2 (c) 3 : 4 (d) 4 : 3
3. $\left(\frac{-2}{3}\right)^{-3}$ equals ....
   (a) $\frac{-27}{8}$ (b) $\frac{-8}{27}$ (c) $\frac{8}{27}$ (d) $\frac{27}{8}$
4. There are 21 boys and 15 girls in a classroom , one pupil is chosen randomly , the probability that the chosen pupil is a girl = ...
   (a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{4}{7}$ (d) $\frac{5}{6}$
5. $\sqrt{(-8)^2 + (-6)^2} =$ ....
   (a) $|-10|$ (b) ± 10 (c) 14 (d) -14
6. 10 % of L.E. $2\frac{1}{2}$ = L.E. ....
   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 25

3 **[a] Simplify to the simplest form :** $\left(-\frac{3}{7}\right)^0 \times \left(\frac{-2}{5}\right)^2 \times \sqrt{6\frac{1}{4}}$

[b] Find the numerical value of the expression :

$3ab + 8a \div (4b)$ when $a = 4$, $b = -2$

4 **[a]** Find in $\mathbb{Q}$ the S.S. of : $3x + 1 = 25$

[b] Find the value of : $\frac{8 \times 8^{-3}}{8^{-4}}$

5 **[a]** A factory of a tire record the distance that traveled by a certain type of them before damage for 800 units of this type as following.

The distance in thousand (km.)	Less than 50	50 to 100	More than 100 till 150	More than 150
The number of damage tire	80	120	280	320

If you bought a tyre of this type , what is the probability of change it :

1. Before traveled 50 thousand km.
2. After traveled more than 100 thousand km.

[b] Find in $\mathbb{Q}$ the S.S. of : $2x + 5 < 16$

Model 2

Answer the following questions :

1 Complete :

1. $\left(\frac{-2}{3}\right)^0 =$ .... 2. $\sqrt{\frac{16}{49}} =$ ....
3. The probability of the impossible event = ...
4. 1 , 2 , 3 , 5 , 8 , ... , ... (In the same pattern)
5. If the probability that the student is absent in a school is 0.15 , if the number of students of this school is 600 , then the number of the present students that day is ....

2 Choose the correct answer :

1. $2^3 \times 2^3 =$ ....

(a) 2^6 (b) 2^8 (c) 2^{15} (d) 2^{53}

2. Which of the following is the greatest ?

(a) 2.3×10^4 (b) 2.3×10^5 (c) 3.2×10^4 (d) 3.2×10^5

3 $(x^2)^{-3} \times x^6 =$ ....

(a) x^{12} (b) x^{-12} (c) x (d) 1

4 Which of the following may be probability of an event ?

(a) -0.35 (b) 87 % (c) 1.05 (d) 130 %

5 If $-x > 4$, then ....

(a) $x > -4$ (b) $x > 4$ (c) $x < -4$ (d) $x < 4$

6 Area of a rectangle of length 120 cm. and width 80 cm. equals .... m^2

(a) 9600 (b) 400 (c) 9.6 (d) 0.96

3 [a] Two integers numbers , the smaller one is $2x$ and the greater is $5x$, if the difference between them is 30 Find the two numbers.

[b] **Find the value of :** $\frac{5^{-4} \times 5^7}{5^3}$ in the simplest form.

4 [a] **Find in $\mathbb{Q}$ the S.S. of each of the following :**

1 $(3x + 2) + 5 = 13$

2 $2x + 15 < 19$

[b] **Find the value of the expression in the simplest form :**

$\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

5 [a] If a regular die is thrown once and observed the number on upper face ,

find the probability of each of the following :

1 Getting a prime even number.

2 Getting an odd number less than 4

[b] If $x = -\frac{1}{2}$, $y = -\frac{3}{4}$, **find in the simplest form :** $\left(\frac{y}{x^2}\right)^{-2}$

Model examination for the merge students

Answer the following questions :

1 Choose the correct answer :

1. $\left(\frac{-2}{3}\right)^2 = \dots\dots$

   (a) $\frac{4}{9}$ (b) $\frac{-4}{9}$ (c) $\frac{4}{6}$ (d) $\frac{-4}{6}$

2. $\left(\frac{4}{7}\right)^0 = \dots\dots$

   (a) 0 (b) 1 (c) $\frac{4}{7}$ (d) -1

3. $2 \times 6 - 4 \times 2 = \dots\dots$

   (a) 4 (b) 8 (c) 10 (d) 2

4. $(7)^{-2} = \dots\dots$

   (a) 49 (b) $\frac{1}{49}$ (c) 14 (d) -14

5. $\sqrt{9 + 16} = \dots\dots$

   (a) 7 (b) 5 (c) 25 (d) -7

2 Complete each of the following :

1. If $x + 2 = 6$, then $x = \dots\dots$
2. When tossing a coin once , then the probability of the appearance of a tail = ……
3. The probability of the impossible event = ……
4. $\sqrt{\left(\frac{2}{5}\right)^2} = \dots\dots$
5. $7\,(6^2 - 5 \times 6) = \dots\dots$

3 Complete the solution to find the result :

1. $12 \times 2^2 \div 24 + 3^2 = 12 \times \dots\dots \div 24 + \dots\dots$

   $= \dots\dots \div 24 + \dots\dots = \dots\dots + \dots\dots = \dots\dots$

2. $\frac{8 + 20 - 4}{8 - 4} = \frac{\dots\dots - 4}{\dots\dots} = \frac{\dots\dots}{\dots\dots} = \dots\dots$

4 Put (✓) or (✗) :

1. If $2x + 3 = 7$, then $x = 2$ ( )
2. $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^5 = \left(\frac{2}{3}\right)^6$ ( )
3. $(x^2)^3 = x^6$ ( )
4. $\left(\frac{3}{2}\right)^2 = -\frac{9}{4}$ ( )
5. $\sqrt{100 - 64} = 2$ ( )

5 A card is drawn randomly from 8 cards numbered from 1 to 8 , join from column (A) to column (B) :

Column (A)	Column (B)
1 The event of getting an even number equals	• $\frac{1}{2}$
2 The probability of getting an even number equals	• $\{8 , 6 , 4 , 2\}$
3 The event of getting a number > 6 equals	• 1
4 The probability of getting a number < 9 equals	• $\frac{1}{8}$
5 The probability of getting a number 8 equals	• $\{8 , 7\}$

Schools Examinations on Algebra and Statistics

1 Cairo Governorate

AL Nozha Directorate of Education
Modern Language Schools

Remark: Some school exams are modified to include what was canceled last year

Answer the following questions :

1 Choose the correct answer :

1. $7x^{-1} = \dots\dots$
   (a) $7x$ (b) $\frac{7}{x}$ (c) $7x^2$ (d) 0
2. If $-x < 2$, then $\dots\dots$
   (a) $x < 2$ (b) $x \le 2$ (c) $x > 2$ (d) $x > -2$
3. $\frac{6a^2x^4}{2a^3x^3} = \dots\dots$ where $a \ne 0$, $x \ne 0$
   (a) $3ax$ (b) $3a^5x^7$ (c) $\frac{3x}{a}$ (d) $\frac{3}{ax}$
4. $\sqrt{(-6)^2 + (-8)^2} = 25 - \dots\dots$
   (a) 15 (b) 10 (c) 6 (d) 8
5. If $5x = 35$, then $2x + 1 = \dots\dots$
   (a) 7 (b) 8 (c) 15 (d) 7
6. $2^3 \times 2^5 = \dots\dots$
   (a) 2^2 (b) 2^8 (c) 2^{15} (d) 2^{53}

2 Complete :

1. $\sqrt{\frac{25x^2y^2}{36}} = \dots\dots$
2. The S.S. of : $x + 21 = 8$ in $\mathbb{Z}$ is $\dots\dots$
3. The additive inverse of $\left(\frac{-2}{5}\right)^2 = \dots\dots$
4. The probability of certain event $= \dots\dots$
5. ✱ $5 + 8 \div 2 - 3 \times 4 = \dots\dots$

3 [a] If $x = \frac{-3}{2}$, $y = \frac{1}{2}$, $z = \frac{-4}{3}$, **find the numerical value of :** $x^2 y^2 z^2$

[b] **Find the S.S. of :** $5x + 8 = 13 - 2x$, $x \in \mathbb{Q}$

4 [a] **Find the S.S. in $\mathbb{Q}$:** $3x - 1 \le 5$

[b] **Find the value of :** $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

5 [a] A box contain 4 white balls , 5 red balls , 6 blue balls. One ball is drawn randomly , **Find the probability of the drawn ball is :**

1 red 2 black 3 white or blue

[b] Two natural numbers , one of them twice the other and their sum is 24

Find the two numbers.

2 Cairo Governorate

El Maadi Directorate
El Orman Smart School

Answer the following questions :

1 Choose the correct answer :

1 If $0 \in \{5 , x - 3\}$, then $x =$ ....

(a) 0 (b) -5 (c) 3 (d) -3

2 $3^4 \times 3^3 =$ ....

(a) 3^{12} (b) 3^7 (c) 3 (d) 3^{-1}

3 The S.S. of the inequality : $x < 3$ in $\mathbb{N}$ is ....

(a) $\{0\}$ (b) $\{0 , 1 , 2\}$ (c) $\{1 , 2\}$ (d) $\varnothing$

4 $\frac{6 a^3 x^4}{3 a^2 x^3} =$ .... where $a \neq 0 , x \neq 0$

(a) $2 a x$ (b) $2 a^2 x^7$ (c) $\frac{3 x}{a}$ (d) $\frac{3}{a x}$

5 $\left(\frac{-2}{3}\right)^{-3} =$ ....

(a) $\frac{27}{8}$ (b) $\frac{-8}{27}$ (c) $\frac{8}{27}$ (d) $\frac{-27}{8}$

6 If the probability of success of a student is 0.6 , then the probability of his failure is ....

(a) 1 (b) $\frac{1}{10}$ (c) $\frac{4}{10}$ (d) $\frac{6}{10}$

2 Complete the following :

1 The multiplicative inverse of 7 is ....

2 0.00025 in scientific notation = ...

3 $\left(\frac{3}{4}\right)^2 \div \left(\frac{3}{4}\right)^3 =$ ....

4 $\sqrt{16 + 9} = 4 +$ ....

5 A class has 36 pupils , 25 of them are boys , if a pupil is chosen randomly , then the probability that the pupil is a girl = ...

3 [a] Find the simplest form of : $\left(\frac{7^{-2} \times 7^{5}}{7^{3}}\right)^{2}$

[b] Simplify and find the value of : $\left(-\frac{3}{2}\right)^{2} \times \sqrt{\frac{64}{9}} \times \left(\frac{2}{5}\right)^{0}$

[c] * Find the value of : $12 \times 2^{2} \div 24 + 3^{2}$

4 [a] 1 Calculate : $\sqrt{100-(-8)^{2}}$

2 If $x = \frac{1}{2}$, $y = \frac{2}{3}$, then find the value of : $(x^{2} y^{2})^{-3}$

[b] Find in $\mathbb{Q}$ the S.S. of the inequality : $3x + 6 > 3$

5 [a] Find the solution set in $\mathbb{Q}$: $4x - 5 = 27$

[b] A fair die is rolled once. Calculate the probability of appearance :

1 an even number. 2 a number greater than 4

3 Cairo Governorate

El-Zeiton Zone
Talaea Gaber El Ansary Language School

Answer the following questions :

1 Choose the correct answer :

1 If $4x = 20$, then $3x - 1 =$ ....

(a) 13 (b) 14 (c) 15 (d) 16

2 $3^{10} + 3^{10} + 3^{10} =$ ....

(a) 9^{30} (b) 3^{30} (c) 3^{10} (d) 3^{11}

3 If $\frac{4x-1}{2x+3} = 0$ where $x \in \mathbb{Q}$, then $x =$ ....

(a) $-\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) 0

4 A class contains 40 students , 36 of them are succeed in a test , then the probability of failed is ....

(a) 0.1 (b) 0.9 (c) 0.3 (d) $\frac{2}{3}$

5 $\sqrt{(8)^{2} + (6)^{2}} = 8 +$ ....

(a) 14 (b) 10 (c) 2 (d) 6

6 $\frac{|-5|+1}{3}$ .... $\mathbb{Z}$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

2 Complete the following :

1 If $0.0000016 = 1.6 \times 10^{n}$, then n = ...

2 The multiplicative invers of $\sqrt{5\frac{4}{9}}$ = ...

3 When a fair die is tossed once , then the probability of getting an even prime number = ...

4 If $\frac{3}{4}X = 75$, then $\sqrt{X}$ = ...

5 25 % from .... = 8

3 [a] **Find in the simplest form :** $\left(\frac{5^5 \times 5^{-2}}{5^4}\right)^{-2}$

[b] **Find the S.S. in $\mathbb{Q}$:** $-2 < 4X + 1 < 6$

4 [a] **Simplify :** $\left(\frac{-2}{7}\right)^{-2} \times \sqrt{\frac{16}{49}} \times \left(\frac{-1}{7}\right)^0$

[b] If the length of a rectangle is 5 cm. more than its width and its perimeter is 42 cm. Find the area of this rectangle.

5 [a] **Simplify :** $\frac{(4X^3y^2)^2}{(2Xy^2)^3}$, then find the value when $X = \frac{1}{2}$, $y = 3$

[b] A team plays 30 matches in national league its drawn probability is 0.2 and its win probability is 0.7 Calculate the number of loss matches.

4 Giza Governorate

El-Dokki Directorate
Modern Narmer language School

Answer the following questions :

1 **Choose the correct answer :**

1 If $\left(\frac{1}{2}\right)^X = 8$, then X = ...

(a) 4 (b) −4 (c) 3 (d) −3

2 $\sqrt{\left(\frac{-2}{9}\right)^2}$ = ...

(a) $\pm\frac{2}{9}$ (b) $\frac{2}{9}$ (c) $\frac{-2}{9}$ (d) $\frac{2}{3}$

3 If $X + 4 = 10$, then $5X$ = ...

(a) 30 (b) 20 (c) 12.5 (d) 25

4 If $(2^X)^y = 8$, then ....

(a) $X + y = 3$ (b) $Xy = 3$ (c) $X - y = 3$ (d) $\frac{X}{y} = 3$

5 If X is a rational number where $-X > 4$, then ....

(a) $X > -4$ (b) $X > 4$ (c) $X < -4$ (d) $X < 4$

6 If a letter is selected randomly from the word "SCHOOL" , then the probability that the letter is O equals ....

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

2 Complete :

1 If $\sqrt{x+3} = 3$, then $x =$ ....

2 $\left(1\frac{1}{2}\right)^{-2} =$ ....

3 $2^5 \times 5^5 = 10^{\cdots\cdots}$

4 ✻ $4 \times 3^2 - 20 =$ ....

5 The probability of the certain event is ....

3 [a] Find in the simplest form : $\left(\frac{-5}{7}\right)^0 \times \left(\frac{-3}{2}\right)^2 \times \sqrt{\frac{16}{9}}$

[b] Write the result of the following in the standard form of $a \times 10^n$, $n \in \mathbb{Z}$:

$(4.4 \times 10^3) \times (3 \times 10^5)$

4 [a] Find in $\mathbb{Q}$ the solution set of the equation : $3x + 1 = 25$

[b] Find the valve of : $\frac{3^{-4} \times 3^7}{3^3}$ in the simplest form.

5 [a] Find in $\mathbb{Q}$ the solution set of the inequality : $2x + 5 < 9$

[b] If a die is rolled once and the number of dots on the upper face is observed, write down the sample space **, then find the probability of the following events :**

1 getting a number greater than 6

2 getting a number satisfies the inequality : $1 < x < 6$

3 getting a number divisible by 3

5 Giza Governorate

Omrania Directorate
El Sadat E.L.S

Answer the following questions :

1 Choose the correct answer :

1 $x^{12} \div x^4 =$ .... where $x \neq 0$

(a) x^8 (b) x^3 (c) x^{16} (d) x^{-8}

2 $\pm\sqrt{\frac{4}{9}} =$ ....

(a) $\frac{-4}{9}$ (b) $\frac{-2}{3}$ (c) $\pm\frac{2}{3}$ (d) $\frac{2}{3}$

3 If the probability that pupils success is 75 % , then the probability of his failure is ....

(a) – 0.75 (b) 0.25 (c) – 0.25 (d) 0.75

4 If $-X < 7$, then X .... -7

(a) < (b) > (c) = (d) ≤

5 $6000 \times 50 =$ ....

(a) 300×10^2 (b) 30×10^5 (c) -3×10^3 (d) 3×10^5

6 If $3X = 6$, then $5X =$ ....

(a) $\frac{5}{2}$ (b) $\frac{2}{5}$ (c) 10 (d) 5

2 Complete each of the following :

1 The probability of impossible event is ....

2 The additive inverse of the number $\left(\frac{-1}{3}\right)^2$ is ....

3 $\sqrt{16+9} =$ ....

4 $\left(\frac{2}{5}\right)^{-2} =$ ....

5 $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, ... (in the same pattern)

3 [a] Find in the simplest from :

1 $\frac{9^{-2} \times 9^5}{9^3}$ 2 $\left(\frac{-3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0$

[b] Find in $\mathbb{Q}$ the solution set of the following equation : $4X + 1 = 21$

4 [a] If $X = \frac{1}{2}$, $y = \frac{2}{3}$, $z = \frac{-3}{2}$ **, then find the value of :** $(X y z)^2$

[b] Find in $\mathbb{Q}$ the solution set of the following inequality : $3X - 1 \le 2X + 3$

5 [a] The sum of three consecutive numbers is 24 , find them.

[b] A fair die is rolled once **, calculate the probability of appearance :**

1 an even number. 2 a number greater than 5

6 Alexandria Governorate

West Educational Zone
Inspectorate of Mathematics

Answer the following questions :

1 Complete each of the following :

1 The multiplicative identity element in $\mathbb{Q}$ is ....

2 $\sqrt{\frac{25}{49}} = \ldots\ldots\ldots$

3 $(x-5)^0 = 1$, if $x \neq \ldots\ldots\ldots$

4 If the probability of success of a student is 0.8 , then the probability of his failure is ....

5 If $X \subset Y$, then $X \cap Y = \ldots\ldots\ldots$

2 Choose the correct answer :

1 $\sqrt{64+36} = 8 + \ldots\ldots\ldots$

(a) 2 (b) 6 (c) 10 (d) -2

2 Half the number 2^{24} equals ....

(a) 2^{12} (b) 1^{23} (c) 2^{23} (d) 1^{12}

3 $0.0000068 = \ldots\ldots\ldots$

(a) 6.8×10^{-6} (b) 6.8×10^{5} (c) 6.8×10^{-7} (d) 6.8×10^{7}

4 If $2x = -12$, then $x^2 = \ldots\ldots\ldots$

(a) 6 (b) 144 (c) -36 (d) 36

5 The S.S. of the equation $5 - x = 3$ in $\mathbb{Q}$ is ....

(a) $\{2\}$ (b) $\{-2\}$ (c) $\{7\}$ (d) $\varnothing$

6 The probability of a certain event equals ....

(a) zero (b) 1 (c) -1 (d) $\frac{1}{2}$

3 [a] Simplify to the simplest form : $\frac{x^3 \times x^{-2}}{x^{-5} \times x}$, then find the value when $x = (-2)$

[b] Find the S.S. in $\mathbb{Q}$ for each of the following :

1 $3x - 5 > 1$ 2 $3x + 6 = 30 - 5x$

4 [a] A fair die is rolled once **, what is the probability of getting :**

1 an even number ? 2 a factor of 6 ?

[b] Find the value of : $\sqrt{6\frac{1}{4}} \times \left(\frac{2}{7}\right)^0 \times \left(\frac{-2}{5}\right)^2$

5 [a] A rectangle whose length is more than its width by 3 and its perimeter equals 26 , find its area.

[b] If $x = \frac{1}{2}$ and $y = \frac{4}{3}$ **find the value of :** $x^3 y^2$

7 Alexandria Governorate

El-Montaza Educational Zone
Math's Supervision

Answer the following questions :

1 Choose the correct answer :

1. $2^3 \times 2^5 =$ ....
   (a) 2^2 (b) 2^8 (c) 2^{15} (d) 2^{53}
2. Which of the following may be a probability of an event ?
   (a) -0.25 (b) $87\ \%$ (c) 1.05 (d) $130\ \%$
3. The multiplicative inverse of the number $\sqrt{\frac{9}{16}}$ is ....
   (a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$
4. $5\ x^{-1} =$ ....
   (a) $-5\ x$ (b) $5\ x$ (c) $\frac{5}{x}$ (d) $\frac{1}{5\ x}$
5. If $-x < 3$, then ....
   (a) $x > -3$ (b) $x > 3$ (c) $x < -3$ (d) $x < 3$
6. If $x + 2 = 5$, then $5\ x =$ ....
   (a) 3 (b) 35 (c) 15 (d) 7

2 Complete :

1. The probability of the certain event = ...
2. If $0.00057 = 5.7 \times 10^n$, then n = ...
3. The additive inverse of $\left(\frac{-2}{3}\right)^2$ is ....
4. $\sqrt{16 + 9} = 4 +$ ....
5. $2^{10} + 2^{10} = 2^{\cdots\cdots}$

3

[a] Find in $\mathbb{Q}$ the solution set of the following : 1 $3\ x + 1 = 16$ 2 $7\ x - 1 < 13$

[b] ✱ Find the value of : $5^2 + [3 \times 8 \div 2^2 - 2 \times 3]$

4

[a] If $x = \frac{1}{3}$, $y = \frac{1}{6}$, **find the numerical value of :** $(x + y)^{-1}$

[b] What is the number that if added to its three times , the result is 24 ?

5

[a] Simplify : $\frac{a^3 \times a^{-8}}{a^{-5}}$, $a \neq 0$

[b] A box contains 4 white , 5 red and 6 blue balls , a ball is drawn randomly from the box , **find the probability of getting the following events :**

1 the ball is blue. 2 the ball is white or red.

8 El-Kalyoubia Governorate

Directorate of Education
Math Supervision

Answer the following questions :

1 Choose the correct answer :

1 The number which is not in the standard form is ....

(a) 6.2×10^5 (b) 7.834×10^{16} (c) 0.8×10^5 (d) 6.7×10^{25}

2 If $3t = 6$, then $6t =$ ....

(a) 16 (b) 14 (c) 12 (d) 10

3 $\frac{1}{4} \times 4^{20} =$ ....

(a) 4^{15} (b) 4^{19} (c) 2^{19} (d) 2^{39}

4 $\frac{6a^2 x^4}{2a^3 x^3} =$ .... where $a \neq 0$, $x \neq 0$

(a) $3ax$ (b) $3a^5 x^7$ (c) $\frac{3x}{a}$ (d) $\frac{3}{ax}$

5 A class formed from 36 students , 16 of them are girls. If a student selected randomly from the class , then the probability that the student is a boy = ...

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{1}{2}$ (d) $\frac{1}{36}$

6 10 % of $2\frac{1}{2}$ L.E. = ... L.E.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) 25

2 Complete :

1 The additive inverse of $\left(\frac{-2}{3}\right)^4$ is ....

2 When a die is tossing twice and observed the upper face in each time the probability of appearance number 5 on the two faces is ....

3 $\sqrt{10^2 - 6^2} =$ ....

4 If $-x + 2 > 6$ and the substitution set $= \{-2 , -5 , -1\}$, then $x =$ ....

5 If $3a = \sqrt{4}\,b$, then $\frac{a}{b} =$ ....

3 **[a] Find in simplest form :** $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{a}{b}\right)^0$ where $a \neq 0$, $b \neq 0$

[b] If $x = \frac{-1}{2}$, $y = \frac{-3}{4}$, **find the value of :** $\left(\frac{y}{x}\right)^{-2}$

4 Find in $\mathbb{Q}$ the S.S. of :

1 $3x + 1 = 25$

2 $15 + 2x < 1$

5 **[a]** Find with steps the value of : $\left(2 \times \sqrt{36} - 2^4\right) \div 4$

[b] If a regular die is thrown once and observing the number on the upper face
Find the probability of getting :

1 a prime even number.
2 an odd number less than 4

9 El-Sharkia Governorate

East Zagazig Educational Administration
Omar AL-Farouk Formal Language School

Answer the following questions :

1 **Complete the following :**

1 $\left(\frac{-2}{3}\right)^0 =$ ……………

2 The probability of certain (sure) event = ……………

3 $\sqrt{a^4 b^2} =$ ……………

4 The number 0.00023 in the standard form is ……………

5 ✻ $7\,(6^2 - 5 \times 6) =$ ……………

2 **Choose the correct answer :**

1 Half the number $2^{10} =$ ……………
(a) 2^{10} (b) 2^{11} (c) 2^9 (d) 2

2 $\sqrt{10^2 - 6^2} =$ ……………
(a) 4 (b) 8 (c) 1 (d) 0

3 If the probability of success of a student is 0.7 , then the probability of his failure is ……………
(a) 0.7 (b) 0.4 (c) 1 (d) 0.3

4 $7^{-1} =$ ……………
(a) $\frac{1}{7}$ (b) 7 (c) $\frac{-1}{7}$ (d) -7

5 $\frac{4}{10} + \frac{3}{100} =$ ……………
(a) 0.34 (b) 0.43 (c) 4.3 (d) 3.4

6 $5^2 \times 5^4 =$ ……………
(a) 5^5 (b) 25^6 (c) 5^6 (d) 25^8

3 **[a]** Find in $\mathbb{Q}$ the solution set of : $2X + 1 = 9$

[b] Find the value of the following in the simplest form : $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} + \left(\frac{3}{7}\right)^0$

4 **[a] Simplify to the simplest form :** $\frac{3^6 \times 3^{-2}}{3^2}$

[b] If $x = \frac{-1}{2}$, $y = \frac{3}{4}$ **, find the value of :** $\frac{y}{x^2}$

5 **[a] Find in $\mathbb{Q}$ the solution set of :** $3x - 2 \le 7$

[b] A box contains 5 black balls , 3 white balls and 6 red balls , if a ball is drawn randomly from the box **Calculate the probability of :**

1 the ball is white. 2 the ball is not red.

10 El-Monofia Governorate

Monof Educational Directorate

Answer the following questions : **(Calculators are Permitted)**

1 **Choose the correct answer :**

1 $(3^2)^3 =$ ....

(a) 3^5 (b) 2^6 (c) 9^3 (d) 3^8

2 $|-3| + 5 =$ ....

(a) -8 (b) -2 (c) 2 (d) 8

3 If $A = 7^x$, $B = 7^{-x}$, then $A \times B =$ ....

(a) 49 (b) 7 (c) 1 (d) 0

4 Quarter of $2^{20} =$ ....

(a) 2^{10} (b) 2^{19} (c) 4^{19} (d) 4^9

5 0.0037 in the standard form is 3.7×10^x , then $x =$ ....

(a) 3 (b) 4 (c) -3 (d) -4

6 Which of the following could be a probability of an event ?

(a) 2 (b) 3 (c) -1 (d) 3 %

2 **Complete each of the following :**

1 1 , 2 , 3 , 5 , 8 , ... (in the same pattern).

2 The S.S. of inequality : $-x > 0$ in $\mathbb{N}$ is ....

3 $\sqrt{3^2 + 4^2} = 2 +$ ....

4 $\frac{8}{27} = \left(\frac{\cdots\cdots}{\cdots\cdots}\right)^3$

5 The probability of impossible event = ...

3 [a] **Find the S.S. of the following in $\mathbb{Q}$:**

1 $5x - 2 = 8$ 2 $2x + 3 > 4$

[b] ✻ **Calculate the value of :** $2\left[(5^2 + 1) - (4^2 - 1)\right]$

4 [a] **Simplify :** $\left(\frac{-2}{3}\right)^3 \times \sqrt{\frac{81}{64}} \times \left(\frac{1}{3}\right)^{zero}$

[b] A box contains 3 red balls , 4 yellow balls and 5 green balls. A ball is drawn randomly from the box. **Find the probability of the drawn ball is :**

1 yellow. 2 not green.

5 [a] **Evaluate :** $\frac{7^5 \times 7^{-2}}{7^3}$

[b] If $x = \frac{1}{2}$, $y = \frac{1}{3}$, **find the value of the expression :** $(4x^2 - y)^2$

11 El-Dakahlia Governorate

Dakahlia Directorate of Education
Math supervision

Answer the following questions :

1 Choose the correct answer from those given :

1 The additive inverse of $(-1)^{10}$ is ....

(a) 0 (b) -1 (c) -10 (d) 1

2 Half milliard $= 5 \times 10^{\cdots\cdots}$

(a) 6 (b) -9 (c) 8 (d) 9

3 Quarter of the number 2^{16} is ....

(a) 1^{16} (b) 4^7 (c) 4^{15} (d) 2^4

4 $\sqrt{144 + \cdots\cdots} = 12 + 5$

(a) 40 (b) 25 (c) 16 (d) 145

5 If the probability of success of a student is 0.7
, then the probability of his failure is ....

(a) 0.03 (b) 1 (c) 30 % (d) 3

6 If $8y = 16$, then $y + 5 =$ ....

(a) 10 (b) 13 (c) 21 (d) 7

2 Complete each of the following :

1 The smallest odd prime number is ....

2 If the area of a square is $169\ k^2\ cm^2$. , then its side length = ... cm.

3 If $y = \frac{1}{4}$, $x = \frac{1}{3}$, then $(x - y)^{-2} =$ ....

4 .... is a subset of the sample space.

5 The S.S. of the inequality $5 \le x \le 6$ in $\mathbb{N}$ is ....

3 [a] **Divide :** $(25\,a\,b + 5\,a) \div 5\,a$, then find the numerical value of the expression when $a = 2$, $b = -1$

[b] **Simplify to the simplest form :** $\left(-\frac{49}{25}\right)^0 \times \left(\frac{-2}{7}\right)^2 \times \sqrt{12\frac{1}{4}}$

4 [a] **Find in $\mathbb{Q}$ the S.S. of the following :**

1 $2x + 7 < 15$

2 $6x + 6 = 6$

[b] **Simplify to the simplest form :** $\frac{(3)^{-6} \times (3)^{11}}{(3)^3}$

5 [a] **Find in $\mathbb{N}$ the S.S. of the inequality :** $5x - 2 \ge 3$, then represent it on the number line.

[b] A ball chosen randomly from a bag contains 5 red balls , 7 blue balls , 3 yellow balls , **find the probability of each of the following :**

1 getting black ball.

2 getting a red ball.

12 El-Ismailia Governorate

Directorate of Education
Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 If $x = y$, then $\left(\frac{3}{4}\right)^{x-y} =$ ....

(a) 0 (b) 1 (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

2 If $4790000 = a \times 10^6$, then a = ...

(a) 479 (b) 47.9 (c) 4.79 (d) 470

3 If $\frac{x - 3}{x + 4} = 0$, then $x =$ ....

(a) 3 (b) -3 (c) 4 (d) -4

4 The ratio between two numbers is 1 : 2 , if the first is 100 , then the second is ....

(a) 50 (b) 25 (c) 10 (d) 200

5 $7x^2 y^{-3} =$ ....

(a) $\frac{7}{x^2 y^3}$ (b) $\frac{7x^2}{y^3}$ (c) $\frac{x^2 y^3}{7}$ (d) $\frac{x^2}{7y^3}$

6 When tossing a die once , the probability of getting an odd number = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{5}{6}$

2 Complete :

1 If $2x - 1 = 5$, then $10x =$ ....

2 $5y^2 \times 3y^2 =$ ....

3 $\sqrt{\frac{25x^4}{y^4}} =$ .... where $y \neq 0$

4 If $3^{10} + 3^{10} + 3^{10} = 3^x$, then $x =$ ....

5 ✻ $4 + 4 \times 4 \div 4 - 2^2 =$ ....

3 [a] Find in the simplest from : $\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

[b] Simplify : $\frac{a^7 \times a^5}{a^4 \times a^6}$, then find the value at $a = -3$

4 [a] Find the S.S. in $\mathbb{Q}$ of : $3x + 4 < 25$

[b] If $x = \frac{1}{2}$, $y = -\frac{2}{3}$, $z = \frac{3}{4}$

Find the value of : $x^2 y^2 \div z$

5 [a] One card is selected randomly from 8 cards numbered from 1 to 8 ,
find the probability of getting :

1 an even number 2 a prime number 3 a number more than 7

[b] Find the S.S. in $\mathbb{Q}$ of : $6x - 8 = 22$

13 Damietta Governorate

Damietta inspection of Mathematic official language schools

Answer the following questions :

1 Choose the correct answer :

1 If $x + 9 = 11$, then the value of $7x =$ ....

(a) 9 (b) 14 (c) 2 (d) 13

2 $\left(\frac{-2}{3}\right)^{-3} =$ ....

(a) $\frac{-27}{8}$ (b) $\frac{-8}{27}$ (c) $\frac{8}{27}$ (d) $\frac{27}{8}$

3 $2^3 + 2^3 =$ ....

(a) 2^6 (b) 2^4 (c) 2^9 (d) 1

4 If $-x > 4$, then ....

(a) $x > -4$ (b) $x > 4$ (c) $x < -4$ (d) $x < 4$

5 $x^2 + x^2 =$ ....

(a) x^4 (b) x^2 (c) $2x^2$ (d) $2x^4$

6 Three times of a number is 48 , then $\frac{1}{4}$ the number is ....

(a) 16 (b) 4 (c) 12 (d) 8

2 Complete each the following :

1 The probability of the certain event equals ....

2 The number 0.000053 in the scientific notation = ...

3 The multiplicative inverse of $\sqrt{\frac{9}{25}}$ is ....

4 A class has 36 pupils , 20 of them are boys. If a pupil is chosen randomly , then the probability that the pupil is a girl = ...

5 1 , 2 , 3 , 5 , 8 , ... (in the same pattern).

3 [a] Simplify to the simplest form : $\left(\frac{-5}{3}\right)^2 \times \left(\frac{-4}{9}\right)^0 \times \sqrt{3\frac{6}{25}}$

[b] Find the solution set of each of the following where $x \in \mathbb{Q}$:

1 $3x + 1 > 25$ 2 $5x + 8 = 15 - 2x$

4 [a] Reduce : $\frac{x^7 \times x^9}{x^6 \times x^8}$ to the simplest form , then find the value of the result when : $x = -3$

[b] The sum of three consecutive even numbers is 60 , Find them.

5 [a] If $x = 3$ and $y = -4$, **find the value of :** $\sqrt{x^2 + y^2}$

[b] A box contains 4 white , 5 red and 6 blue balls , a ball is drawn randomly from the box. **Calculate the probability of getting :**

1 a blue ball. 2 a white or red ball. 3 a green ball.

14 El-Fayoum Governorate

Fayoum west Administration

Answer the following questions :

1 Choose the correct answer :

1 The multiplicative inverse of $\sqrt{\frac{100}{81}} =$ ....

(a) $\pm \frac{10}{9}$ (b) $\pm \frac{9}{10}$ (c) $\frac{10}{9}$ (d) $\frac{9}{10}$

2 The probability of the impossible event = ...

(a) zero (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

3 If a = b , then $\left(\frac{2}{3}\right)^{a-b}$ = ...

(a) zero (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

4 $3^{10} + 3^{10} + 3^{10}$ = ...

(a) 3^{30} (b) 3^{11} (c) 9^{10} (d) 9^{11}

5 If $3x + 1 = 16$, then the value of $5x$ = ...

(a) 10 (b) 15 (c) 25 (d) 26

6 5^{-1} = ...

(a) -5 (b) $\frac{1}{5}$ (c) 5 (d) 25

2 Complete the following :

1 If $2ab = 10$, then $a^2 b^2$ = ...

2 ✱ $3 \times 4 - 21 \div 3$ = ...

3 $x = \frac{1}{2}$, $y = \frac{3}{4}$, $y \div x$ = ...

4 $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, ... , ... (in the same pattern)

5 $(5 \times 10^2) \times (9 \times 10^3)$ = ... (in the standard form)

3 [a] Write the following in the simplest form :

1 $\frac{10^{-3} \times 10^6}{10^2}$ 2 $\frac{\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{-5}}{\frac{1}{2}}$

[b] Find in $\mathbb{Q}$ the solution set of the following :

1 $8 + 2x = 14$ 2 $3x - 1 = -10$

4 [a] Find in $\mathbb{Q}$ the solution set of each of the following :

1 $2(x - 3) = -x + 12$ 2 $5x - 1 = 29$

[b] Calculate the following when a = 2 , b = 5 : 1 $\frac{9-b}{a^3}$ 2 $\frac{6^2}{a+1}$

5 [a] Find in $\mathbb{Z}$ the solution set of the following :

1 $3x - 4 \geq -10$ 2 $x + 2 \geq 2$

[b] A box contains 15 cards numbered from 1 to 15 A card is drawn randomly

Find the probability of the drawn card carries :

1 an odd prime number. 2 a number less than or equals 1

3 a number more than 15 4 the number 15

15 Qena Governorate

Qena Directorate of Education
Math's supervision

Answer the following questions :

1 Choose the correct answer :

1. If $x + 5 = 11$, then $3x =$ ....
   (a) 16 (b) 18 (c) 1.8 (d) 6
2. The probability of the certain event = ...
   (a) 1 (b) 2 (c) 0 (d) 0.5
3. $\sqrt{13^2 - 5^2} =$ ....
   (a) 11 (b) 12 (c) 13 (d) 14
4. If the standard form of 0.00000058 is 5.8×10^n , then n = ...
   (a) 7 (b) − 7 (c) 5 (d) 8
5. $10^{-3} =$ ....
   (a) 10 (b) 1000 (c) 0.001 (d) 0.01
6. ❋ $3^2 \times 6 \div 3 + (2^4 - 6) =$ ....
   (a) 18 (b) 28 (c) 42 (d) 32

2 Complete :

1. If the age of Amir now is x years , then his age after 5 years is .... years.
2. $\sqrt{\left(\frac{-4}{9}\right)^2} =$ ....
3. The probability of the impossible event ....
4. $\left(\frac{4}{9}\right)^{-2} = \left(\frac{9}{4}\right)^n$, then n = ...
5. If half of $2^{40} = 2^n$, then n = ...

3 [a] **Find the S.S. in $\mathbb{Q}$ for :** $\frac{3}{5}x + 4 < 28$

[b] If $a = \frac{-2}{3}$, $b = \frac{3}{4}$ **find the value of :** $a^3 b^3$

4 [a] **Simplify :** $\left(\frac{4}{9}\right)^{-2} \times \left(\frac{4}{9}\right)^6$

[b] Three consecutive even numbers their sum is 156 Find the numbers

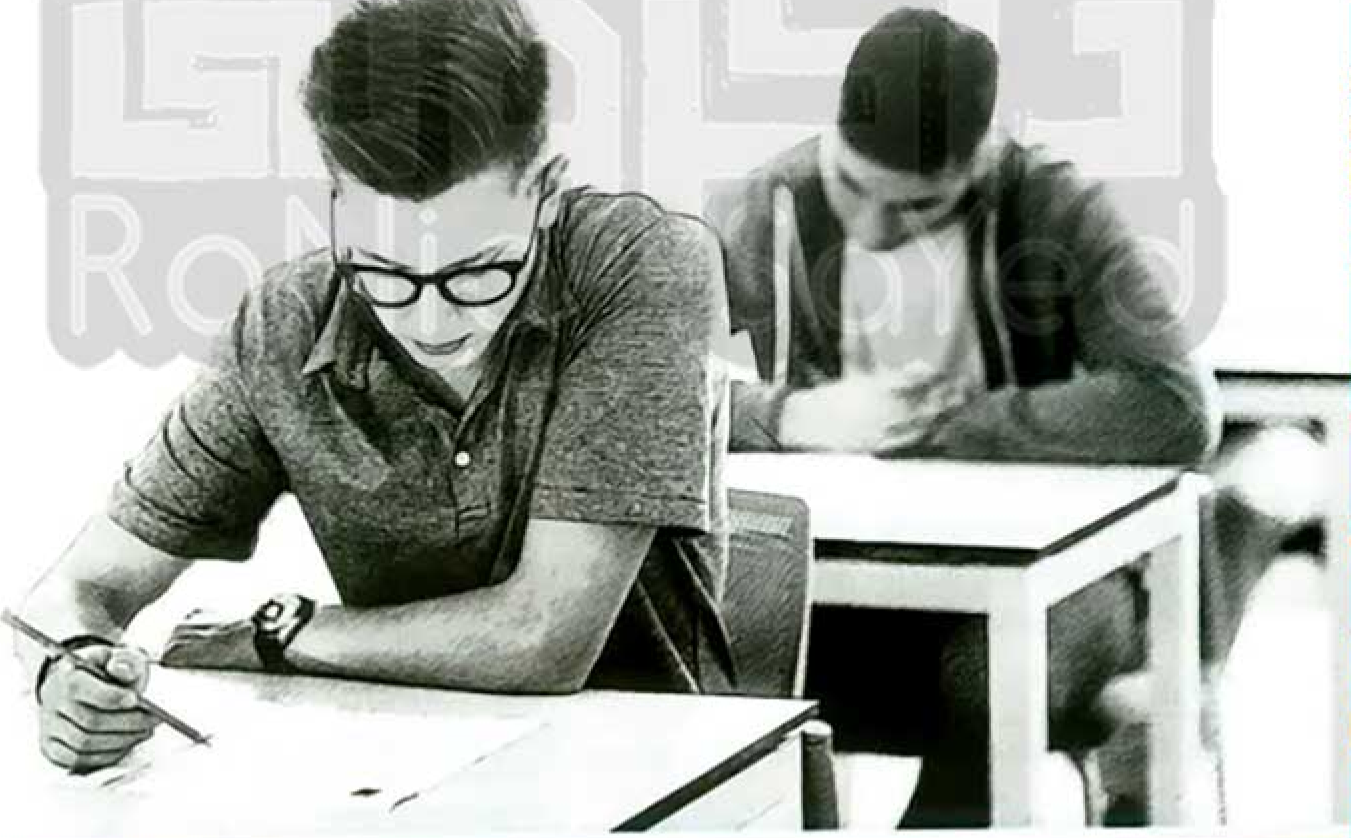
5 [a] If $3^x = 7$, $3^y = 5$ **Find :** 3^{x+y}

[b] A fair die is rolled once. **Calculate the probability of appearance of :**

1. an even number. 2. a number greater than 3 3. the number 5

Final Examinations

on Algebra and Statistics



Model Examinations of the School Book on Algebra and Statistics

Model 1

Answer the following questions :

1 **Complete :**

1. $\frac{81}{625} = \left(\frac{25}{9}\right)^{\dots\dots}$
2. If $7 - 2X = 3$, then $X =$ .... where $X \in \mathbb{N}$
3. $3^{-1} + 4^{-1} =$ ....
4. The standard form of the number $0.7 \times 0.005 =$ ....
5. The probability of the certain event = ...

2 **Choose the correct answer :**

1. The sum of the probabilities for all possible outcomes of a randomly experiment is ....

   (a) zero (b) 1 (c) > 1 (d) < 1

2. If $3a = \sqrt[3]{4}\,b$, then $\frac{a}{b} =$ ....

   (a) 2 : 3 (b) 3 : 2 (c) 3 : 4 (d) 4 : 3

3. $\left(\frac{-2}{3}\right)^{-3}$ equals ....

   (a) $\frac{-27}{8}$ (b) $\frac{-8}{27}$ (c) $\frac{8}{27}$ (d) $\frac{27}{8}$

4. There are 21 boys and 15 girls in a classroom , one pupil is chosen randomly , the probability that the chosen pupil is a girl = ...

   (a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{4}{7}$ (d) $\frac{5}{6}$

5. $\sqrt{(-8)^2 + (-6)^2} =$ ....

   (a) $|-10|$ (b) ± 10 (c) 14 (d) -14

6. 10 % of L.E. $2\frac{1}{2}$ = L.E. ....

   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 25

3 **[a] Simplify to the simplest form :** $\left(-\frac{3}{7}\right)^0 \times \left(\frac{-2}{5}\right)^2 \times \sqrt{6\frac{1}{4}}$

[b] Find the numerical value of the expression :

$3ab + 8a \div (4b)$ when $a = 4$, $b = -2$

4 [a] Find in $\mathbb{Q}$ the S.S. of : $3X + 1 = 25$

[b] Find the value of : $\frac{8 \times 8^{-3}}{8^{-4}}$

5 [a] A factory of a tire record the distance that traveled by a certain type of them before damage for 800 units of this type as following.

The distance in thousand (km.)	Less than 50	50 to 100	More than 100 till 150	More than 150
The number of damage tire	80	120	280	320

If you bought a tyre of this type , what is the probability of change it :

1 Before traveled 50 thousand km.

2 After traveled more than 100 thousand km.

[b] Find in $\mathbb{Q}$ the S.S. of : $2X + 5 < 16$

Model 2

Answer the following questions :

1 Complete :

1 $\left(\frac{-2}{3}\right)^0 =$ .... 2 $\sqrt{\frac{16}{49}} =$ ....

3 The probability of the impossible event = ...

4 1 , 2 , 3 , 5 , 8 , ... , ... (In the same pattern)

5 If the probability that the student is absent in a school is 0.15 , if the number of students of this school is 600 , then the number of the present students that day is ....

2 Choose the correct answer :

1 $2^3 \times 2^3 =$ ....

(a) 2^6 (b) 2^8 (c) 2^{15} (d) 2^{53}

2 Which of the following is the greatest ?

(a) 2.3×10^4 (b) 2.3×10^5 (c) 3.2×10^4 (d) 3.2×10^5

3 $(x^2)^{-3} \times x^6 =$ ....

(a) x^{12} (b) x^{-12} (c) x (d) 1

4 Which of the following may be probability of an event ?

(a) -0.35 (b) 87 % (c) 1.05 (d) 130 %

5 If $-x > 4$, then ....

(a) $x > -4$ (b) $x > 4$ (c) $x < -4$ (d) $x < 4$

6 Area of a rectangle of length 120 cm. and width 80 cm. equals .... m^2.

(a) 9600 (b) 400 (c) 9.6 (d) 0.96

3 [a] Two integers numbers , the smaller one is $2x$ and the greater is $5x$, if the difference between them is 30 Find the two numbers.

[b] Find the value of : $\frac{5^{-4} \times 5^7}{5^3}$ in the simplest form.

4 [a] Find in $\mathbb{Q}$ the S.S. of each of the following :

1 $(3x + 2) + 5 = 13$

2 $2x + 15 < 19$

[b] Find the value of the expression in the simplest form :

$$\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$$

5 [a] If a regular die is thrown once and observed the number on upper face ,

find the probability of each of the following :

1 Getting a prime even number.

2 Getting an odd number less than 4

[b] If $x = -\frac{1}{2}$, $y = -\frac{3}{4}$, find in the simplest form : $\left(\frac{y}{x^2}\right)^{-2}$

Model examination for the merge students

Answer the following questions :

1 Choose the correct answer :

1. $\left(\frac{-2}{3}\right)^2 =$ ....

   (a) $\frac{4}{9}$ (b) $\frac{-4}{9}$ (c) $\frac{4}{6}$ (d) $\frac{-4}{6}$

2. $\left(\frac{4}{7}\right)^0 =$ ....

   (a) 0 (b) 1 (c) $\frac{4}{7}$ (d) -1

3. $2 \times 6 - 4 \times 2 =$ ....

   (a) 4 (b) 8 (c) 10 (d) 2

4. $(7)^{-2} =$ ....

   (a) 49 (b) $\frac{1}{49}$ (c) 14 (d) -14

5. $\sqrt{9 + 16} =$ ....

   (a) 7 (b) 5 (c) 25 (d) -7

2 Complete each of the following :

1. If $X + 2 = 6$, then $X =$ ....
2. When tossing a coin once , then the probability of the appearance of a tail = ...
3. The probability of the impossible event = ...
4. $\sqrt{\left(\frac{2}{5}\right)^2} =$ ....
5. $7 (6^2 - 5 \times 6) =$ ....

3 Complete the solution to find the result :

1. $12 \times 2^2 \div 24 + 3^2 = 12 \times$ .... $\div 24 +$ ....

   $=$ .... $\div 24 +$ .... $=$ .... $+$ .... $=$ ....

2. $\frac{8 + 20 - 4}{8 - 4} = \frac{\text{.............} - 4}{\text{.............}} = \frac{\text{.............}}{\text{.............}} =$ ....

4 Put (✓) or (✗) :

1. If $2x + 3 = 7$, then $x = 2$ ( )
2. $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^5 = \left(\frac{2}{3}\right)^6$ ( )
3. $(x^2)^3 = x^6$ ( )
4. $\left(\frac{3}{2}\right)^2 = -\frac{9}{4}$ ( )
5. $\sqrt{100 - 64} = 2$ ( )

5 A card is drawn randomly from 8 cards numbered from 1 to 8

, join from column (A) to column (B) :

Column (A)	Column (B)
1 The event of getting an even number equals ……………	• $\frac{1}{2}$
2 The probability of getting an even number equals ……………	• $\{8, 6, 4, 2\}$
3 The event of getting a number > 6 equals ……………	• 1
4 The probability of getting a number < 9 equals ……………	• $\frac{1}{8}$
5 The probability of getting a number 8 equals ……………	• $\{8, 7\}$

Schools Examinations on Algebra and Statistics

1 Cairo Governorate

Western Cairo Educational Zone
Mathematics Inspection

Answer the following questions :

1 Choose the correct answer from the given :

1. $\sqrt{9} =$ ....
   (a) 3 (b) -3
   (c) ± 3 (d) 81
2. If the probability of success of a student is 0.8 , then the probability of his failure is ....
   (a) 1 (b) -1 (c) 0.2 (d) 0.8
3. $5^{-1} =$ ....
   (a) -5 (b) $\frac{1}{5}$ (c) $-\frac{1}{5}$ (d) -15
4. If $x > y$, then $x + z$ .... $y + z$
   (a) $<$ (b) $>$ (c) $=$ (d) $\le$
5. $\frac{4}{10} + \frac{3}{100} =$ ....
   (a) 0.34 (b) 0.43 (c) 4.3 (d) 3.4
6. $6000 \times 50 =$ ....
   (a) 300×10^2 (b) 3×10^5 (c) 30×10^5 (d) -3×10^3

2 Complete each of the following :

1. The probability of the impossible event = ...
2. $(3^2)^{-1} =$ ....
3. Twice the number $\frac{1}{2} =$ ....
4. Quarter of the number 4^{20} is $4^{\cdots\cdots}$
5. $3x + x + 2y + y$ in the simplest form is ....

3 [a] **Find in $\mathbb{Q}$ the S.S. of the equation : $2x + 1 = 9$**

[b] **Simplify to the simplest form : $\frac{5^4 \times 5^{-2}}{5^2}$**

[c] If $x = 6$, $y = 3$, **then find the value of : $\left(\frac{x}{y}\right)^{-2}$**

4 **[a] Find the S.S. of the following inequality in $\mathbb{Q}$: $4x - 3 < 7$**

[b] Find the value of : $12 \times 2 \div 24 + 9$

5 **[a] Find the value of the expression in the simplest form : $\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$**

[b] A box contains 4 white , 5 red and 6 blue balls. A ball is drawn randomly from the box.

Calculate the probabilities of the following events :

1 The ball is red. 2 The ball is blue or white. 3 The ball is not blue.

2 Cairo Governorate

El-Zeiton Zone
Talaea Gaber El Antary Language School

Answer the following questions :

1 **Choose the correct answer :**

1 If $4x = 20$, then $3x - 1 =$ ....

(a) 13 (b) 14 (c) 15 (d) 16

2 $3^{10} + 3^{10} + 3^{10} =$ ....

(a) 9^{30} (b) 3^{30} (c) 3^{10} (d) 3^{11}

3 If $\frac{4x-1}{2x+3} = 0$ where $x \in \mathbb{Q}$, then $x =$ ....

(a) $-\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) 0

4 A class contains 40 students , 36 of them are succeed in a test , then the probability of failed is ....

(a) 0.1 (b) 0.9 (c) 0.3 (d) $\frac{2}{3}$

5 $\sqrt{(8)^2 + (6)^2} = 8 +$ ....

(a) 14 (b) 10 (c) 2 (d) 6

6 $\frac{|-5|+1}{3}$ .... $\mathbb{Z}$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

2 **Complete the following :**

1 If $0.0000016 = 1.6 \times 10^{n}$, then n = ...

2 The multiplicative invers of $\sqrt{5\frac{4}{9}} =$ ....

3 When a fair die is tossed once , then the probability of getting an even prime number = ...

4 If $\frac{3}{4}x = 75$, then $\sqrt{x} =$ ....

5 25 % from .... = 8

3 [a] **Find in the simplest form :** $\left(\frac{5^5 \times 5^{-2}}{5^4}\right)^{-2}$

[b] **Find the S.S. in** $\mathbb{Q}$ **:** $-2 < 4x + 1 < 6$

4 [a] **Simplify :** $\left(\frac{-2}{7}\right)^{-2} \times \sqrt{\frac{16}{49}} \times \left(\frac{-1}{7}\right)^0$

[b] If the length of a rectangle is 5 cm. more than its width and its perimeter is 42 cm. Find the area of this rectangle.

5 [a] **Simplify :** $\frac{(4x^3 y^2)^2}{(2xy^2)^3}$ where $x \neq 0$, $y \neq 0$, then find the value when $x = \frac{1}{2}$, $y = 3$

[b] A team plays 30 matches in national league its drawn probability is 0.2 and its win probability is 0.7 Calculate the number of loss matches.

3 Cairo Governorate

El Maadi Directorate
El Orman Smart School

Answer the following questions :

1 Choose the correct answer :

1 If $0 \in \{5 , x - 3\}$, then $x =$ ....

(a) 0 (b) -5 (c) 3 (d) -3

2 $3^4 \times 3^3 =$ ....

(a) 3^{12} (b) 3^7 (c) 3 (d) 3^{-1}

3 The S.S. of the inequality : $x < 3$ in $\mathbb{N}$ is ....

(a) $\{0\}$ (b) $\{0 , 1 , 2\}$ (c) $\{1 , 2\}$ (d) $\varnothing$

4 $\frac{6a^3 x^4}{3a^2 x^3} =$ .... where $a \neq 0$, $x \neq 0$

(a) $2ax$ (b) $2a^2 x^7$ (c) $\frac{3x}{a}$ (d) $\frac{3}{ax}$

5 $\left(\frac{-2}{3}\right)^{-3} =$ ....

(a) $\frac{27}{8}$ (b) $\frac{-8}{27}$ (c) $\frac{8}{27}$ (d) $\frac{-27}{8}$

6 If the probability of success of a student is 0.6 , then the probability of his failure is ....

(a) 1 (b) $\frac{1}{10}$ (c) $\frac{4}{10}$ (d) $\frac{6}{10}$

2 Complete the following :

1 The multiplicative inverse of 7 is ....

2 0.00025 in scientific notation = ...

3 $\left(\frac{3}{4}\right)^2 \div \left(\frac{3}{4}\right)^3 =$ ....

4 $\sqrt{16+9} = 4 +$ ....

5 A class has 36 pupils , 25 of them are boys , if a pupil is chosen randomly , then the probability that the pupil is a girl = ...

3 [a] **Find the simplest form of :** $\left(\frac{7^{-2} \times 7^5}{7^3}\right)^2$

[b] **Simplify and find the value of :** $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{2}{5}\right)^0$

4 [a] 1 **Calculate :** $\sqrt{100-(-8)^2}$

2 If $x = \frac{1}{2}$, $y = \frac{2}{3}$ **, then find the value of :** $(x^2 y^2)^{-3}$

[b] **Find in $\mathbb{Q}$ the S.S. of the inequality :** $3x + 6 > 3$

5 [a] **Find the solution set in $\mathbb{Q}$:** $4x - 5 = 27$

[b] A fair die is rolled once. **Calculate the probability of appearance :**

1 an even number. 2 a number greater than 4

4 Giza Governorate

Mathe Inspection

Answer the following questions :

1 **Choose the correct answer from those given :**

1 $\left(\frac{2}{3}\right)^2 \times \frac{2}{3} =$ ....

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$ (c) $\frac{-4}{9}$ (d) $\frac{8}{27}$

2 $5^{-1} =$ ....

(a) 5 (b) -5 (c) $\frac{-1}{5}$ (d) $\frac{1}{5}$

3 $\sqrt{\frac{4}{9}} =$ ....

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{2}{9}$ (d) $\frac{4}{3}$

4 Quarter of 16 = ...

(a) 1 (b) 4 (c) 8 (d) 16

5 The age of Omar is x year , then his age 5 years ago is ....

(a) $5x$ (b) $5 + x$ (c) $5 - x$ (d) $x - 5$

6 A letter is selected at random from the name (ZAMALEK) the probability of selecting the letter A is ....

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

2 Complete the following :

1 $\left(\frac{3}{4}\right)^4 \div \left(\frac{3}{4}\right)^3 =$ ....

2 $0.000735 =$ .... $\times$ .... (In standard form)

3 If $2x - 7 = 3$, then $x =$ ....

4 If a = b , then $\left(\frac{4}{5}\right)^{a-b} =$ ....

5 The probability of any event not less than .... and not more than ....

3 [a] Calculate the following :

1 $6 \times (2)^2 \div 24 + 3^2$ 2 $\sqrt{100 - (-6)^2}$

[b] Find the result in the simplest form : 1 $\frac{7^{-2} \times 7^5}{7^3}$ 2 $\left(\left(\frac{1}{3}\right)^2\right)^2$

4 [a] Find in $\mathbb{Q}$ the S.S. of the following :

1 $x + 4 = 14$ 2 $3x + 1 = 25$

[b] If $x = \frac{1}{2}$, $y = \frac{2}{3}$, $z = \frac{-3}{2}$, then find the value of : $(x y z)^2$

5 [a] Find in $\mathbb{Q}$ the S.S. of : $2x + 5 < 15$

[b] A box contains 3 white , 5 red and 7 blue balls , a ball is drawn randomly from the box.
Calculate the probabilities of the following :

1 The ball is red. 2 The ball is white or blue.

5 Giza Governorate

Dokki Zone
Talaee Islamic School

Answer the following questions :

1 Choose the correct answer from those given :

1 $3^3 \times 3^4 =$ ....

(a) 3^{12} (b) 3 (c) 3^7 (d) 3^{-1}

2 $3x^{-1} =$ .... where $x \neq 0$

(a) $-3x$ (b) $\frac{3}{x}$ (c) $3x$ (d) $\frac{1}{3x}$

3 The multiplicative inverse of the number $\sqrt{\frac{4}{9}}$ = ...

(a) $\frac{-3}{2}$ (b) $\frac{2}{3}$ (c) $-\frac{2}{3}$ (d) $\frac{3}{2}$

4 If $2a = \sqrt{9}\,b$, then $\frac{a}{b}$ = ...

(a) $\frac{3}{2}$ (b) $-\frac{2}{3}$ (c) $-\frac{3}{2}$ (d) $\frac{2}{3}$

5 If $-x < 2$, then ....

(a) $x > -2$ (b) $x > 2$ (c) $x < -2$ (d) $x < 2$

2 Complete each of the following :

1 $\sqrt{16+9} = 4 +$ ....

2 The additive inverse of $\left(-\frac{2}{3}\right)^2$ is ....

3 $3^5 + 3^5 + 3^5 = 3^{\cdots}$

4 $\frac{6a^2x^4}{2a^3x^3}$ = ... where $a \neq 0$, $x \neq 0$

5 $(2x)^2 \times \frac{1}{x}$ = ... where $x \neq 0$

6 $\left(2\sqrt{3}\right)^2$ = ...

3 Find in Q the solution set of the following :

1 $8x + 4 = 12$ 2 $3x - 1 \leq 2x + 3$

4 [a] If the length of a rectangle exceeds its width by 4 metres and its perimeter is 108 metres. Find the dimentions of the rectangle.

[b] **Simplify :** $\frac{a^4 \times a^{-7}}{a^{-6}}$, $a \neq 0$

5 [a] **Write the following number in the standard form :** 0.75×10^8

[b] If $x = -\frac{1}{2}$, $y = \frac{3}{4}$, $z = -\frac{3}{2}$, **find the numerical value of :** $x^3 \div y z^2$

6 Giza Governorate

Omrania Directorate
El Sadat E.L.S.

Answer the following questions :

1 Choose the correct answer :

1 $x^{12} \div x^4$ = ... where $x \neq 0$

(a) x^8 (b) x^3 (c) x^{16} (d) x^{-8}

2 $\pm\sqrt{\frac{4}{9}} =$ ....

(a) $\frac{-4}{9}$ (b) $\frac{-2}{3}$ (c) $\pm\frac{2}{3}$ (d) $\frac{2}{3}$

3 If the probability that pupils success is 75 % , then the probability of his failure is ....

(a) -0.75 (b) 0.25 (c) -0.25 (d) 0.75

4 If $-x < 7$, then x .... -7

(a) $<$ (b) $>$ (c) $=$ (d) $\leq$

5 $6000 \times 50 =$ ....

(a) 300×10^2 (b) 30×10^5 (c) -3×10^3 (d) 3×10^5

6 If $3x = 6$, then $5x =$ ....

(a) $\frac{5}{2}$ (b) $\frac{2}{5}$ (c) 10 (d) 5

2 Complete each of the following :

1 The probability of impossible event is ....

2 The additive inverse of the number $\left(\frac{-1}{3}\right)^2$ is ....

3 $\sqrt{16+9} =$ ....

4 $\left(\frac{2}{5}\right)^{-2} =$ ....

5 $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, ... (in the same pattern)

3 [a] Find in the simplest from :

1 $\frac{9^{-2} \times 9^5}{9^3}$ 2 $\left(\frac{-3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0$

[b] Find in $\mathbb{Q}$ the solution set of the following equation : $4x + 1 = 21$

4 [a] If $x = \frac{1}{2}$, $y = \frac{2}{3}$, $z = \frac{-3}{2}$ **, then find the value of :** $(x\,y\,z)^2$

[b] Find in $\mathbb{Q}$ the solution set of the following inequality : $3x - 1 \leq 2x + 3$

5 [a] The sum of three consecutive numbers is 24 , find them.

[b] A fair die is rolled once **, calculate the probability of appearance :**

1 An even number. 2 A number greater than 5

7 Alexandria Governorate

West Educational Zone
Inspectorate of Mathematics

Answer the following questions :

1 Complete each of the following :

1. The multiplicative identity element in $\mathbb{Q}$ is ....
2. $\sqrt{\frac{25}{49}} =$ ....
3. $(x-5)^0 = 1$, if $x \neq$ ....
4. If the probability of success of a student is 0.8 , then the probability of his failure is ....
5. If $X \subset Y$, then $X \cap Y =$ ....

2 Choose the correct answer :

1. $\sqrt{64+36} = 8 +$ ....
   (a) 2 (b) 6 (c) 10 (d) -2
2. Half the number 2^{24} equals ....
   (a) 2^{12} (b) 1^{23} (c) 2^{23} (d) 1^{12}
3. $0.0000068 =$ ....
   (a) 6.8×10^{-6} (b) 6.8×10^{5} (c) 6.8×10^{-7} (d) 6.8×10^{7}
4. If $2x = -12$, then $x^2 =$ ....
   (a) 6 (b) 144 (c) -36 (d) 36
5. The S.S. of the equation : $5 - x = 3$ in $\mathbb{Q}$ is ....
   (a) $\{2\}$ (b) $\{-2\}$ (c) $\{7\}$ (d) $\varnothing$
6. The probability of a certain event equals ....
   (a) zero (b) 1 (c) -1 (d) $\frac{1}{2}$

3 **[a] Simplify to the simplest form :** $\frac{x^3 \times x^{-2}}{x^{-5} \times x}$, then find the value when $x = -2$

[b] Find the S.S. in $\mathbb{Q}$ for each of the following :

1. $3x - 5 > 1$
2. $3x + 6 = 30 - 5x$

4 **[a]** A fair die is rolled once **, what is the probability of getting :**

1. an even number ?
2. a factor of 6 ?

[b] Find the value of : $\sqrt{6\frac{1}{4}} \times \left(\frac{2}{7}\right)^0 \times \left(\frac{-2}{5}\right)^2$

5 [a] A rectangle whose length is more than its width by 3 cm. and its perimeter equals 26 cm. , find its area.

[b] If $x = \frac{1}{2}$ and $y = \frac{4}{3}$ find the value of : $x^3 y^2$

8 Alexandria Governorate

Mid Educational Zone
Math Inspection

Answer the following questions :

1 **Choose the correct answer :**

1. $x^9 \div x^3 = \dots\dots$, $x \neq 0$
   (a) x^{12} (b) x^3 (c) x^6 (d) x^{-3}
2. If $0.0035 = 3.5 \times 10^n$, then n = ……
   (a) 2 (b) -3 (c) -2 (d) 3
3. $\left(\frac{4}{7}\right)^0 = \dots\dots$
   (a) 1 (b) $\frac{4}{7}$ (c) 0 (d) -1
4. $5^2 + 5^2 = \dots\dots$
   (a) 10^4 (b) 50 (c) 5^4 (d) 10^2
5. If $-x > 4$, then ……
   (a) $x > -4$ (b) $x < 4$ (c) $x > 4$ (d) $x < -4$
6. The sum of all probabilities of all possible events of a random experiment ……
   (a) $= 0$ (b) $= 1$ (c) > 1 (d) < 1

2 **Complete each of the following :**

1. The probability of the certain event = ……
2. 1 , 2 , 3 , 5 , 8 , …… (in the same pattern)
3. $(a^2)^4 = \dots\dots$
4. $\sqrt{9 + 16} = \dots\dots$
5. $2 \times 6 - 4 \div 2 = \dots\dots$

3 **Find in the simplest form :**

1. $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$
2. $\frac{7^{-2} \times 7^7}{7^3}$

4 [a] Find in $\mathbb{Q}$ the solution set of : $3x + 1 = 25$

[b] If $x = \frac{3}{2}$, $y = \frac{4}{3}$, find the value of : $x^2 y^2$

5 **[a] Find in $\mathbb{Q}$ the solution set of :** $2X + 15 < 19$

[b] A bag contains 5 red , 3 yellow and 2 black balls. A ball is drawn randomly from the box.

Calculate the probability of getting :

1 A yellow ball. **2** A green ball.

9 El-Kalyoubia Governorate

Directorate of Education
Math Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The number which is not in the standard form is ....

(a) 6.2×10^5 (b) 7.834×10^{16} (c) 0.8×10^5 (d) 6.7×10^{25}

2 If 3 t = 6 , then 6 t = ...

(a) 16 (b) 14 (c) 12 (d) 10

3 $\frac{1}{4} \times 4^{20} =$ ....

(a) 4^{15} (b) 4^{19} (c) 2^{19} (d) 2^{39}

4 $\frac{6a^2X^4}{2a^3X^3} =$ .... where $a \neq 0$, $X \neq 0$

(a) $3aX$ (b) $3a^5X^7$ (c) $\frac{3X}{a}$ (d) $\frac{3}{aX}$

5 A class formed from 36 students , 16 of them are girls. If a student selected randomly from the class , then the probability that the student is a boy = ...

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{1}{2}$ (d) $\frac{1}{36}$

6 10 % of $2\frac{1}{2}$ L.E. = ... L.E.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) 25

2 **Complete :**

1 The additive inverse of $\left(\frac{-2}{3}\right)^4$ is ....

2 When a die is tossing twice and observed the upper face in each time the probability of appearance number 5 on the two faces is ....

3 $\sqrt{10^2 - 6^2} =$ ....

4 If $-X + 2 > 6$ and the substitution set = $\{-2 , -5 , -1\}$, then $X =$ ....

5 If $3a = \sqrt{4}\,b$, then $\frac{a}{b} =$ ....

3 [a] Find in the simplest form : $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{a}{b}\right)^0$ where $a \neq 0$, $b \neq 0$

[b] If $x = \frac{-1}{2}$, $y = \frac{-3}{4}$, find the value of : $\left(\frac{y}{x}\right)^{-2}$

4 Find in $\mathbb{Q}$ the S.S. of :

1. $3x + 1 = 25$
2. $15 + 2x < 1$

5 [a] Find with steps the value of : $\left(2 \times \sqrt{36} - 2^4\right) \div 4$

[b] If a regular die is thrown once and observing the number on the upper face
Find the probability of getting :

1. a prime even number.
2. an odd number less than 4

10 El-Kalyoubia Governorate

Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. The side length of a square whose area $9x^2$ is ....
   (a) $3x$ (b) $3x^2$ (c) $9x$ (d) $9x^2$
2. $\left(\frac{-2}{3}\right)^{-3} =$ ....
   (a) $\frac{-27}{8}$ (b) $\frac{-8}{27}$ (c) $\frac{8}{27}$ (d) $\frac{27}{8}$
3. $2^3 \times 2^3 =$ ....
   (a) 1 (b) 2^9 (c) 2^4 (d) 2^6
4. $\sqrt{9 + 16} =$ ....
   (a) 7 (b) 5 (c) 25 (d) -7
5. If $-x > 4$, then ....
   (a) $x > -4$ (b) $x > 4$ (c) $x < 4$ (d) $x < -4$
6. There are 21 boys and 15 girls in a classroom , one pupil is chosen randomly , then the probability that the chosen pupil is a girl = ...
   (a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{4}{7}$ (d) $\frac{5}{6}$

2 **Complete :**

1. $7\,(6^2 - 5 \times 6) =$ ....
2. 1 , 2 , 3 , 5 , 8 , ... , ... (in the same pattern).

3 The additive inverse of $\left(\frac{-2}{5}\right)^2$ is ……………

4 $\left(\frac{-2}{5}\right)^{zero}$ = ……………

5 The probability of the certain event = ……………

3 **[a] Simplify to the simplest form : $\left(\frac{-3}{7}\right)^0 \times \left(\frac{-2}{5}\right)^2 \times \sqrt{6\frac{1}{4}}$**

[b] Write the following number in the standard form 720×10^6

4 **[a] Find in $\mathbb{Q}$ the S.S. of : $2X + 15 < 19$**

[b] Simplify to the simplest form : $\frac{5^{-4} \times 5^7}{5^3}$

5 **[a] Find in $\mathbb{Q}$ the S.S. of : $3X + 1 = 25$**

[b] The set $\{2, 3, 5\}$ is used in writing a 2-digit number.

Find the probability of the following events :

1 The tens digit is odd. 2 The units digit is even.

11 El-Monofia Governorate

Shiben Elkom Educational Zone
Language Formal Schools

Answer the following questions :

1 **Choose the correct answer :**

1 The S.S. of the inequality : $X < 2$ in $\mathbb{N}$ is ……………

(a) $\{0\}$ (b) $\{1\}$ (c) $\{0, 1\}$ (d) $\varnothing$

2 $3^{10} + 3^{10} + 3^{10}$ = ……………

(a) 3^{10} (b) 3^{30} (c) 9^{10} (d) 3^{11}

3 $\sqrt{(-8)^2 + (-6)^2}$ = ……………

(a) $|-10|$ (b) ± 10 (c) 14 (d) -14

4 $\frac{6a^2X^4}{2a^3X^3}$ = …………… where $aX \neq 0$

(a) $3aX^2$ (b) $3a^5X^7$ (c) $\frac{3X}{a}$ (d) $\frac{3}{aX}$

5 Which of the following is the greatest ?

(a) 2.3×10^4 (b) 2.3×10^5 (c) 3.2×10^4 (d) 3.2×10^5

6 $2 \times 6 - 4 \div 2$ = ……………

(a) 10 (b) 2 (c) 12 (d) 6

2 **Complete the following :**

1. A class has 36 pupils , 20 of them are boys. If a pupil is chosen randomly , then the probability that the pupil is a girl = ...
2. If $7 - 2x = 3$, then $3x =$ ....
3. $2\frac{1}{4} = \left(\frac{-3}{2}\right)^{\cdots\cdots}$
4. $\sqrt{100 - 64} = 10 -$ ....
5. The standard form of the number $0.7 \times 0.005 =$ ....

3 **[a] Find in $\mathbb{Q}$ the S.S. of the inequality :** $3x + 5 > 2$

[b] Find in $\mathbb{Q}$ the S.S. of the equation : $3x - 4 = 2x + 5$

4 **[a] Find the value of :** $12 \times (2)^2 \div 24 + 3^2$

[b] Find in the simplest form : $\left(\frac{7^4 \times 7^{-2}}{7^3}\right)^{-2}$

5 **[a] Simplify to the simplest form :** $\left(\frac{-5}{3}\right)^2 \times \left(\frac{-4}{9}\right)^0 \times \sqrt{3\frac{6}{25}}$

[b] The set $\{2 , 3 , 5\}$ is used in writing a 2-digit number.

Find the probability of each of the following events :

1. The sum of the two digits is 7
2. Both digits are equal.

12 El-Dakahlia Governorate

Dakahlia Directorate of Education
Math supervision

Answer the following questions :

1 **Choose the correct answer from those given :**

1. The additive inverse of $(-1)^{10}$ is ....
   (a) 0 (b) -1 (c) -10 (d) 1
2. Half milliard $= 5 \times 10^{\cdots\cdots}$
   (a) 6 (b) -9 (c) 8 (d) 9
3. Quarter of the number 2^{16} is ....
   (a) 1^{16} (b) 4^7 (c) 4^{15} (d) 2^4
4. $\sqrt{144 + \cdots} = 12 + 5$
   (a) 40 (b) 25 (c) 16 (d) 145

5 If the probability of success of a student is 0.7 , then the probability of his failure is ....

(a) 0.03 (b) 1 (c) 30 % (d) 3

6 If $8\ y = 16$, then $y + 5 =$ ....

(a) 10 (b) 13 (c) 21 (d) 7

2 Complete each of the following :

1 The smallest odd prime number is ....

2 If the area of a square is $169\ k^2\ cm^2$, then its side length = ... cm.

3 If $y = \frac{1}{4}$, $X = \frac{1}{3}$, then $(X - y)^{-2} =$ ....

4 .... is a subset of the sample space.

5 The S.S. of the inequality $5 \le X \le 6$ in $\mathbb{N}$ is ....

3 [a] **Divide :** $(25\ a\ b + 5\ a) \div 5\ a$ where $a \ne 0$, then find the numerical value of the expression when $a = 2$, $b = -1$

[b] **Simplify to the simplest form :** $\left(-\frac{49}{25}\right)^0 \times \left(\frac{-2}{7}\right)^2 \times \sqrt{12\frac{1}{4}}$

4 [a] **Find in $\mathbb{Q}$ the S.S. of the following :**

1 $2X + 7 < 15$ 2 $6X + 6 = 6$

[b] **Simplify to the simplest form :** $\frac{(3)^{-6} \times (3)^{11}}{(3)^3}$

5 [a] **Find in $\mathbb{N}$ the S.S. of the inequality :** $5X - 2 \ge 3$, then represent it on the number line.

[b] A ball chosen randomly from a bag contains 5 red balls , 7 blue balls , 3 yellow balls , **find the probability of each of the following :**

1 getting black ball. 2 getting a red ball.

13 Port Said Governorate

Educational Directorate
Math Inspection

Answer the following questions :

1 Choose the correct answer :

1 $6 \times 2 - 4 \div 2 =$ ....

(a) 1 (b) 2 (c) 10 (d) 12

2 $|-3|+|5|=$ ....

(a) -8 (b) -2 (c) 2 (d) 8

3 There are 21 boys and 15 girls in a classroom , one pupil is chosen randomly , the probability that the chosen pupil is a girl = ...

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{4}{7}$ (d) $\frac{5}{6}$

4 The S.S. of the inequality : $x<0$ in $\mathbb{N}$ is ....

(a) $\{0\}$ (b) $\{1\}$ (c) $\{0 , 1\}$ (d) $\varnothing$

5 Quarter of 4^{20} = ...

(a) 4^5 (b) 4^{10} (c) 4^{19} (d) 2^{10}

6 $\frac{6\,a^2\,x^4}{2\,a^3\,x^3}$ = ... where $a \neq 0$, $x \neq 0$

(a) $3\,a\,x$ (b) $3\,a^5\,x^7$ (c) $\frac{3\,x}{a}$ (d) $\frac{3}{a\,x}$

2 Complete :

1 $\sqrt{16+9}=2+$ ....

2 $10^{-3}=\frac{1}{\text{..............}}$

3 The probability of the impossible event = ...

4 1 , 8 , 27 , ... (in the same pattern).

5 The multiplicative inverse of 7 is ....

3 [a] Solve : $x+2=8$ in $\mathbb{Z}$

[b] Evaluate : $\frac{7^{-2}\times 7^5}{7^3}$

4 [a] Find the S.S. in $\mathbb{Q}$: $3\,x-1 \leq 2\,x+3$

[b] Find the value of the expression in the simplest form : $\left(\frac{-1}{3}\right)^2+\sqrt{\frac{64}{81}}-\left(\frac{3}{7}\right)^0$

5 [a] A fair die is rolled once , calculate the probability of getting :

1 An even number.

2 A prime number.

3 A number greater than 3

[b] If $x=3$ find the numerical value of the expression : $2\left(\frac{5\,x+3}{4\,x-3}\right)$

14 Damietta Governorate

Damietta inspection of Mathematic official language schools

Answer the following questions :

1 Choose the correct answer :

1. If $x + 9 = 11$, then the value of $7x =$ ....

   (a) 9 (b) 14 (c) 2 (d) 13

2. $\left(\frac{-2}{3}\right)^{-3} =$ ....

   (a) $\frac{-27}{8}$ (b) $\frac{-8}{27}$ (c) $\frac{8}{27}$ (d) $\frac{27}{8}$

3. $2^3 + 2^3 =$ ....

   (a) 2^6 (b) 2^4 (c) 2^9 (d) 1

4. If $-x > 4$, then ....

   (a) $x > -4$ (b) $x > 4$ (c) $x < -4$ (d) $x < 4$

5. $x^2 + x^2 =$ ....

   (a) x^4 (b) x^2 (c) $2x^2$ (d) $2x^4$

6. Three times of a number is 48 , then $\frac{1}{4}$ the number is ....

   (a) 16 (b) 4 (c) 12 (d) 8

2 Complete each the following :

1. The probability of the certain event equals ....
2. The number 0.000053 in the scientific notation = ...
3. The multiplicative inverse of $\sqrt{\frac{9}{25}}$ is ....
4. A class has 36 pupils , 20 of them are boys. If a pupil is chosen randomly , then the probability that the pupil is a girl = ...
5. 1 , 2 , 3 , 5 , 8 , ... (in the same pattern).

3 [a] **Simplify to the simplest form :** $\left(\frac{-5}{3}\right)^2 \times \left(\frac{-4}{9}\right)^0 \times \sqrt{3\frac{6}{25}}$

[b] **Find the solution set of each of the following where $x \in \mathbb{Q}$:**

1. $3x + 1 > 25$
2. $5x + 8 = 15 - 2x$

4 [a] **Reduce :** $\frac{x^7 \times x^9}{x^6 \times x^8}$ to the simplest form , then find the value of the result when : $x = -3$

[b] The sum of three consecutive even numbers is 60 , find them.

5 [a] If $x = 3$ and $y = -4$, **find the value of :** $\sqrt{x^2 + y^2}$

[b] A box contains 4 white , 5 red and 6 blue balls , a ball is drawn randomly from the box. **Calculate the probability of getting :**

1 a blue ball. 2 a white or red ball. 3 a green ball.

15 El-Fayoum Governorate

Directorate of Education

Answer the following questions : **(Calculator is allowed)**

1 Choose the correct answer :

1 The multiplicative inverse of $\sqrt{\frac{100}{36}}$ = ...

(a) $\pm \frac{10}{6}$ (b) $\pm \frac{6}{10}$ (c) $\frac{10}{6}$ (d) $\frac{6}{10}$

2 The probability of the certain event = ...

(a) zero (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

3 If a = b , then : $\left(\frac{2}{3}\right)^{(a-b)}$ = ...

(a) zero (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

4 $3^{10} + 3^{10} + 3^{10}$ = ...

(a) 3^{30} (b) 3^{11} (c) 9^{10} (d) 9^{11}

5 If $3x + 1 = 16$, then the value of $5x$ = ...

(a) 10 (b) 15 (c) 25 (d) 26

6 5^{-1} = ...

(a) -5 (b) 25 (c) 5 (d) $\frac{1}{5}$

2 Complete the following :

1 The probability of the impossible event = ...

2 $(3 \times 10^2) \times (15 \times 10^3)$ = ... in the standard form.

3 If $\frac{x}{5} = 30\%$, then x = ...

4 $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{16}$, ... , ... (in the same pattern)

5 If $x = \frac{1}{2}$, $y = \frac{3}{4}$, then $y \div x$ = ...

3 **[a] Write the following in the simplest form :** $\frac{7^{-3} \times 7^6}{7^2}$

[b] Find in $\mathbb{Q}$ the solution set of the following : $8 + 2\,x = 14$

4 **[a] Find the result of the following :** $10 \times 4 - (2 \times 6 - 8)$

[b] Calculate the following when : $a = 2$, $b = 5$

1. $\frac{b - a}{b^3}$
2. $\frac{a^2}{b - 1}$

5 **[a]** A box contains 15 cards numbered from 1 to 15 , a card is drawn randomly **, find the probability of :**

1. The drawn card carries a prime number.
2. The drawn card carries a number divisible by 3

[b] Find in $\mathbb{Q}$ the solution set of the following : $3\,x - 4 \geq -10$

Second

Geometry and Measurement

Quizzes

on Geometry and Measurement

Quiz 1 on lesson 1 – unit 3

time 15 min.

1 Complete :

1. If two straight lines intersect , then the measures of each two vertically opposite angles are ……………
2. The sum of the measures of the accumulative angles at a point is equal to ……………
3. If two straight lines are parallel to a third , then the two straight lines are ……………

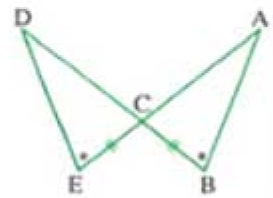
2 [a] In the opposite figure :

$\overline{AE} \cap \overline{BD} = \{C\}$

, BC = CE

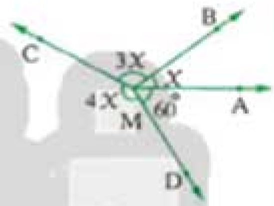
m (∠ B) = m (∠ E)

Prove that : AB = DE



[b] In the opposite figure :

Find by proof the value of X



Quiz 2 till lesson 2 – unit 3

time 15 min.

1 Complete the following :

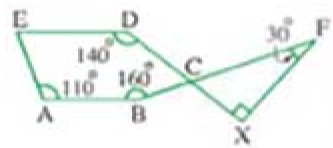
1. The sum of measures of the interior angles of heptagon = ……………°
2. The sum of measures of the exterior angles of the triangle = ……………
3. If the measure of an interior angle of a regular polygon = 120° then the number of its sides = ……………

2 [a] In the opposite figure :

$\overline{BF} \cap \overline{DX} = \{C\}$, $\overline{FX} \perp \overline{DX}$, m (∠ F) = 30°

, m (∠ B) = 160° , m (∠ D) = 140° , m (∠ A) = 110°

Prove that : $\overline{AB} \,/\!/\, \overline{ED}$

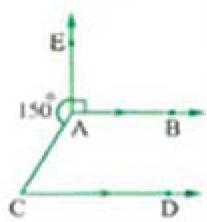


[b] In the opposite figure :

$\overrightarrow{AB} \,/\!/\, \overrightarrow{CD}$, m (∠ EAC) = 150°

, $\overrightarrow{AB} \perp \overrightarrow{AE}$

Find : 1 m (∠ BAC) 2 m (∠ C)



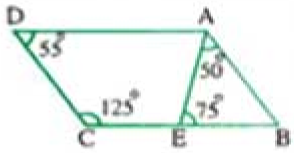
Quiz 3 till lesson 3 – unit 3

time 15 min.

1 Complete the following :

1. If ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 110° , then m (∠ D) = ... °
2. The quadrilateral in which only two sides are parallel is called ....
3. The measure of the interior angle of the regular hexagon = ...

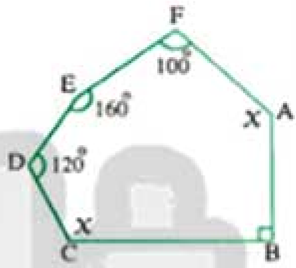
2 [a] In the opposite figure :



E ∈ $\overline{BC}$, m (∠ BAE) = 50° , m (∠ AEB) = 75° , m (∠ D) = 55° , m (∠ C) = 125°

Prove that : the figure ABCD is a parallelogram.

[b] The opposite figure :



ABCDEF is a hexagon and m (∠ A) = m (∠ C) = X

Find : the value of X

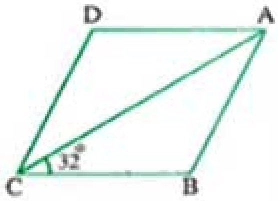
Quiz 4 till lesson 4 – unit 3

time 15 min.

1 Complete the following :

1. If two adjacent sides in a rectangle are equal in length , then it will be ....
2. If ABCD is a square , then m (∠ CAB) = ...
3. The polygon which has an interior reflex angle is called ....

2 [a] In the opposite figure :



ABCD is a rhombus , $\overline{AC}$ is a diagonal in it m (∠ ACB) = 32°

Find : m (∠ D)

[b] If the measure of the exterior angle of a regular polygon equals 30° , what is the number of sides of this polygon and what is the sum of measures of its interior angles ?

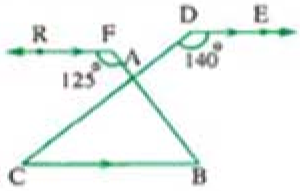
Quiz 5 till lesson 5 – unit 3

time 15 min.

1 Complete the following :

1. The sum of measures of the interior angles of the triangle = ...
2. The measure of the exterior angle of the equilateral triangle = ...
3. In the parallelogram ABCD , if m (∠ A) = $\frac{1}{2}$ m (∠ B) , then m (∠ B) = ...

2 [a] In the opposite figure :

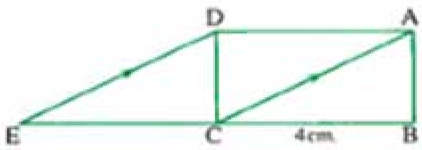


$\overleftrightarrow{DE}$ // $\overrightarrow{FR}$ // $\overline{BC}$, m (∠ D) = 140°

, m (∠ F) = 125°

Calculate the measures of the angles of : Δ ABC

[b] In the opposite figure :



ABCD is a rectangle ,

$\overline{AC}$ // $\overline{DE}$, E ∈ $\overrightarrow{BC}$

1. **Prove that :** ACED is a parallelogram.
2. **Find :** The length of $\overline{CE}$

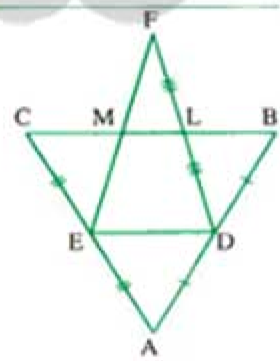
Quiz 6 till lesson 6 – unit 3

time 15 min.

1 Complete the following :

1. The ray which is drawn from the midpoint of a side of a triangle parallel to one of the other sides ....
2. If the measure of an angle in a triangle is greater than the sum of measure of the other two angles , then the triangle is ....
3. The length of the line segment joining the midpoints of two sides of a triangle equals .... the length of the third side.

2 [a] In the opposite figure :



D is the midpoint of $\overline{AB}$

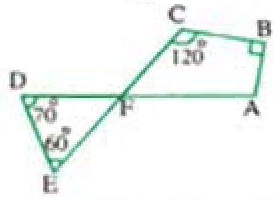
, E is the midpoint of $\overline{AC}$

, L is the midpoint of $\overline{FD}$

, BC = 20 cm.

Find : The length of $\overline{LM}$

[b] In the opposite figure :



$\overline{AD}$ ∩ $\overline{CE}$ = {F} , m (∠ B) = 90°

, m (∠ C) = 120° , m (∠ E) = 60° , m (∠ D) = 70°

Find : m (∠ A)

Quiz 7 till lesson 7 – unit 3

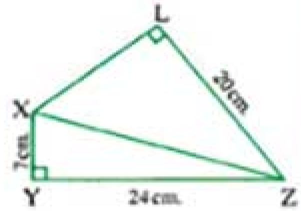
time 15 min.

1 Complete the following :

1. A rectangle whose length = 4 cm. and width = 3 cm. , then the length of its diagonal = ... cm.
2. In Δ ABC , if m ($\angle$ B) = m ($\angle$ A) + m ($\angle$ C) , then $\angle$ B is ....
3. In the right-angled triangle , the area of the square on the hypotenuse equals ....

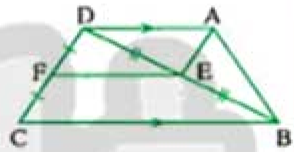
2 [a] In the opposite figure :

m ($\angle$ Y) = m ($\angle$ L) = 90°
, XY = 7 cm. , YZ = 24 cm.
and L Z = 20 cm.
Find : The length of $\overline{XL}$



[b] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, AD = $\frac{1}{2}$ BC
, E is the midpoint of $\overline{BD}$
and F is the midpoint of $\overline{CD}$
Prove that : AEFD is a parallelogram.

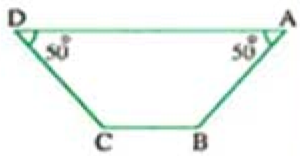


Quiz 8 till lesson 8 – unit 3

time 15 min.

1 Complete the following :

1. The image of the point (1 , 2) by the transformation (x , y) $\longrightarrow$ (x , – y) is ....
2. If Δ XYZ is a right-angled triangle at Z , then $(XZ)^2$ = ...
3. **In the opposite figure :**
   m ($\angle$ A) = m ($\angle$ D) = 50°
   $\overrightarrow{AB}$ and $\overrightarrow{DC}$ intersect at E
   not shown in the figure.
   , then m ($\angle$ E) = ...



2 [a] If the ratio among the measures of the interior angles of a quadrilateral is 2 : 2 : 3 : 5 , find the measure of the greatest angle in the quadrilateral.

[b] On lattice , draw Δ ABC where A (1 , 2) , B (4 , 2) , C (4 , 4) , then map its image by the transformation (x , y) $\longrightarrow$ (y , – x)

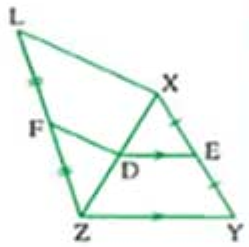
Quiz 9 till lesson 9 – unit 3

time 15 min.

1 Complete :

1. The image of the point (3 , – 5) by the reflection in x-axis is ....
2. The triangle contains two .... angles at least.
3. The image of the point (2 , 3) by the reflection in y-axis is ....

2 [a] In the opposite figure :



XE = EY , $\overline{ED}$ // $\overline{YZ}$

, F is the midpoint of $\overline{ZL}$

Prove that : $\overline{DF}$ // $\overline{XL}$

[b] On lattice , Draw the image of Δ ABO where

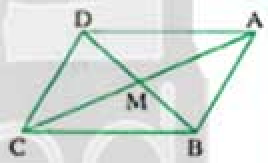
A (2 , 2) , B (4 , 2) , O (0 , 0) by reflection in y-axis.

Quiz 10 till lesson 10 – unit 3

time 15 min.

1 Complete the following :

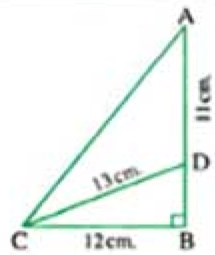
1. The point (2 , – 4) is the image of the point .... by reflection in the origin point.
2. **In the opposite figure :**



If ABCD is a parallelogram , then the image of Δ AMD by reflection in the point M is Δ ....

3. If the reflection in a straight line transforms the figure to itself , then this straight line is called ....

2 [a] In the opposite figure :



ABC is a triangle in which m ($\angle$ B) = 90°

, D $\in$ $\overline{AB}$, where AD = 11 cm. , BC = 12 cm.

and DC = 13 cm.

Find : The length of each of $\overline{BD}$ and $\overline{AC}$

[b] Draw on a lattice Δ ABC , where A (5 , 1) , B (2 , 1) and C (5 , 3) , then draw its image by reflection in the origin point.

Quiz 11 till lesson 11 – unit 3

time 15 min.

1 Complete the following :

1. The translation reserves .... , ...
2. The translation is determined by two things , they are .... and ....
3. If $\grave{A}$ (3 , 5) is the image of the point A by the translation $(x , y) \longrightarrow (x + 4 , y - 2)$, then the point A is ....

2 On lattice , draw Δ ABC where A (– 3 , 2) , B (– 1 , 1) and C (– 2 , 4) then draw its image.

1. by the translation $(x , y) \longrightarrow (x + 4 , y - 3)$
2. by reflection in x-axis.

Quiz 12 till lesson 12 – unit 3

time 15 min.

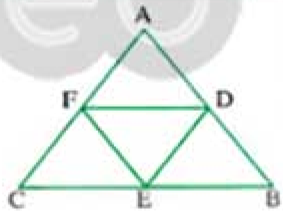
1 Complete the following :

1. The point (3 , 4) is the image of the point (4 , – 3) by rotation about the origin point with an angle of measure ....
2. The image of the point (2 , – 1) by a translation of magnitude 3 units in the negative direction of y-axis is ....
3. If M is the midpoint of $\overline{LN}$, then the image of $\overline{LN}$ by a rotation R (M , 180°) is ....

2 [a] In the opposite figure :

Each of the triangles ADF , DBE , DEF , FEC is equilateral

Find the image of Δ ADF :



1. By the translation AD in the direction of $\overrightarrow{AD}$
2. By reflection in $\overleftrightarrow{DF}$
3. By a rotation R (D , 60°)

[b] On lattice , draw Δ ABC where : A (1 , 1) , B (3 , 1) , C (2 , 3) , **then draw :**

1. The image of Δ ABC by reflection in x-axis.
2. The image of Δ ABC by rotation R (O , 180°)

Final Revision

of Geometry and Measurement

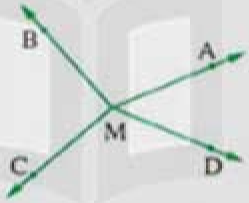
Revision for the important theorems , corollaries and rules of geometry and measurement

Some relations between the angles

If two straight lines intersect , then the measures of each two vertically opposite angles are equal.

If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, then
m (∠ AMC) = m (∠ BMD)
"Vertically opposite angles"
, m (∠ AMD) = m (∠ CMB)
"Vertically opposite angles"

The sum of the measures of the accumulative angles at a point is equal to 360°

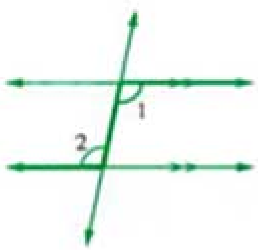


m (∠ AMD) + m (∠ DMC)
+ m (∠ CMB) + m (∠ BMA)
= 360°

Parallelism

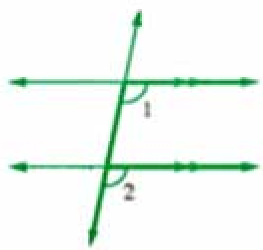
If a straight line intersects two parallel straight lines , then

Each two alternate angles are equal in measure.



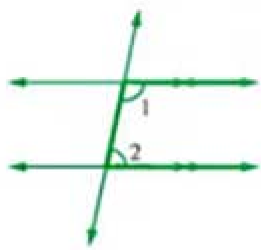
For example :
m (∠ 1) = m (∠ 2)
(alternate angles)

Each two corresponding angles are equal in measure.



For example :
m (∠ 1) = m (∠ 2)
(corresponding angles)

Each two interior angles in the same side of the transversal are supplementary.



For example :
m (∠ 1) + m (∠ 2) = 180°

Remember — How to prove the parallelism of two straight lines

The two straight lines are parallel if a third straight line intersects them (as a transversal) and **one** of the following cases is satisfied :

1. Two alternate angles have the same measure.

or 2. Two corresponding angles have the same measure.

or 3. Two interior angles in the same side of the transversal are supplementary.

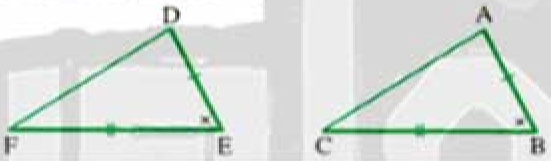
Congruence of triangles

The cases of congruence of two triangles

First case :

Two sides and the included angle (S.A.S.)

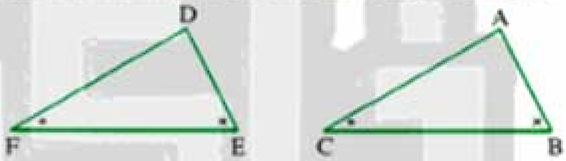
Two triangles are congruent if two sides and the inculded angle of one triangle are congruent to the corresponding parts of the other triangle.



Second case :

Two angles and one side (A.S.A.)

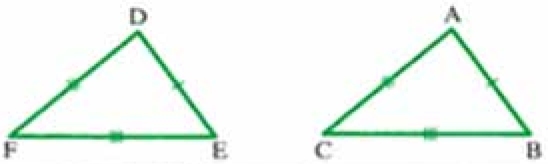
Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle.



Third case :

Three sides (S.S.S.)

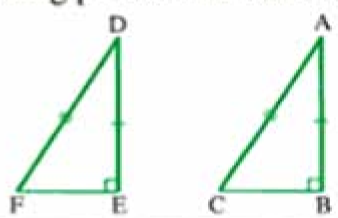
Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle.



Fourth case :

Hypotenuse and one side in the right-angled triangle (R.H.S.)

Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.



The polygon

Rules of the convex polygon

First rule

The sum of measures of the interior angles of a polygon of n sides $= (n - 2) \times 180°$

Second rule

The measure of each interior angle of the regular polygon of n sides $= \frac{(n-2) \times 180°}{n}$

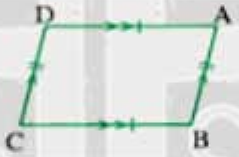
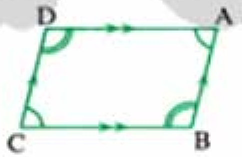
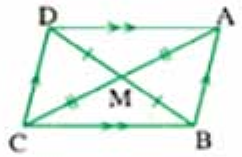
Third rule

The sum of measures of the exterior angles of any convex polygon = 360°

Fourth rule

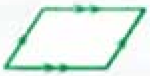
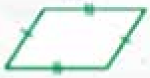
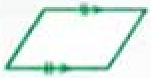
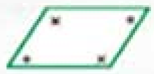
The number of sides of the regular polygon in which the measure of one of its interior is $x° = \frac{360°}{180° - x}$

The parallelogram and its properties

1 Each two opposite sides are equal in length.		• AB = DC • AD = BC
2 Each two opposite angles are equal in measure.		• m (∠ A) = m (∠ C) • m (∠ B) = m (∠ D)
3 The sum of measures of each two consecutive angles is 180°		• m (∠ A) + m (∠ B) = 180° • m (∠ B) + m (∠ C) = 180° • m (∠ C) + m (∠ D) = 180° • m (∠ D) + m (∠ A) = 180°
4 The two diagonals bisect each other.		• AM = CM • BM = DM

When does a quadrilateral represent a parallelogram ?

A quadrilateral represents a parallelogram if one of the following conditions satisfies

Each two opposite sides are parallel.	Each two opposite sides are equal in length.	Two opposite sides are parallel and equal in length.	Each two opposite angles are equal in measure.	The two diagonals bisect each other.
				

The parallelogram is

a rectangle if :

One of its angles is a right angle

or

Its two diagonals are equal in length.

a rhombus if :

Two adjacent sides are equal in length.

or

Its two diagonals are perpendicular.

a square if :

One of its angles is right and two adjacent sides are equal in length.

or

One of its angles is right and its two diagonals are perpendicular.

or

Its two diagonals are perpendicular and equal in length.

or

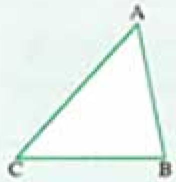
Two adjacent sides are equal in length and its two diagonals are equal in length.

Notice that

* A square is a rectangle with two adjacent sides equal in length or with two perpendicular diagonals.
* A square is a rhombus with a right angle or with two diagonals equal in length.
* To prove that the quadrilateral is a rectangle , a rhombus or a square , we must first prove that it is a parallelogram.

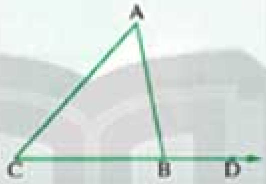
The triangle

- The sum of the measures of the interior angles of a triangle is 180°



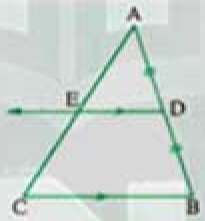
$m(\angle A) + m(\angle B) + m(\angle C) = 180°$

- The measure of the exterior angle of a triangle is equal to the sum of the measures of its non adjacent interior angles.



If $D \in \overrightarrow{CB}$, then $\angle ABD$ is an exterior angle of ΔABC
, $m(\angle ABD) = m(\angle A) + m(\angle C)$

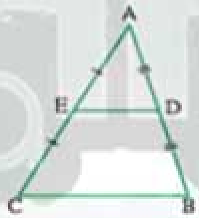
- The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.



If D is the midpoint of $\overline{AB}$, $\overrightarrow{DE} \parallel \overline{BC}$, then E is the midpoint of $\overline{AC}$

- The line segment joining the midpoints of two sides of a triangle is parallel to the third side.

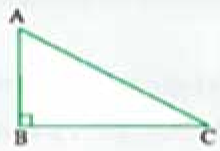
If D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$, then $\overline{DE} \parallel \overline{BC}$



- The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side.

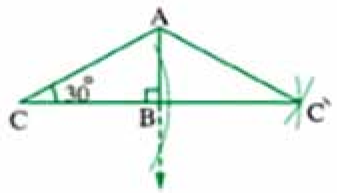
If D the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$, then $DE = \frac{1}{2}BC$

Pythagoras' theorem :
The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.

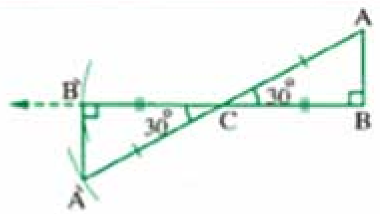


In ΔABC :
If $m(\angle B) = 90°$
, then $(AC)^2 = (AB)^2 + (BC)^2$
, $(AB)^2 = (AC)^2 - (BC)^2$
, $(BC)^2 = (AC)^2 - (AB)^2$

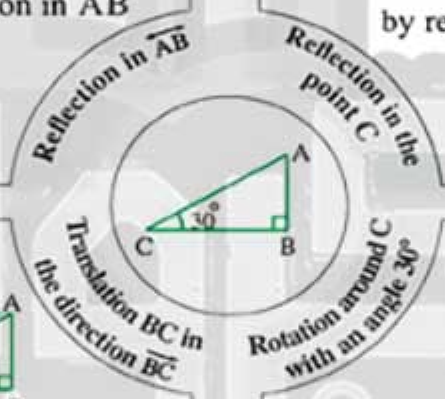
Geometric transformations

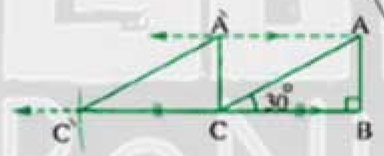


- Image of each of A and B are them.
- To find the image of C : We place the compasses at C , then we draw an arc cuts $\overleftrightarrow{AB}$ at two points , then we place at each of them and draw two arcs intersect at C' , then $\Delta ABC'$ is the image of ΔABC by reflection in $\overleftrightarrow{AB}$

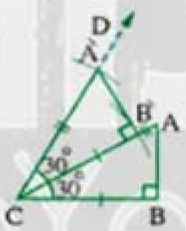


- Image of C is itself.
- To find the image of A : We draw $\overrightarrow{AC}$, and by using the compasses take the point A' on $\overrightarrow{AC}$ where $AC = A'C$
- Similarly , we find image of B , then $\Delta A'B'C$ is the image of ΔABC by reflection in the point C





- Image of B is C
- To find the image of C : We draw $\overrightarrow{BC}$ and by using the compasses , we take point C' on $\overrightarrow{BC}$ where $CC' = BC$
- To find the image of A : We draw from A a ray parallels to $\overrightarrow{BC}$, then by using the compasses we determine on it A' where $AA' = BC$, then $\Delta A'CC'$ is the image of ΔABC by translation BC in the direction $\overrightarrow{BC}$



- Image of C is itself.
- To find the image of A : Put the protractor with its straight edge on $\overline{CA}$ and in the antic lockwise direction , draw $\overrightarrow{CD}$ where $m(\angle ACD) = 30°$, then take the point A' on $\overrightarrow{CD}$ where $CA' = CA$
- Similarly , we find B' which is the image of B , $\Delta A'B'C$ is the image of ΔABC by rotation around C with angle of measure 30°

Summary for geometrical transformations (reflection , translation , rotation) in the Cartesian plane :

The image of the point (x , y)	
by reflection in the x-axis	The point $(x , -y)$
by reflection in the y-axis	The point $(-x , y)$
by reflection in the origin point	The point $(-x , -y)$
by translation $(x, y) \Rightarrow (x + k , y + \ell)$	The point $(x + k , y + \ell)$
by rotation R (O , 90°) $\left(\frac{1}{4} \text{ turn}\right)$	The point $(-y , x)$
by rotation about O with an angle of measure (– 90°) or (270°)	The point $(y , -x)$
by rotation R (O , ± 180°) $\left(\frac{1}{2} \text{ turn}\right)$	The point $(-x , -y)$
by rotation R (O , ± 360°) (identity rotation)	The point (x , y)

Final Examinations

on Geometry and Measurement

Model Examinations of the School Book on Geometry and Measurement

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 Circumference of a circle of radius 7 cm. = ... cm. $\left(\pi = \frac{22}{7}\right)$

(a) 11 (b) 22 (c) 44 (d) 88

2 The image of the point (– 1 , 3) by translation (4 , – 2) is ....

(a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)

3 The measure of the exterior angle of the equilateral triangle is ....

(a) 30° (b) 45° (c) 60° (d) 120°

4 In a parallelogram if the adjacent sides are equal in the length , then the shape is ....

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

5 The number of the diagonals of a pentagon is ....

(a) 3 (b) 5 (c) 7 (d) 9

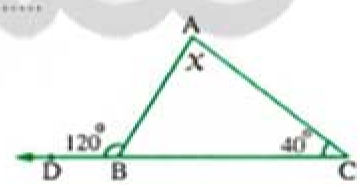
6 The number of axes of symmetry of an isosceles triangle = ...

(a) zero (b) 1 (c) 2 (d) 3

2 Complete the following :

1 The image of the point (2 , 1) by reflection in X-axis is ....

2 **In the opposite figure :**



x = ...°

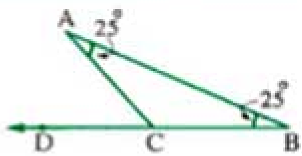
3 XYZ is a triangle in which m (∠ Y) = 90° , XY = 3 cm.

, XZ = 5 cm. , then YZ = ... cm.

4 ABCD is a parallelogram in which m (∠ A) = 100° , then m (∠ B) + m (∠ D) = ...°

5 The sum of the measures of the interior angles of a triangle = ...°

3 [a] In the opposite figure :

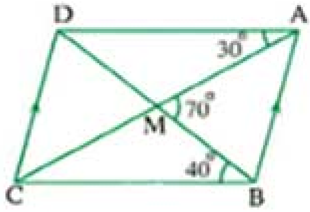


m (∠ A) = m (∠ B) = 25°

Find : m (∠ ACD)

[b] Draw a triangle ABC in which AB = 5 cm. , AC = 3 cm. and m (∠ A) = 40°
, then draw $\grave{C}$ is the image of C under rotation R (A , 40°) , $\grave{B}$ is the image of B under rotation R (A , – 40°)

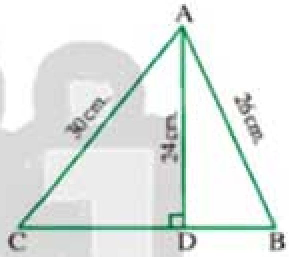
4 **[a] In the opposite figure :**



$\overline{AB} \parallel \overline{DC}$, $\overline{AC} \cap \overline{BD} = \{M\}$,
m (∠ DAC) = 30° , m (∠ DBC) = 40°
and m (∠ AMB) = 70°
Prove that : ABCD is a parallelogram.

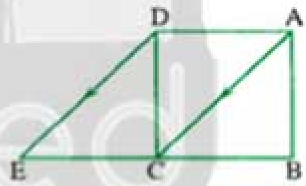
[b] Use the translation : $(x , y) \longrightarrow (x + 2 , y + 3)$
to find the point whose image is (2 , 3)

5 **[a] In the opposite figure :**



$\overline{AD} \perp \overline{BC}$, if AD = 24 cm. , AB = 26 cm. , AC = 30 cm.
1 **Find :** The length of $\overline{BC}$
2 **Find :** The area of Δ ABC

[b] In the opposite figure :



ABCD is a square , E ∈ $\overrightarrow{BC}$, $\overline{AC} \parallel \overline{DE}$
Prove that : ACED is a parallelogram.

Model 2

Answer the following questions :

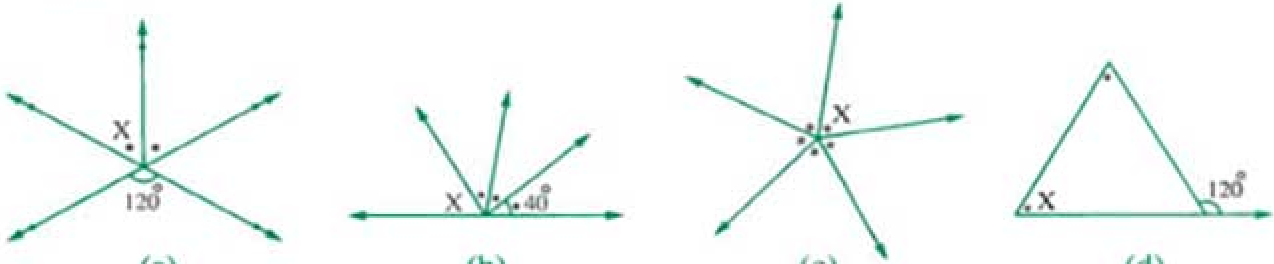
1 **Choose the correct answer from those given :**

1 ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , then AC = ... cm.
(a) 10 (b) 28 (c) 100 (d) 160

2 The measure of each angle of regular hexagon equals ....
(a) 60° (b) 108° (c) 120° (d) 135°

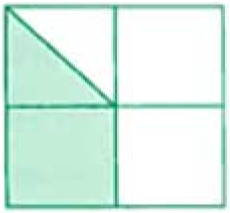
3 The two diagonals are equal in length and not perpendicular in ....
(a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

4 In all the following shapes m (∠ X) = 60° except the shape ……………



(a) (b) (c) (d)

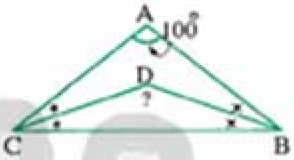
5 **In the opposite figure :**



The area of the shaded part from the area of all shape equals ……………

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

6 **In the opposite figure :**

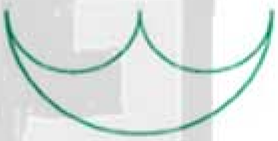


m (∠ BDC) = ……………°

(a) 60 (b) 80 (c) 100 (d) 140

2 Complete the following :

1 **In the opposite figure :**



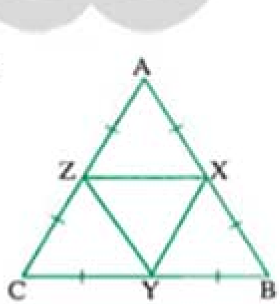
Semicircle of diameter 14 cm. and two semicircles the diameter of each is 7 cm.
, then the perimeter of the figure equals …………… cm. $\left(\pi = \frac{22}{7}\right)$

2 The image of the point (2 , 3) by translation MN , in direction $\overrightarrow{MN}$, where M (2 , – 1) , N (5 , 1) is ……………

3 The volume of a cube of side length 1.2 m. = …………… cm^3.

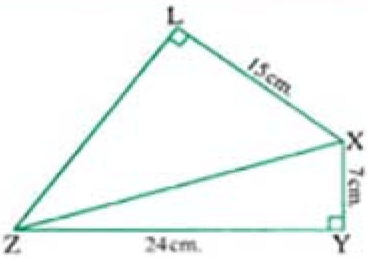
4 The ray drawn parallel to one side of a triangle and passing through the midpoint of another side ……………

5 **In the opposite figure :**



The image of the triangle XBY by translation XZ in direction $\overrightarrow{XZ}$ is ……………

3 [a] In the opposite figure :

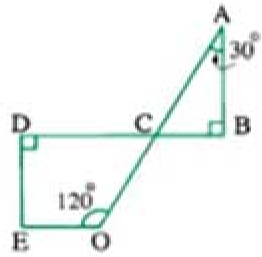


XYZL is a quadrilateral in which
m (∠ Y) = m (∠ L) = 90° , XY = 7 cm. ,
YZ = 24 cm. , XL = 15 cm.
Find : The length of each of $\overline{XZ}$ and $\overline{LZ}$

[b] Using the square lattice , draw $\overline{AB}$ where A (4 , 3) , B (– 1 , 1)
then find the image of $\overline{AB}$ by translation $(x , y) \longrightarrow (x + 2 , y - 1)$

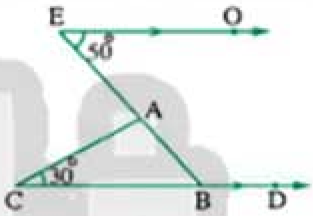
4 [a] Draw the image of triangle ABC where A (1 , 1) , B (3 , 4) , C (5 , 2) by reflection in x-axis.

[b] **In the opposite figure :**



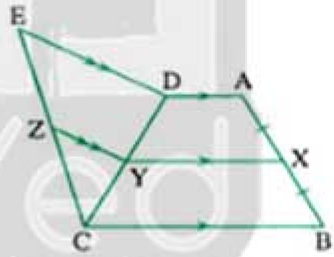
$\overline{AB}$ and $\overline{ED}$ are perpendicular to $\overline{BD}$
, $\overline{BD} \cap \overline{AO} = \{C\}$,
m (∠ A) = 30°
, m (∠ EOC) = 120° ,
Find : m (∠ E)

5 [a] **In the opposite figure :**



$\overrightarrow{EO} \parallel \overrightarrow{CD}$, m (∠ E) = 50°
, m (∠ C) = 30° ,
Find the measures of angles of Δ ABC , m (∠ ABD)

[b] **In the opposite figure :**



X is the midpoint of $\overline{AB}$
, Y ∈ $\overline{CD}$, Z ∈ $\overline{CE}$
, $\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, $\overline{YZ} \parallel \overline{DE}$
Is CZ = ZE ? giving reason

Model examination for the merge students

Answer the following questions :

1 Choose the correct answer :

1 The sum of the measures of the interior angles of a triangle = ...°

(a) 90 (b) 360 (c) 180 (d) 540

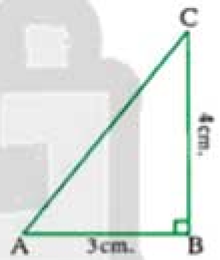
2 The image of the point (3 , – 2) by reflection in the y-axis is the point ....

(a) (3 , 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (– 2 , 3)

3 The diagonals are equal and perpendicular in ....

(a) rhombus. (b) square. (c) recangle. (d) parallelogram.

4 **In the opposite figure :**

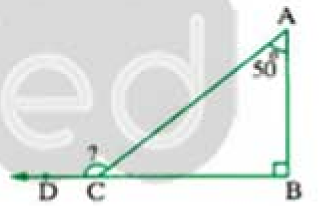


AC = ... cm.

(a) 5 (b) 7

(c) 25 (d) 625

5 **In the opposite figure :**



m (∠ ACD) = ...°

(a) 40 (b) 140

(c) 90 (d) 50

2 Complete each of the following :

1 The length of the line segment that joins two midpoints of two sides of a triangle equals .... the length of the third side.

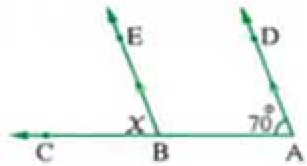
2 The rectangle is a parallelogram in which one of it's angles is ....

3 The length of the side of a rhombus whose perimeter is 24 cm. equals .... cm.

4 The image of the point A (– 3 , 2) by reflection in the origin point is the point Á (.............. , ...)

5 **In the opposite figure :**

x = ...°



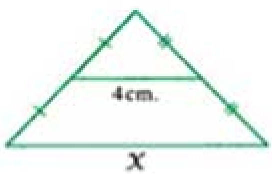
3 Put (✓) or (✗) :

1. The image of the point (4 , 3) by reflection in the x-axis is the point (3 , – 4) ( )
2. If ABC is a right-angled triangle at B , then $(AB)^2 = (BC)^2 + (AC)^2$ ( )
3. The pentagon has 5 diagonals. ( )
4. ABCD is a parallelogram , in which m (∠ A) = 70° , then m (∠ C) = 110° ( )
5. Any triangle contains at least two acute angles. ( )

4 Join from the column (A) to the suitable in the column (B) :

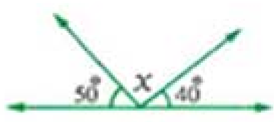
Column (A)	Column (B)
1 The sum of the measures of the interior angles of a quadrilateral = ...	• 120°
2 The measure of each angle of a regular hexagon = ...	• 360°
3 The image of the point (3 , 2) by translation (1 , – 2) is the point	• (– 1 , – 3)
4 The image of the point (1 , 3) by rotation about the origin point , of angle of measure 180° is the point (............... , ...)	• 45
5 The diagonal of the square divides the angle of the vertex into two angles , the measure of each = ...°	• (4 , 0)

5 Find the value of x :



x = ... cm.

Fig. (1)



x = ...°

Fig. (2)

Schools Examinations on Geometry and Measurement

1 Cairo Governorate

East Nasr City Zone
Manaret Heliopolis School

Answer the following questions :

1 Complete :

1. The sum of measures of the exterior angles of the heptagon is ....°
2. If two lines intersect , then each two vertically opposite angles are ....
3. In Δ ABC , if m ($\angle$ A) + m ($\angle$ C) = m ($\angle$ B) , then m ($\angle$ B) = ...°
4. The image of (2 , 3) by translation $(x , y) \longrightarrow (x + 4 , y - 2)$ is ....
5. The length of the line segment joining two midpoints of two sides of a triangle equals .... the length of the third side.

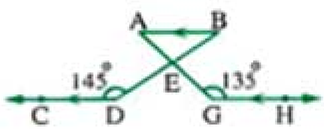
2 Choose the correct answer :

1. ABCD is a parallelogram in which m ($\angle$ A) = 80° , then m ($\angle$ C) = ...
   (a) 80° (b) 100° (c) 120° (d) 60°
2. If the image of the point (5 , – 3) by rotation about the origin point is itself , then the measure of the rotation angle is ....°
   (a) 90 (b) 180 (c) 270 (d) 360
3. The line segment joining two midpoints of two sides of a triangle is .... the third side.
   (a) intersecting (b) parallel to (c) perpendicular to (d) congestive
4. The sum of measures of the accumulative angles at a point is ....°
   (a) 90 (b) 180 (c) 270 (d) 360
5. If ABC is a right-angled triangle at B and AB = 4 cm. , BC = 3 cm. , then AC = ... cm.
   (a) 16 (b) 25 (c) 9 (d) 5
6. The sum of measures of the interior angles of a hexagon is ....°
   (a) 360 (b) 540 (c) 720 (d) 120

3 [a] In the opposite figure :

ABE is a triangle , $\overline{AB}$ // $\overline{GH}$ // $\overline{CD}$, m ($\angle$ CDB) = 145°
, m ($\angle$ AGH) = 135°

Calculate : m ($\angle$ DEG)

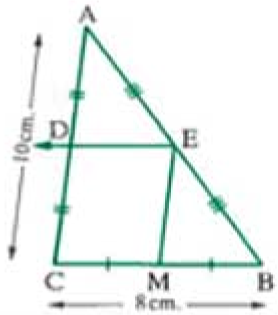


[b] In the opposite figure :

ABC is a triangle in which E , M and D are the midpoints of the sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively

, BC = 8 cm. , AC = 10 cm.

Prove that : DEMC is a parallelogram.



4 **[a] In the opposite figure :**

m ($\angle$ ACB) = 90° , AB = 13 cm. , AD = 3 cm.

, CD = 4 cm. , BC = 12 cm.

Find : **1** The length of $\overline{AC}$

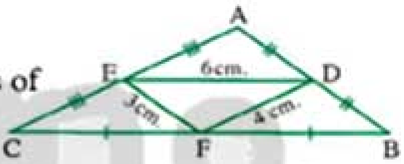
2 The perimeter of the figure ABCD

[b] In the opposite figure :

ABC is a triangle in which : D , F and E are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$

respectively such that : DF = 4 cm. , FE = 3 cm. , DE = 6 cm.

Calculate : The perimeter of Δ ABC



5 **[a]** Using the lattice , find the images of the points :

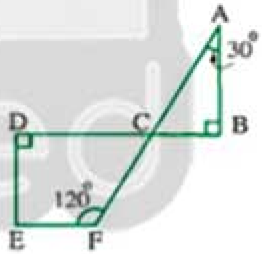
A (– 4 , 1) , B (0 , 4) and C (– 2 , 2) by reflection in X-axis.

[b] In the opposite figure :

$\overline{AB} \perp \overline{BC}$, $\overline{BD} \cap \overline{AF} = \{C\}$

, m ($\angle$ A) = 30° , m ($\angle$ F) = 120°

Find with proof : **1** m ($\angle$ ACB) **2** m ($\angle$ E)



2 Cairo Governorate

Abdeen Directorate
Patriarchal College

Answer the following questions :

1 **Choose the correct answer :**

1 The sum of the measures of the interior angles of the pentagon equals .... °

(a) 360 (b) 540 (c) 720 (d) 108

2 The two diagonals are equal in length and perpendicular in the ....

(a) parallelogram (b) rectangle (c) rhombus (d) square

3 The image of the point $(4, -7)$ by reflection in X-axis is ....

(a) $(4, 7)$ (b) $(-4, -7)$ (c) $(-4, 7)$ (d) $(4, -7)$

4 The line segment joining the midpoints of two sides of a triangle is .... the third side.

(a) parallel to (b) perpendicular to (c) bisect to (d) equal to

5 In ΔABC, if $m(\angle B) = 90°$, $AB = 6$ cm. and $BC = 8$ cm., then $AC =$ .... cm.

(a) 100 (b) 8 (c) 14 (d) 10

6 The image of the point $(-3, 4)$ by rotation about the origin with an angle of measure $90°$ is ....

(a) $(-4, 3)$ (b) $(3, -4)$ (c) $(-4, -3)$ (d) $(4, 3)$

2 Complete :

1 A quadrilateral in which only two opposite sides are parallel is called ....

2 The measure of each angle of the regular octagon equals ....°

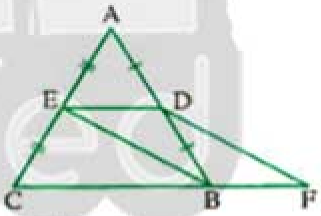
3 The image of the point $(1, -5)$ by translation $(-4, 6)$ is ....

4 If the measure of each interior angle of a regular polygon is $140°$, then the number of its sides is ....

5 ABCD is a parallelogram in which $m(\angle A) + m(\angle C) = 100°$, $m(\angle B) =$ ....°

3

[a] **In the opposite figure :**



D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

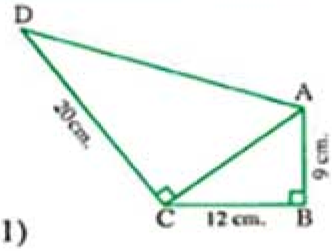
, $F \in \overrightarrow{CB}$, where $BF = \frac{1}{2} CB$

Prove that : DFBE is a parallelogram.

[b] Draw the image of ΔXYZ in which $XY = 3$ cm., $YZ = 5$ cm. and $ZX = 7$ cm. by reflection in the straight line containing the longest side.

4

[a] **In the opposite figure :**



$m(\angle B) = m(\angle ACD) = 90°$

, $AB = 9$ cm., $BC = 12$ cm. and $CD = 20$ cm.

Find by proof : The length of each of $\overline{AC}$ and $\overline{AD}$

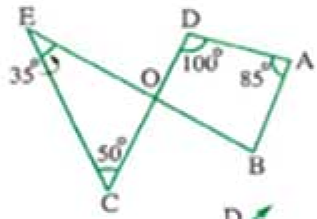
[b] In the square lattice draw ΔABC where : $A(-4, 2)$, $B(-1, 1)$ and $C(-2, 5)$, then draw its image by the translation : $(X, y) \longrightarrow (X + 5, y - 3)$

5 **[a] In the opposite figure :**

$\overline{DC} \cap \overline{BE} = \{O\}$, $m(\angle A) = 85°$

, $m(\angle D) = 100°$, $m(\angle E) = 35°$ and $m(\angle C) = 50°$

Find : $m(\angle B)$

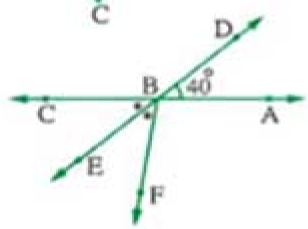


[b] In the opposite figure :

$\overleftrightarrow{AC} \cap \overleftrightarrow{DE} = \{B\}$, $m(\angle ABD) = 40°$

, $\overrightarrow{BE}$ bisects $\angle CBF$

Find : $m(\angle ABF)$



3 Cairo Governorate

Hadayek El Koba Educational Zone

Answer the following questions : (Calculators are Permitted)

1 **Choose the correct answer :**

1. The measure of the interior angle of the regular octagon equals ....
   (a) 1080° (b) 180° (c) 135° (d) 108°
2. If ABCD is a parallelogram in which $m(\angle A) + m(\angle C) = 140°$, then $m(\angle B) =$ ....
   (a) 40° (b) 110° (c) 70° (d) 60°
3. In ΔABC , if $m(\angle B) = 90°$, AB = 20 cm. , AC = 25 cm. , then BC = ... cm.
   (a) 625 (b) 12 (c) 15 (d) 225
4. If the perimeter of a square is 20 cm. , then its area equals .... cm^2.
   (a) 25 (b) 100 (c) 5 (d) 16
5. The image of point (– 3 , 5) by rotation about origin point with an angle of measure 90° is ....
   (a) (– 5 , 3) (b) (– 5 , – 3) (c) (5 , 3) (d) (3 , – 5)
6. The image of point (– 1 , 3) by translation (4 , – 2) is ....
   (a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)

2 **Complete the following :**

1. The line segment joining the midpoints of two sides of a triangle is parallel to ....
2. The image of the point (2 , – 4) by reflection in y-axis is ....
3. The number of axis of symmetry of a rectangle is ....

4 The measure of exterior angle of an equilateral triangle equals ....°

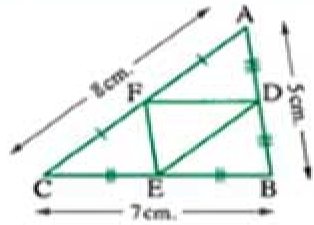
5 The volume of a cube of side length 6 cm. is .... cm^3.

3 **[a] In the opposite figure :**

ABC is a triangle , D , E and F are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ respectively

where AB = 5 cm. , BC = 7 cm. and CA = 8 cm.

Calculate : The perimeter of Δ DEF



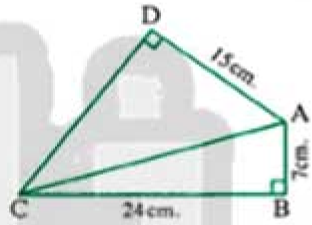
[b] On square lattice , draw the square ABCD where A (1 , 2) , B (– 2 , 2) , C (– 2 , 5) and D (1 , 5) , then draw its image by reflection in X-axis.

4 **[a] In the opposite figure :**

m ($\angle$ B) = m ($\angle$ D) = 90°

AD = 15 cm. , AB = 7 cm. and BC = 24 cm.

Find : The length of each of $\overline{AC}$, $\overline{DC}$



[b] Draw Δ ABC where A (1 , 5) , B (3 , 1) and C (5 , 3) , then draw its image by rotation about the origin point with an angle of measure 180°

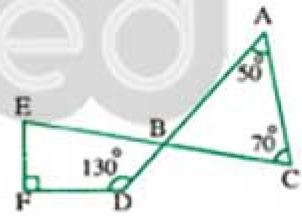
5 **[a] In the opposite figure :**

$\overline{CE} \cap \overline{AD} = \{B\}$

, m ($\angle$ A) = 50° , m ($\angle$ C) = 70°

, m ($\angle$ D) = 130° and m ($\angle$ F) = 90°

Find : m ($\angle$ E)



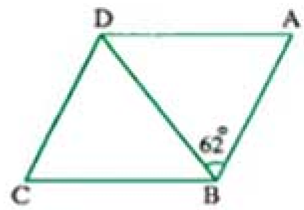
[b] In the opposite figure :

ABCD is a rhombus

, $\overline{BD}$ is a diagonal in it

, m ($\angle$ ABD) = 62°

Find with proof : m ($\angle$ A)



4 Giza Governorate

El-Haram Zone
Hefr El-Bahn Language School British Academy

Answer the following questions :

1 Choose the correct answer :

1. The measure of each angle of the regular pentagon equals ....
   (a) 100° (b) 120° (c) 108° (d) 110°
2. The sum of measures of accumulative angles at a point equals ....
   (a) 90° (b) 100° (c) 360° (d) 180°
3. Any triangle has at least two .... angles.
   (a) acute (b) obtuse (c) right (d) reflex
4. ABCD is a parallelogram in which m (∠ A) = 70° , then m (∠ B) = ...°
   (a) 70 (b) 110 (c) 90 (d) 180
5. (– 4 , 5) is the image of (5 , 4) by rotation in the origin point with an angle of measure ....
   (a) 90° (b) – 90° (c) 180° (d) 360°
6. The image of the point (– 2 , 2) by a translation of magnitude 3 units in the positive direction of y-axis is ....
   (a) (– 1 , 2) (b) (– 2 , 5) (c) (– 1 , 5) (d) (1 , 5)

2 Complete the following :

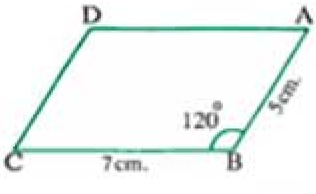
1. The sum of measures of the interior angles of any triangle equals ....
2. Each two opposite angles of a parallelogram are ....
3. The length of the line segment joining the midpoint of two sides of a triangle is equal to .... the length of the third side.
4. The image of the point (3 , – 5) by rotation 360° about the origin point is ....
5. The image of (– 3 , 0) by reflection in the y-axis is ....

3

[a] A regular hexagon of side length 15 cm.

Find : 1 its perimeter. 2 the measure of each angle.

[b] In the opposite figure :



ABCD is a parallelogram , $\overline{AB}$ = 5 cm.

, BC = 7 cm. , m (B) = 120°

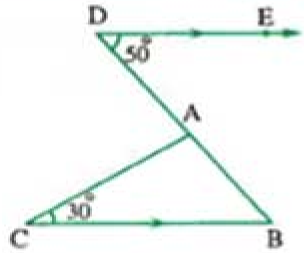
Find : m (∠ C) , m (∠ D) , the length of each of $\overline{AD}$ and $\overline{DC}$

4 [a] **In the opposite figure :**

$\overline{DE} \parallel \overline{BC}$, m (∠ D) = 50°

, m (∠ C) = 30°

Find : m (∠ B) and m (∠ BAC)



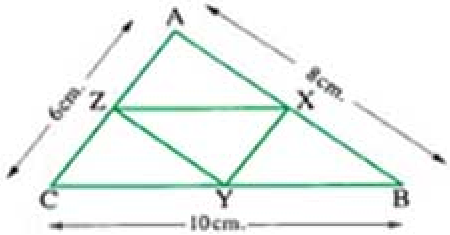
[b] **In the opposite figure :**

X , Y and Z

are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively

AB = 8 cm. , AC = 6 cm. , BC = 10 cm.

Find : The perimeter of Δ XYZ



5 [a] **In the opposite figure :**

m (∠ B) = 90° , AB = 7 cm.

BC = 24 cm.

Find : The length of $\overline{AC}$

[b] Draw ABC in which A (1 , 1) , B (5 , 1) and C (3 , 3) , then find its image by reflection in y-axis.

5 Giza Governorate

Experimental Schools Math inspector

Answer the following questions :

1 **Complete the following :**

1 The image of the point (5 , – 3) under rotation of angle of measure 90° about the origin point is ....

2 The ray which is drawn from the midpoint of a side in a triangle parallel to another side .... the third side.

3 In the rhombus the two diagonals are ....

4 The sum of measures of the exterior angles of any convex polygon equals ....°

5 In Δ ABC , if m (∠ A) + m (∠ B) = m (∠ C) , then Δ ABC is ....

2 **Choose the correct answer :**

1 If the measure of an interior angle of a regular polygon is 135° , then the number of its sides is ....

(a) 6 (b) 4 (c) 7 (d) 8

2 The image of the point $(-1 , 4)$ by the translation $(x , y) \longrightarrow (x+3 , y-2)$ followed by reflection in the x-axis is ……………

(a) $(2 , 2)$ (b) $(-2 , 2)$ (c) $(-2 , -2)$ (d) $(2 , -2)$

3 ABCD is a parallelogram in which $m(\angle A) + m(\angle C) = 110°$, then $m(\angle B) =$ ……………

(a) 125° (b) 80° (c) 100° (d) 110°

4 The concave polygon should have …………… angle.

(a) an acute (b) a right (c) an obtuse (d) a reflex

5 The image of the point …………… is itself by reflection in y-axis.

(a) $(0 , 3)$ (b) $(3 , 0)$ (c) $(3 , 3)$ (d) $(-3 , 3)$

6 The identity rotation about the origin point with an angle of measure ……………°

(a) 90 (b) 180 (c) 360 (d) -90

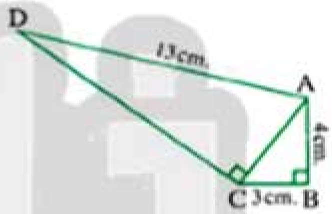
3 [a] **In the opposite figure :**

ABCD is a quadrilateral in which

$m(\angle B) = m(\angle ACD) = 90°$

, AB = 4 cm. , BC = 3 cm. and AD = 13 cm.

Find : AC and DC



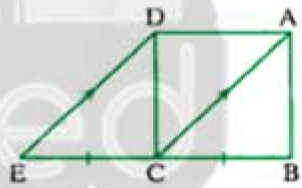
[b] **In the opposite figure :**

ABCD is a square , $E \in \overrightarrow{BC}$

such that BC = CE

1 **Prove that :** The figure ACED is a parallelogram.

2 **Find :** $m(\angle E)$

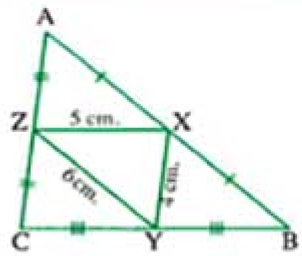


4 [a] **In the opposite figure :**

Δ ABC in which X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ respectively

, XZ = 5 cm. , XY = 4 cm. , YZ = 6 cm.

Find with proof : The perimeter of Δ ABC

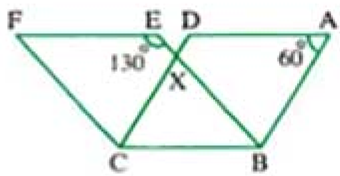


[b] **In the opposite figure :**

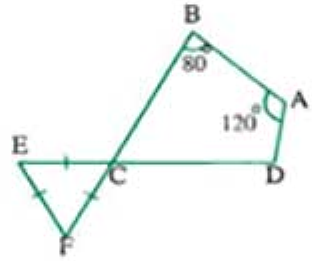
ABCD and EBCF are two parallelograms

, $m(\angle BAD) = 60°$, $m(\angle BEF) = 130°$

Find with proof : $m(\angle BXC)$



5 **[a] In the opposite figure :**



ABCD is a quadrilateral in which m (∠ A) = 120°

, m (∠ B) = 80° , Δ CEF is an equilateral triangle

Find : m (∠ D)

[b] Using the square lattice draw Δ ABC where :

A (2 , 4) , B (4 , 2) and C (3 , 1) , then map its image by the translation

$(x , y) \longrightarrow (x + 2 , y + 3)$

6 Alexandria Governorate

East Educational Zone
Math's Supervision (A)

Answer the following questions :

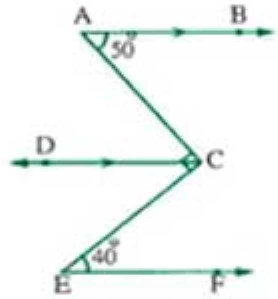
1 **Complete each of the following :**

1. The angle whose measure is 89° is .... angle.
2. The length of the line segment that joins two midpoints of two sides of a triangle equals ....
3. The area of one face of a cube is 25 cm^2 then its volume equals .... cm^3
4. The image of the point (2 , 1) by reflection in x-axis is ....
5. In the parallelogram XYZL , if m (∠ X) = $\frac{1}{2}$ m (∠ Y) , then m (∠ Y) = ...

2 **Choose the correct answer from those given :**

1. The supplementary of the angle whose measure is 30° is an angle of measure ....°
   (a) 30 (b) 60 (c) 120 (d) 150
2. The number of the diagonals of a pentagon is ....
   (a) 3 (b) 5 (c) 7 (d) 9
3. The image of the point (– 1 , 3) by translation (4 , – 2) is ....
   (a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)
4. The sum of the measures of the interior angles of the triangle equals ....°
   (a) 90 (b) 360 (c) 180 (d) 540
5. ABC is right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , then AC = ... cm.
   (a) 10 (b) 28 (c) 100 (d) 160
6. The measure of the exterior angle of the equilateral triangle is ....°
   (a) 30 (b) 45 (c) 60 (d) 120

3 **[a] In the opposite figure :**



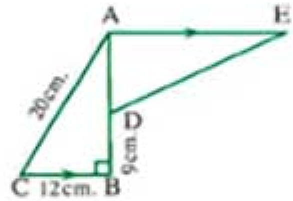
$\overrightarrow{AB} \parallel \overrightarrow{CD}$, m ($\angle$ A) = 50°

, $\angle$ ACE is right angle and m ($\angle$ E) = 40°

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{EF}$

[b] Use the translation : $(x , y) \longrightarrow (x + 2 , y + 3)$ to locate the point whose image is (2 , 3)

4 **[a] In the opposite figure :**



ABC is a triangle , m ($\angle$ B) = 90°

, $\overline{AE} \parallel \overline{BC}$, if BC = 12 cm. , AC = 20 cm.

, D $\in$ AB where BD = 9 cm. and AE = 2 BC

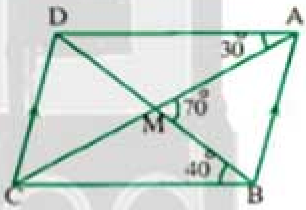
Find : The length of each of $\overline{AD}$, $\overline{ED}$

[b] Using square lattice , draw Δ ABC , where :

A (– 2 , 4) , B (5 , 0) , C (3 , – 3)

, then draw the reflected image of Δ ABC in the origin point.

5 **[a] In the opposite figure :**



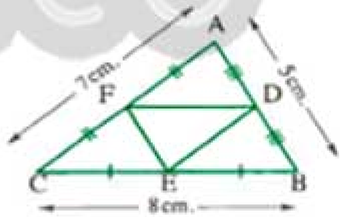
$\overline{AB} \parallel \overline{DC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

m ($\angle$ DAC) = 30° , m ($\angle$ DBC) = 40°

and m ($\angle$ AMB) = 70°

Prove that : ABCD is a parallelogram

[b] In the opposite figure :



AB = 5 cm. , BC = 8 cm. , AC = 7 cm. D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

Calculate : The perimeter of Δ DEF

7 Alexandria Governorate

Middle Educational Zone
Math's Supervision

Answer the following questions :

1 **Complete each of the following :**

1. The measure of the straight angle equals ....°
2. The image of the point (4 , – 2) by reflection in x-axis is ....

3 The ray drawn from the midpoint of a side parallel to another side in a triangle ....

4 The image of the point (– 2 , 1) by rotation with an angle of measure 180° about origin point is ....

5 The length of line segment joining the midpoints of two sides of a triangle equals ....

6 A rhombus is a .... with two adjacent equal sides in length.

2 Choose the correct answer :

1 The perimeter of a square with side length 1 cm. equals .... cm.

(a) 5 (b) 1 (c) 4 (d) 2

2 The number of diagonals in a pentagon is ....

(a) 0 (b) 3 (c) 4 (d) 5

3 The edge length of a cube whose volume is 27 cm^3 is .... cm.

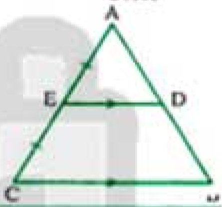
(a) 9 (b) 3 (c) 27 (d) 6

4 In Δ ABC , if m (∠ B) = 90° , AB = 3 cm. , BC = 4 cm. , then AC = ... cm.

(a) 7 (b) 12 (c) 1 (d) 5

5 In the opposite figure : CB : ED = ...

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4

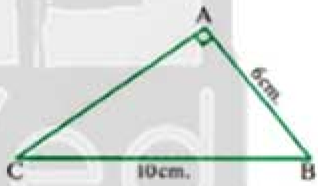


3 [a] In the opposite figure :

ABC is a right-angled triangle at A

, BC = 10 cm. , AB = 6 cm.

Find : The length of $\overline{AC}$

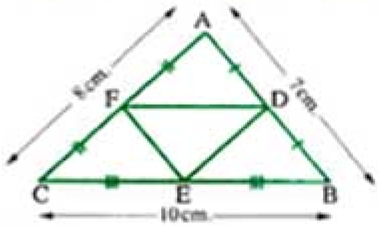


[b] Draw the image of the rectangle XYZL where X (– 5 , 1) , Y (– 5 , 4) , Z (– 1 , 4) , L (– 1 , 1) by reflection in the X-axis.

4 [a] In the opposite figure :

BC = 10 cm. , AB = 7 cm. , AC = 8 cm.

Calculate : The perimeter of the Δ FDE



[b] The ratio between the measures of the angles of a quadrilateral is 2 : 2 : 3 : 5

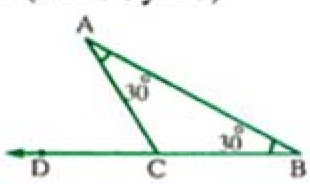
Calculate : The measure of the biggest angle.

5 [a] Find the image of the point A (5 , – 5) by translation (X , y) ⟶ (X – 2 , y – 3)

[b] In the opposite figure :

m (∠ A) = m (∠ B) = 30°

Find showing steps : m (∠ ACD)



8 El-Kalyoubia Governorate

Directorate of Education
Central Maths Supervision

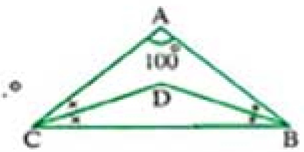
Answer the following questions :

1 Choose the correct answer :

1. If the perimeter of a square is 20 cm. , then the length of its side is .... cm.
   (a) 3 (b) 5 (c) 7 (d) 9
2. The number of diagonals of a pentagon is ....
   (a) 3 (b) 5 (c) 7 (d) 9
3. If ABCD is a parallelogram , m (∠ A) + m (∠ C) = 140° , then m (∠ D) = ...
   (a) 40° (b) 70° (c) 180° (d) 110°
4. The image of the point (– 3 , 5) by rotation about the origin point by an angle of measure 90° is ....
   (a) (– 3 , 5) (b) (5 , – 3) (c) (– 5 , – 3) (d) (3 , – 5)
5. The measure of the exterior angle of the equilateral triangle is ....
   (a) 30° (b) 45° (c) 60° (d) 120°
6. If ABC is a right-angled triangle at B , AB = 6 cm. and BC = 8 cm. , then AC = ...
   (a) 5 cm. (b) 10 cm. (c) 15 cm. (d) 20 cm.

2 Complete :

1. The number of axis of symmetry of a rectangle equals ....
2. The volume of a cube of edge length 0.2 m. equals .... cm^3.
3. The image of (3 , – 2) by reflection in the y-axis is ....
4. In the opposite figure : m (∠ BDC) = ...°
5. The measure of each interior angle of a regular hexagon is ....°



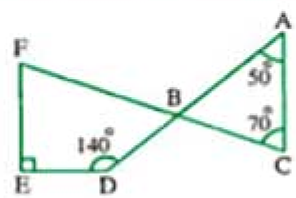
3 [a] In the opposite figure :

$\overline{AD} \cap \overline{FC} = \{B\}$

, m (∠ A) = 50° , m (∠ C) = 70°

, m (∠ D) = 140° , $\overline{EF} \perp \overline{ED}$

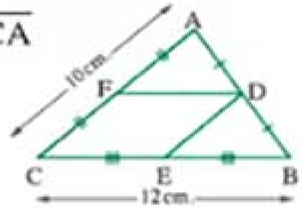
Find : m (∠ F)



[b] In the opposite figure :

ABC is a triangle , D , E , F are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ respectively , BC = 12 cm. , AC = 10 cm.

Find : The perimeter of the quadrilateral DECF

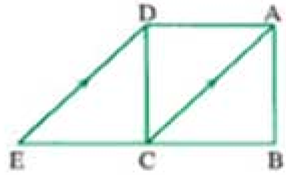


4 **[a] In the opposite figure :**

ABCD is a square , $E \in \overrightarrow{BC}$ and $\overline{AC} // \overline{DE}$

1 **Prove that :** ACED is a parallelogram.

2 **Find :** m (∠ ACE)



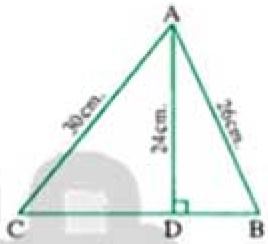
[b] In the opposite figure :

m (∠ ADB) = 90° , AD = 24 cm.

, AB = 26 cm. , AC = 30 cm.

Find : 1 The length of $\overline{BC}$

2 The area of Δ ABC



5 **[a]** Using square lattice , draw $\overline{AB}$ where A (4 , 3) , B (– 1 , 1) , then find the image of $\overline{AB}$ by translation $(x , y) \longrightarrow (x + 2 , y - 1)$

[b] Draw the image of the triangle ABC where A (1 , 1) , B (3 , 4) , C (5 , 2) by reflection in x-axis.

9 El-Gharbia Governorate

Official Language Schools
The Central Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The measure of the exterior angle of the equilateral triangle equals ....°

(a) 30 (b) 45 (c) 60 (d) 120

2 If the image of the point (5 , – 3) by rotation around the origin point is itself , then the measure of the rotation angle equals ....°

(a) 90 (b) 270 (c) 180 (d) 360

3 The image of a square by rotation around the origin point with an angle of measure 90° is a ....

(a) rectangle (b) square (c) rhombus (d) trapezium

4 Any triangle has at least .... acute angle.

(a) 0 (b) 1 (c) 2 (d) 3

5 The measure of the interior angle of the regular hexagon equals ....°

(a) 60 (b) 108 (c) 120 (d) 135

6 The image of the point (– 5 , 0) by reflection on X-axis is ....

(a) (5 , 0) (b) (0 , 5) (c) (– 5 , 0) (d) (0 , – 5)

2 Complete :

1 The line segment joining the midpoints of two sides of a triangle is ....

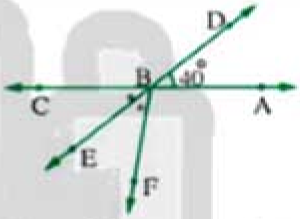
2 If ABCD is a parallelogram in which m (∠ A) = 60° , then m (∠ B) = ...°

3 The image of the point (3 , 2) by reflection in the origin point is ....

4 Each two opposite angles in a parallelogram are ....

5 The rectangle is a parallelogram in which one of its angles is ....

3 [a] **In the opposite figure :**

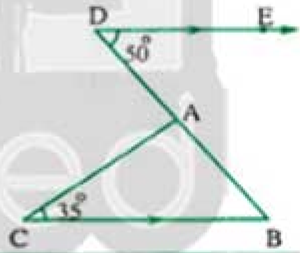


$\overleftrightarrow{AC} \cap \overleftrightarrow{DE} = \{B\}$, m (∠ ABD) = 40°

and $\overrightarrow{BE}$ bisects ∠ CBF

Find with proof : m (∠ ABF)

[b] **In the opposite figure :**

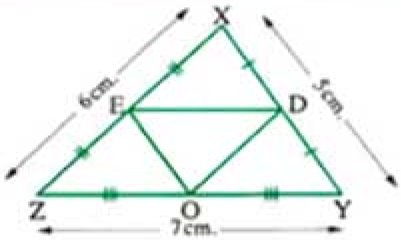


$\overrightarrow{DE} \parallel \overrightarrow{CB}$, m (∠ D) = 50° , m (∠ C) = 35°

Find with proof :

m (∠ B) , m (∠ BAC)

4 [a] **In the opposite figure :**



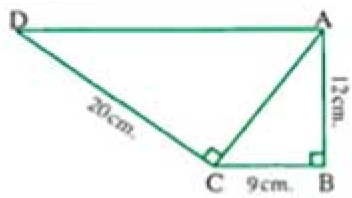
XYZ is a triangle in which XY = 5 cm.

, XZ = 6 cm. , YZ = 7 cm.

Find with proof : The perimeter of Δ DOE

[b] On the square lattice , draw the triangle whose vertices are A (4 , 4) , B (4 , 2) , C (1 , 2) , then find its image by the translation (– 4 , – 2)

5 [a] **In the opposite figure :**



m (∠ B) = m (∠ ACD) = 90°

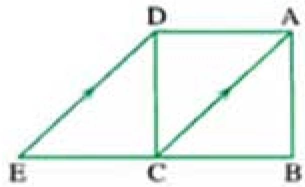
, AB = 12 cm. , BC = 9 cm. , CD = 20 cm.

Find : The length of $\overline{AD}$

[b] In the opposite figure :

ABCD is a square , $E \in \overrightarrow{BC}$, $\overline{AC} \parallel \overline{DE}$

Prove that : ACED is a parallelogram.



10 El-Dakahlia Governorate

Directorate of Education
Maths Supervision

Answer the following questions :

1 Choose the correct answer :

1 The sum of the measures of an exterior angles of a triangle equals ....

(a) 90° (b) 180° (c) 270° (d) 360°

2 The side length of a rhombus whose perimeter 36 cm. equals ....

(a) 144 cm. (b) 6 cm. (c) 9 cm. (d) 18 cm.

3 The measure of each angle of regular octagon equals ....

(a) 60° (b) 108° (c) 120° (d) 135°

4 The parallelogram whose diagonals are not perpendicular but equal in length is called ....

(a) rhombus (b) square (c) rectangle (d) trapezium

5 The image of the point (– 3 , 5) by rotation about the origin point with an angle of measure – 270° is ....

(a) (5 , 3) (b) (– 5 , 3) (c) (3 , 5) (d) (– 5 , – 3)

6 The number of the acute angles in any triangle equals at least ....

(a) zero (b) 1 (c) 2 (d) 3

2 Complete each of the following :

1 The measure of any of the exterior angles of an equilateral triangle equals ....

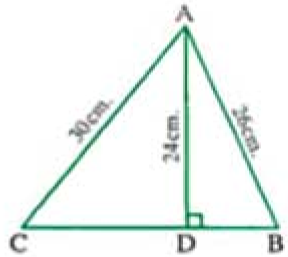
2 If Δ ABC is right-angled triangle at B , m ($\angle$ C) = 55° , then m ($\angle$ A) = ... °

3 The parallelogram whose perimeter 32 cm. and the length of one of its sides is 7 cm. , then the length of its adjacent side equals ....

4 If the image of the point (– 5 , 4) by a translation is (1 , 4) , then the image of the point (3 , – 6) by the same translation is ....

5 The rectangle is a parallelogram in which one of its angles is ....

3 **[a] In the opposite figure :**



ABC is a triangle in which $D \in \overline{BC}$

, m (∠ ADB) = 90° , AB = 26 cm.

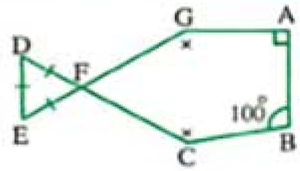
, AD = 24 cm. , AC = 30 cm.

Find : **1** The length of $\overline{BC}$

2 The area of the triangle ABC

[b] Draw the image of triangle ABC where A (2 , 2) , B (5 , 2) , C (3 , 5) by reflection in y-axis.

4 **[a] In the opposite figure :**

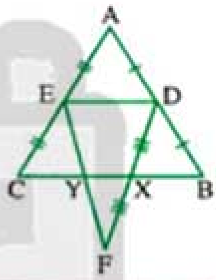


DFE is an equilateral triangle

, m (∠ A) = 90° , m (∠ B) = 100° , m (∠ C) = m (∠ G)

Find : m (∠ C)

[b] In the opposite figure :

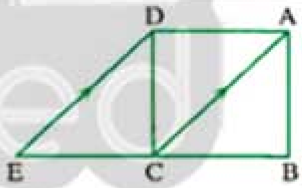


ABC is a triangle in which D , E are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively

, CB = 12 cm. and DX = XF

Find : The length of $\overline{XY}$

5 **[a] In the opposite figure :**



ABCD is a square , $E \in \overrightarrow{BC}$, $\overrightarrow{AC} \parallel \overrightarrow{DE}$

Prove that : ACED is a parallelogram **, then find :** m (∠ ACE)

[b] Draw Δ OBC on a square lattice where

O (0 , 0) , B (4 , 0) , C (0 , 3) , then find its image by rotation about the origin point with an angle of measure 180°

11 Suez Governorate

Directorate of Education
Math Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The two diagonals are equal in length and not perpendicular in a ....

(a) parallelogram (b) rectangle (c) square (d) rhombus

2 The image of (1 , 3) by translation (4 , 2) is ....

(a) (3 , 1) (b) (5 , 5) (c) (5 , 1) (d) (5 , – 5)

3 The measure of an exterior angle in an equilateral triangle is ....

(a) 30° (b) 45° (c) 60° (d) 120°

4 In a parallelogram if the two adjacent sides are equal in length , then the shape is a ....

(a) square (b) rhombus (c) rectangle (d) trapezium

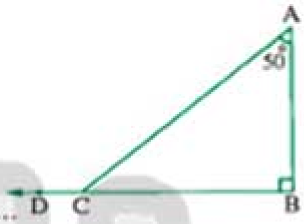
5 The number of axes of symmetry of the equilateral triangle is ....

(a) 0 (b) 1 (c) 2 (d) 3

6 The sum of measures of the interior angles in a triangle is ....°

(a) 90 (b) 360 (c) 180 (d) 540

2 Complete :



1 In the opposite figure : m (∠ ACD) = ...°

2 The ray drawn from the midpoint of one side of a triangle parallel to another side ....

3 The image of the point (1 , – 2) by reflection in X-axis is ....

4 The sum of measures of the interior angles of the pentagon is ....°

5 If ABCD is a rhombus , then .... ⊥ ....

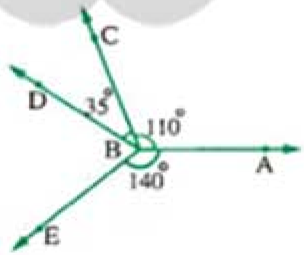
3 [a] In the opposite figure :

B ∈ $\overleftrightarrow{AC}$, m (∠ CBE) = 116°

, $\overrightarrow{BD}$ bisects ∠ ABE

Find : m (∠ ABD)

[b] In the opposite figure :

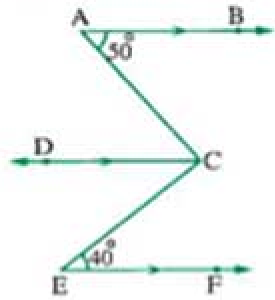


m (∠ ABC) = 110° , m (∠ CBD) = 35°

, m (∠ ABE) = 140°

Find : m (∠ DBE)

4 [a] In the opposite figure :



$\overrightarrow{AB}$ // $\overrightarrow{CD}$ // $\overrightarrow{EF}$

, m (∠ A) = 50° , m (∠ E) = 40°

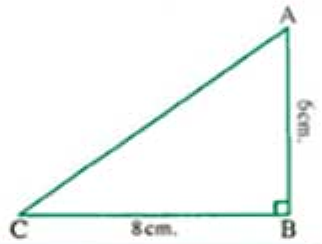
Find by proof : m (∠ ACE)

[b] In the opposite figure :

Δ ABC is right-angled at B

, AB = 6 cm. , BC = 8 cm.

Find : The length of $\overline{AC}$



5 **[a] Complete :**

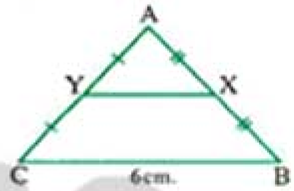
1 The image of (3 , 2) by rotation about the origin point with an angle of measure 90° is ……………

2 The image of (3 , 2) by reflection in the origin point is ……………

3 The image of (3 , 2) by rotation about the origin point with an angle of measure 180° is ……………

4 In the opposite figure :

XY = …………… cm.

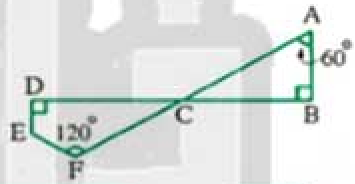


[b] In the opposite figure :

$\overline{AB}$, $\overline{DE}$ are perpendicular to $\overline{BD}$

, $\overline{BD} \cap \overline{AF} = \{C\}$, m ($\angle$ A) = 60° , m ($\angle$ CFE) = 120°

Find with proof : m ($\angle$ E)



12 Port Said Governorate

East Educational Zone
Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The image of the point (2 , – 5) by reflection in x-axis is ……………

(a) (2 , – 5) (b) (2 , 5) (c) (– 2 , – 5) (d) (5 , 2)

2 The measure of each interior angle of a regular hexagon equals ……………

(a) 60° (b) 108° (c) 120° (d) 135°

3 The two diagonals are equal in length and not perpendicular in the ……………

(a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

4 The sum of the measures of the interior angles of a triangle equals ……………

(a) 90° (b) 180° (c) 270° (d) 360°

5 The image of the point (3 , – 2) by reflection in the y-axis is the point ……………

(a) (3 , 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (– 2 , 3)

6 The measure of the exterior angle of the equilateral triangle is ……………

(a) 30° (b) 45° (c) 60° (d) 120°

2 **Complete each of the following :**

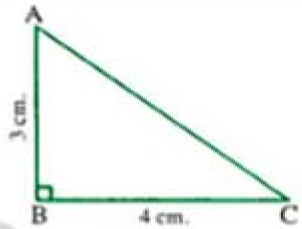
1. The length of the line segment that joins two midpoints of two sides of a triangle equals .... the length of the third side.
2. The image of the point A (– 3 , 2) by reflection in the origin point is the point ....
3. The sum of the measures of the angles of a quadrilateral equals ....°
4. If ABCD is a parallelogram in which m (∠ A) = 60° , then m (∠ B) = ...°
5. The length of the side of a rhombus whose perimeter is 24 cm. equals .... cm.

3 **[a] In the opposite figure :**

ABC is a right-angled triangle at B

, AB = 3 cm. , BC = 4 cm.

Find : The length of $\overline{AC}$



[b] On the lattice , find the image of the triangle LMN where L (– 4 , – 1) , M (– 1 , – 3) , N (0 , – 1) by reflection in the x-axis.

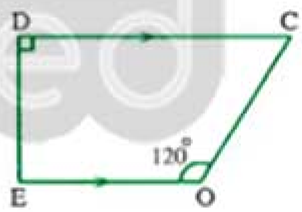
4 **[a]** Using the square lattice , draw $\overline{AB}$ where A (4 , 3) , B (– 1 , 1) , then find the image of $\overline{AB}$ by reflection in the origin point.

[b] In the opposite figure :

$\overline{DC}$ // $\overline{EO}$, m (∠ D) = 90°

, m (∠ O) = 120°

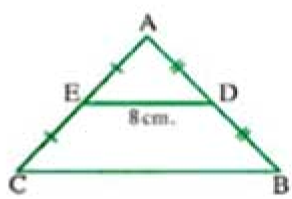
Find : m (∠ C)



5 **[a] In the opposite figure :**

ABC is a triangle in which ED = 8 cm.

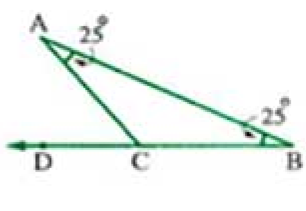
Find : The length of $\overline{CB}$



[b] In the opposite figure :

m (∠ A) = m (∠ B) = 25°

Find : m (∠ ACD)



13 Kafr El-Sheikh Governorate

General Math Supervision

Answer the following questions :

1 Choose the correct answer :

1 The sum of measures of the interior angles of a hexagon equals .... °

(a) 180 (b) 540 (c) 720 (d) 360

2 The two diagonals are equal in length and perpendicular in the ....

(a) trapezium. (b) square. (c) rectangle. (d) parallelogram.

3 In ΔXYZ , if $m(\angle X) > m(\angle Y) + m(\angle Z)$, then $\angle X$ is .... angle.

(a) an acute (b) an obtuse (c) a right (d) a straight

4 The image of $(3 , -5)$ by rotation $R(O , 90°)$ is ....

(a) $(-3 , 5)$ (b) $(-3 , -5)$ (c) $(5 , 3)$ (d) $(5 , -3)$

5 The sum of the measures of the exterior angles of a triangle equals .... °

(a) 60 (b) 120 (c) 270 (d) 360

6 The measure of the exterior angle of the equilateral triangle is .... °

(a) 90 (b) 120 (c) 360 (d) 60

2 Complete :

1 The image of $A(-2 , 3)$ by translation $(x + 3 , y - 2)$ is ....

2 The rhombus with a right angle is ....

3 The image of $(-1 , 2)$ by reflection in the origin point is ....

4 The length of the line segment joining two midpoints of two sides of a triangle is equal to .... the length of the third side.

5 If ABCD is a parallelogram in which BC = 8 cm. and CD = 6 cm. , then its perimeter = ... cm.

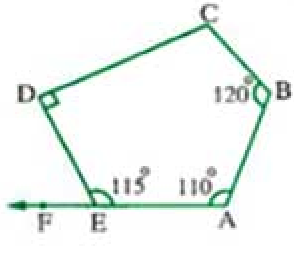
3 [a] In the opposite figure :

ABCDE is a pentagon in which $F \in \overrightarrow{AE}$

, $m(\angle A) = 110°$, $m(\angle B) = 120°$, $m(\angle DEA) = 115°$

, $m(\angle D) = 90°$

Find with proof : $m(\angle C)$

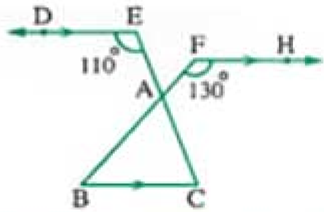


[b] In the opposite figure :

$\overrightarrow{ED} \parallel \overrightarrow{BC} \parallel \overrightarrow{FH}$

, m (∠ E) = 110° , m (∠ F) = 130°

Find with proof : m (∠ BAC)



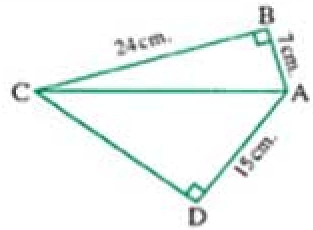
4 **[a] In the opposite figure :**

ABCD is a quadrilateral in which

m (∠ B) = m (∠ D) = 90°

, AB = 7 cm. , BC = 24 cm. and AD = 15 cm.

Find : The length of each of $\overline{AC}$ and $\overline{DC}$

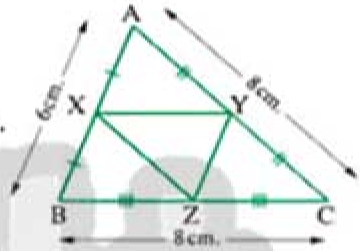


[b] In the opposite figure :

AB = 6 cm. , BC = 8 cm. , AC = 8 cm.

, X , Y , Z are the midpoints of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ respectively.

Find with proof : The perimeter of Δ XYZ



5 **[a]** On a square lattice , draw Δ ABC , where A (2 , 4) , B (2 , 1) and C (6 , 1)

, then draw its image by :

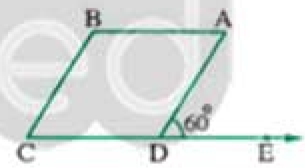
1 reflection in the y-axis. 2 rotation (O , 180°)

[b] In the opposite figure :

ABCD is a parallelogram , E ∈ $\overrightarrow{CD}$

, m (∠ ADE) = 60°

Find with proof : m (∠ B)



14 Beni Suef Governorate

Directorate of official Language Schools
Education Administration

Answer the following questions :

1 **Choose the correct answer :**

1 The measure of the interior angle of a regular polygon of n sides equals ....

(a) $\frac{(n-2) \times 90°}{n}$ (b) $\frac{(n-2) \times 180°}{2}$ (c) $\frac{(n-2) \times 180°}{n}$ (d) 180° × (n − 1)

2 In Δ XYZ , if m (∠ X) = 50° , m (∠ Y) = 100° , then m (∠ Z) = ...

(a) 30° (b) 50° (c) 80° (d) 100°

3 The measure of the exterior angle of the equilateral triangle at any one of its vertices equals ....

(a) 30° (b) 60° (c) 120° (d) 150°

4 If the point $\grave{A}$ (– 4 , 5) is the image of the point A by translation (– 2 , 3) , then the point A is ....

(a) (– 6 , 8) (b) (– 2 , 8) (c) (– 2 , 2) (d) (– 6 , 2)

5 The angle whose measure is 179° , its type is ....

(a) acute (b) right (c) obtuse (d) straight

6 If Δ ABC ≡ Δ XYZ , then ....

(a) AB = YZ (b) BC = XZ (c) YX = CA (d) ZY = CB

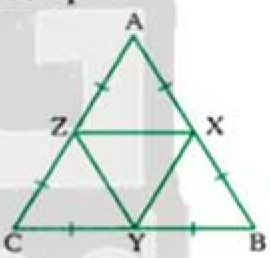
2 Complete :

1 The parallelogram whose two diagonals are .... is called a rectangle.

2 The ray drawn from the midpoint of a side of a triangle parallel to another side ....

3 In the right-angled triangle , the area of the square drawn on the hypotenuse equals ....

4 **In the opposite figure :**



Δ ABC is an equilateral triangle in which

X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively

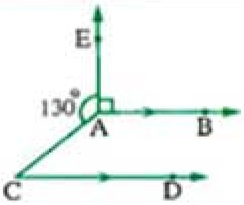
, then the image of Δ XBY

by a translation of magnitude XZ in

the direction of $\overrightarrow{XZ}$ is ....

5 If a straight line intersects two parallel straight lines , then every two corresponding angles are ....

3 [a] In the opposite figure :

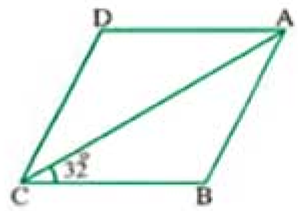


$\overrightarrow{AB}$ // $\overrightarrow{CD}$, m ($\angle$ EAC) = 130°

and m ($\angle$ EAB) = 90°

Find each of : m ($\angle$ BAC) and m ($\angle$ C)

[b] In the opposite figure :



ABCD is a rhombus

, $\overline{AC}$ is a diagonal in it , m ($\angle$ ACB) = 32°

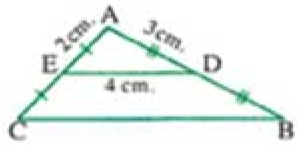
Find : m ($\angle$ D)

4 **[a] In the opposite figure :**

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, AD = 3 cm. , AE = 2 cm. and DE = 4 cm.

Find : The perimeter of the figure DBCE

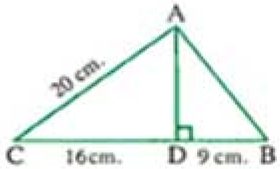


[b] In the opposite figure :

$\overline{AD} \perp \overline{BC}$, BD = 9 cm.

, DC = 16 cm. and AC = 20 cm.

Find : The length of each of $\overline{AD}$ and $\overline{AB}$



5 On a square lattice , draw Δ ABC where

A (– 6 , – 1) , B (– 2 , – 1) and C (– 5 , – 6) , then draw :

1 The image of Δ ABC by reflection in the X-axis.

2 The image of Δ ABC by rotation about the origin point with an angle of measure (– 180°)

15 Souhag Governorate

Math Supervision

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The sum of the measures of the interior angles of a pentagon equals .... °

(a) 108 (b) 180 (c) 540 (d) 720

2 In the .... the two diagonals are perpendicular and not equal in length.

(a) square (b) rectangle (c) rhombus (d) parallelogram

3 The image of the point (– 2 , 3) by reflection in the y-axis is the point ....

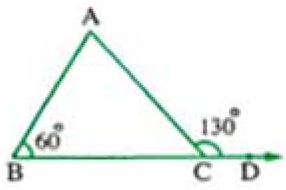
(a) (3 , 2) (b) (– 3 , 2) (c) (2 , 3) (d) (– 3 , – 2)

4 **In the opposite figure :**

m (∠ A) = ...

(a) 40° (b) 50°

(c) 60° (d) 70°



5 The image of the point (– 1 , 3) by translation (4 , – 2) is ....

(a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)

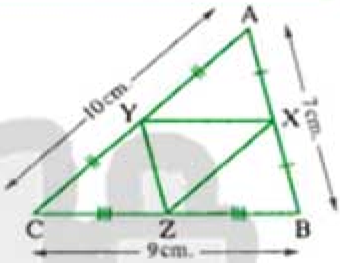
6 In the triangle ABC , if m (∠ A) = 50° , m (∠ B) = 100° , then m (∠ C) = ...

(a) 30° (b) 180° (c) 32° (d) 23°

2 **Complete each of the following :**

1. The length of the line segment joining the midpoints of two sides of a triangle is .... and ....
2. If Δ XYZ is a right-angled triangle at X , XY = 12 cm. and XZ = 9 cm. , then YZ = ... cm.
3. The image of the point (– 1 , 2) by rotation about the origin point with an angle of measure 90° is ....
4. If Δ ABC $\equiv$ Δ XYZ , then m ($\angle$ B) = m ($\angle$ ....)
5. The ray drawn from the midpoint of a side of a triangle parallel to another side ....

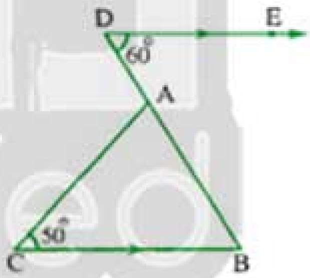
3 **[a] In the opposite figure :**



X , Y , Z are the midpoints of $\overline{AB}$, $\overline{AC}$, $\overline{BC}$ respectively , AB = 7 cm. , BC = 9 cm. , AC = 10 cm.

Find : The perimeter of Δ XYZ

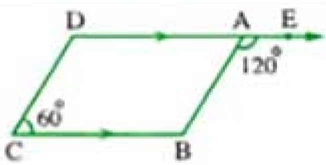
[b] In the opposite figure :



$\overline{DE}$ // $\overline{CB}$, m ($\angle$ D) = 60° , m ($\angle$ C) = 50°

Find : m ($\angle$ DAC)

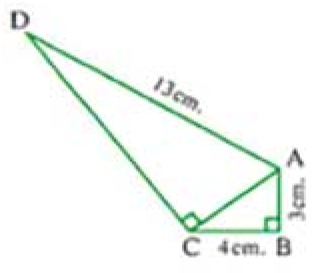
4 **[a] In the opposite figure :**



$\overrightarrow{DA}$ // $\overline{BC}$, m ($\angle$ DCB) = 60° , m ($\angle$ EAB) = 120°

Prove that : ABCD is a parallelogram.

[b] In the opposite figure :



m ($\angle$ B) = m ($\angle$ ACD) = 90° , AB = 3 cm. , BC = 4 cm. and AD = 13 cm.

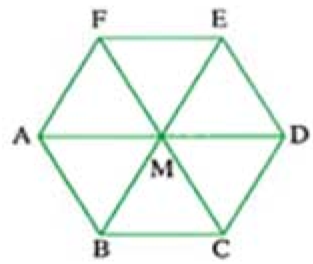
Find : The length of each of $\overline{AC}$, $\overline{CD}$

5 [a] On the square lattice draw ΔABC , where A (1 , 2) , B (4 , 2) , C (4 , – 1) , then find its image by rotation about the origin point with an angle of measure 180°

[b] **In the opposite figure :**

ABCDEF is a regular hexagon

Find the image of ΔABM by :



1. reflection in $\overleftrightarrow{EB}$
2. translation FE in direction of $\overrightarrow{FE}$
3. rotation (M , 120°)
4. reflection in M

Final Examinations

on Geometry and Measurement

Model Examinations of the School Book on Geometry and Measurement

Model 1

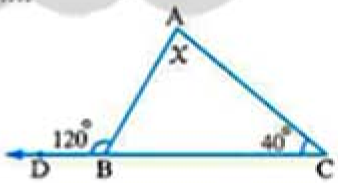
Answer the following questions :

1 Choose the correct answer from those given :

1. Circumference of a circle of radius 7 cm. = ... cm. $\left(\pi = \frac{22}{7}\right)$
   (a) 11 (b) 22 (c) 44 (d) 88
2. The image of the point (– 1 , 3) by translation (4 , – 2) is ....
   (a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)
3. The measure of the exterior angle of the equilateral triangle is ....
   (a) 30° (b) 45° (c) 60° (d) 120°
4. In a parallelogram if the adjacent sides are equal in the length , then the shape is ....
   (a) square. (b) rhombus. (c) rectangle. (d) trapezium.
5. The number of the diagonals of a pentagon is ....
   (a) 3 (b) 5 (c) 7 (d) 9
6. The number of axes of symmetry of an isosceles triangle = ...
   (a) zero (b) 1 (c) 2 (d) 3

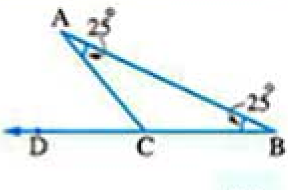
2 Complete the following :

1. The image of the point (2 , 1) by reflection in X-axis is ....
2. **In the opposite figure :**



   X = ...°
3. XYZ is a triangle in which m (∠ Y) = 90° , XY = 3 cm. , XZ = 5 cm. , then YZ = ... cm.
4. ABCD is a parallelogram in which m (∠ A) = 100° , then m (∠ B) + m (∠ D) = ...°
5. The sum of the measures of the interior angles of a triangle = ...°

3 [a] In the opposite figure :

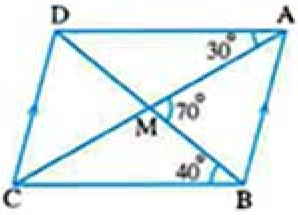


m (∠ A) = m (∠ B) = 25°

Find : m (∠ ACD)

[b] Draw a triangle ABC in which AB = 5 cm. , AC = 3 cm. and m (∠ A) = 40°
, then draw $\grave{C}$ is the image of C under rotation R (A , 40°) , $\grave{B}$ is the image of B under rotation R (A , – 40°)

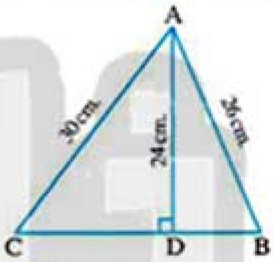
4 **[a] In the opposite figure :**



$\overline{AB} \parallel \overline{DC}$, $\overline{AC} \cap \overline{BD} = \{M\}$,
m (∠ DAC) = 30° , m (∠ DBC) = 40°
and m (∠ AMB) = 70°
Prove that : ABCD is a parallelogram.

[b] Use the translation : $(x , y) \longrightarrow (x + 2 , y + 3)$
to find the point whose image is (2 , 3)

5 **[a] In the opposite figure :**

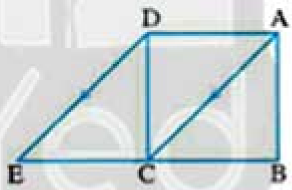


$\overline{AD} \perp \overline{BC}$, if AD = 24 cm. , AB = 26 cm. , AC = 30 cm.

1 **Find :** The length of $\overline{BC}$

2 **Find :** The area of Δ ABC

[b] In the opposite figure :



ABCD is a square , E ∈ $\overrightarrow{BC}$, $\overline{AC} \parallel \overline{DE}$
Prove that : ACED is a parallelogram.

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , then AC = ... cm.

(a) 10 (b) 28 (c) 100 (d) 160

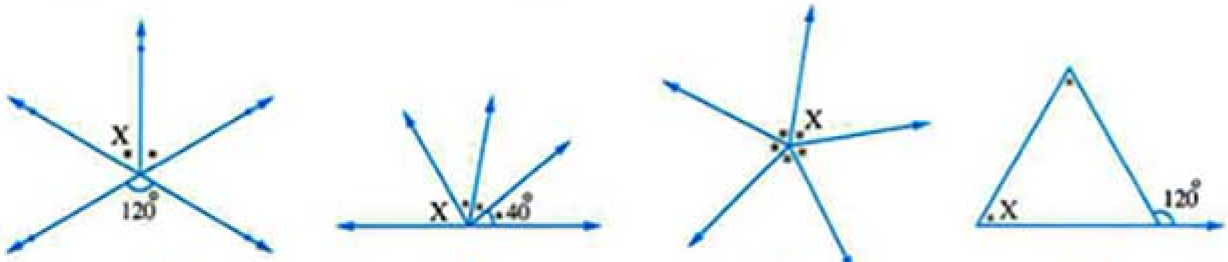
2 The measure of each angle of regular hexagon equals ....

(a) 60° (b) 108° (c) 120° (d) 135°

3 The two diagonals are equal in length and not perpendicular in ....

(a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

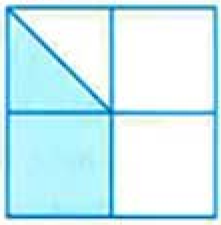
4 In all the following shapes m (∠ X) = 60° except the shape ....



(a) (b) (c) (d)

5 **In the opposite figure :**

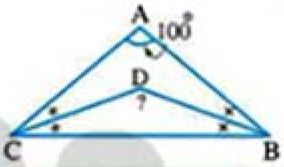
The area of the shaded part from the area of all shape equals ....



(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

6 **In the opposite figure :**

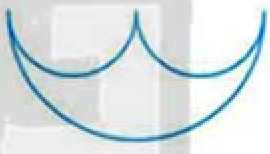
m (∠ BDC) = ...°



(a) 60 (b) 80 (c) 100 (d) 140

2 Complete the following :

1 **In the opposite figure :**



Semicircle of diameter 14 cm. and two semicircles the diameter of each is 7 cm.

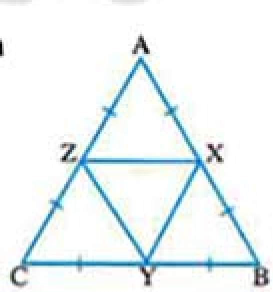
, then the perimeter of the figure equals .... cm. $\left(\pi = \frac{22}{7}\right)$

2 The image of the point (2 , 3) by translation MN , in direction $\overline{MN}$, where M (2 , – 1) , N (5 , 1) is ....

3 The volume of a cube of side length 1.2 m. = ... cm^3.

4 The ray drawn parallel to one side of a triangle and passing through the midpoint of another side ....

5 **In the opposite figure :**

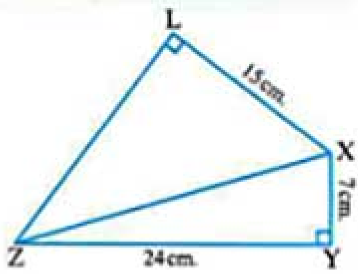


The image of the triangle XBY by translation XZ in direction $\overrightarrow{XZ}$ is ....

3 [a] In the opposite figure :

XYZL is a quadrilateral in which m (∠ Y) = m (∠ L) = 90° , XY = 7 cm. , YZ = 24 cm. , XL = 15 cm.

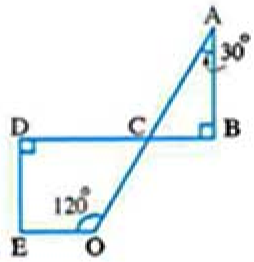
Find : The length of each of $\overline{XZ}$ and $\overline{LZ}$



[b] Using the square lattice , draw $\overline{AB}$ where A (4 , 3) , B (– 1 , 1)
then find the image of $\overline{AB}$ by translation $(x , y) \longrightarrow (x + 2 , y - 1)$

4 [a] Draw the image of triangle ABC where A (1 , 1) , B (3 , 4) , C (5 , 2) by reflection in x-axis.

[b] **In the opposite figure :**



$\overline{AB}$ and $\overline{ED}$ are perpendicular to $\overline{BD}$

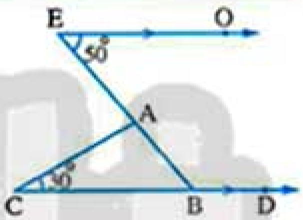
, $\overline{BD} \cap \overline{AO} = \{C\}$,

m (∠ A) = 30°

, m (∠ EOC) = 120° ,

Find : m (∠ E)

5 [a] **In the opposite figure :**

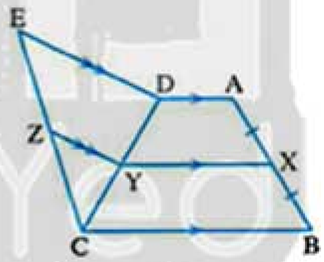


$\overrightarrow{EO} \parallel \overrightarrow{CD}$, m (∠ E) = 50°

, m (∠ C) = 30° ,

Find the measures of angles of Δ ABC , m (∠ ABD)

[b] **In the opposite figure :**



X is the midpoint of $\overline{AB}$

, Y ∈ $\overline{CD}$, Z ∈ $\overline{CE}$

, $\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, $\overline{YZ} \parallel \overline{DE}$

Is CZ = ZE ? giving reason

Model examination for the merge students

Answer the following questions :

1 Choose the correct answer :

1. The sum of the measures of the interior angles of a triangle = ... °

   (a) 90 (b) 360 (c) 180 (d) 540

2. The image of the point (3 , – 2) by reflection in the y-axis is the point ....

   (a) (3 , 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (– 2 , 3)

3. The diagonals are equal and perpendicular in ....

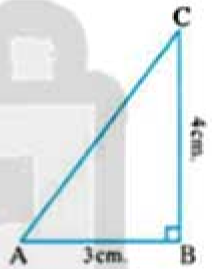
   (a) rhombus. (b) square. (c) recangle. (d) parallelogram.

4. **In the opposite figure :**

   AC = ... cm.

   (a) 5 (b) 7

   (c) 25 (d) 625

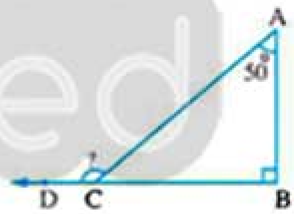


5. **In the opposite figure :**

   m (∠ ACD) = ... °

   (a) 40 (b) 140

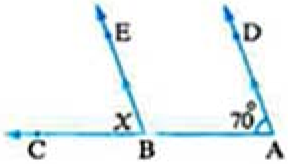
   (c) 90 (d) 50



2 Complete each of the following :

1. The length of the line segment that joins two midpoints of two sides of a triangle equals .... the length of the third side.
2. The rectangle is a parallelogram in which one of it's angles is ....
3. The length of the side of a rhombus whose perimeter is 24 cm. equals .... cm.
4. The image of the point A (– 3 , 2) by reflection in the origin point is the point À (............... , ...)

5 **In the opposite figure :**



$x =$ ....°

3 Put (✓) or (✗) :

1 The image of the point (4 , 3) by reflection in the x-axis is the point (3 , – 4) ( )

2 If ABC is a right-angled triangle at B , then $(AB)^2 = (BC)^2 + (AC)^2$ ( )

3 The pentagon has 5 diagonals. ( )

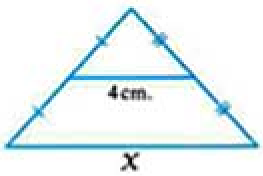
4 ABCD is a parallelogram , in which m (∠ A) = 70° , then m (∠ C) = 110° ( )

5 Any triangle contains at least two acute angles. ( )

4 Join from the column (A) to the suitable in the column (B) :

Column (A)	Column (B)
1 The sum of the measures of the interior angles of a quadrilateral = ...	• 120°
2 The measure of each angle of a regular hexagon = ...	• 360°
3 The image of the point (3 , 2) by translation (1 , – 2) is the point	• (– 1 , – 3)
4 The image of the point (1 , 3) by rotation about the origin point , of angle of measure 180° is the point (.............. , ...)	• 45
5 The diagonal of the square divides the angle of the vertex into two angles , the measure of each = ...°	• (4 , 0)

5 Find the value of x :



$x =$ .... cm.

Fig. (1)

$x =$ ....°

Fig. (2)

Schools Examinations on Geometry and Measurement

1 Cairo Governorate

Hadayek El Koba Educational Zone

Answer the following questions : (Calculators are Permitted)

1 Choose the correct answer :

1. The measure of the interior angle of the regular octagon equals ....
   (a) 1080° (b) 180° (c) 135° (d) 108°
2. If ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 140° , then m (∠ B) = ...
   (a) 40° (b) 110° (c) 70° (d) 60°
3. In Δ ABC , if m (∠ B) = 90° , AB = 20 cm. , AC = 25 cm. , then BC = ... cm.
   (a) 625 (b) 12 (c) 15 (d) 225
4. If the perimeter of a square is 20 cm. , then its area equals .... cm^2.
   (a) 25 (b) 100 (c) 5 (d) 16
5. The image of the point (– 3 , 5) by rotation about the origin point with an angle of measure 90° is ....
   (a) (– 5 , 3) (b) (– 5 , – 3) (c) (5 , 3) (d) (3 , – 5)
6. The image of the point (– 1 , 3) by translation (4 , – 2) is ....
   (a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)

2 Complete the following :

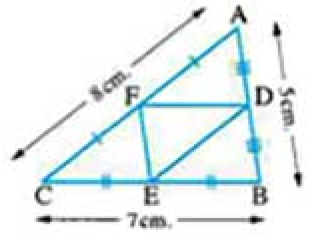
1. The line segment joining the midpoints of two sides of a triangle is parallel to ....
2. The image of the point (2 , – 4) by reflection in y-axis is ....
3. The number of axes of symmetry of a rectangle is ....
4. The measure of the exterior angle of an equilateral triangle equals ....°
5. The volume of a cube of edge length 6 cm. is .... cm^3.

3 [a] In the opposite figure :

ABC is a triangle , D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ respectively

where AB = 5 cm. , BC = 7 cm. and CA = 8 cm.

Calculate : The perimeter of Δ DEF



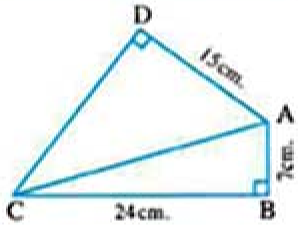
[b] On square lattice , draw the square ABCD where A (1 , 2) , B (– 2 , 2) , C (– 2 , 5) and D (1 , 5) , then draw its image by reflection in X-axis.

4 **[a] In the opposite figure :**

m (∠ B) = m (∠ D) = 90°

AD = 15 cm. , AB = 7 cm. and BC = 24 cm.

Find : The length of each of $\overline{AC}$, $\overline{DC}$



[b] Draw Δ ABC where A (1 , 5) , B (3 , 1) and C (5 , 3)

, then draw its image by rotation about the origin point with an angle of measure 180°

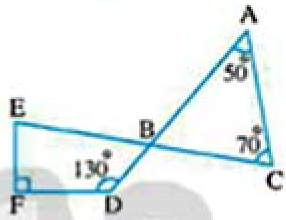
5 **[a] In the opposite figure :**

$\overline{CE} \cap \overline{AD} = \{B\}$

, m (∠ A) = 50° , m (∠ C) = 70°

, m (∠ D) = 130° and m (∠ F) = 90°

Find : m (∠ E)



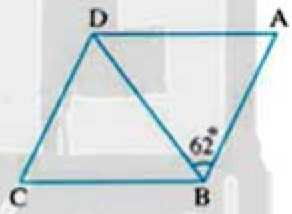
[b] In the opposite figure :

ABCD is a rhombus

, $\overline{BD}$ is a diagonal in it

, m (∠ ABD) = 62°

Find with proof : m (∠ A)



2 Cairo Governorate

East Nasr City Zone
Manaret Heliopolis School

Answer the following questions :

1 **Complete :**

1. The sum of measures of the exterior angles of the heptagon is ....°
2. If two lines intersect , then each two vertically opposite angles are ....
3. In Δ ABC , if m (∠ A) + m (∠ C) = m (∠ B) , then m (∠ B) = ...°
4. The image of (2 , 3) by translation $(X , y) \longrightarrow (X + 4 , y - 2)$ is ....
5. The length of the line segment joining two midpoints of two sides of a triangle equals .... the length of the third side.

2 Choose the correct answer :

1 ABCD is a parallelogram in which m (∠ A) = 80° , then m (∠ C) = ...

(a) 80° (b) 100° (c) 120° (d) 60°

2 If the image of the point (5 , – 3) by rotation about the origin point is itself , then the measure of the rotation angle is ....°

(a) 90 (b) 180 (c) 270 (d) 360

3 The line segment joining two midpoints of two sides of a triangle is .... the third side.

(a) intersecting (b) parallel to (c) perpendicular to (d) congruent to

4 The sum of measures of the accumulative angles at a point is ....°

(a) 90 (b) 180 (c) 270 (d) 360

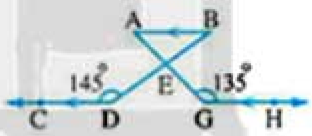
5 If ABC is a right-angled triangle at B and AB = 4 cm. , BC = 3 cm. , then AC = ... cm.

(a) 16 (b) 25 (c) 9 (d) 5

6 The sum of measures of the interior angles of a hexagon is ....°

(a) 360 (b) 540 (c) 720 (d) 120

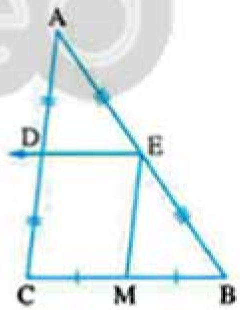
3 [a] In the opposite figure :



ABE is a triangle , $\overline{AB} \parallel \overline{GH} \parallel \overline{CD}$, m (∠ CDB) = 145° , m (∠ AGH) = 135°

Calculate : m (∠ DEG)

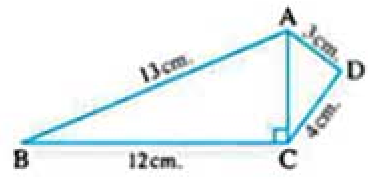
[b] In the opposite figure :



ABC is a triangle in which E , M and D are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively

Prove that : DEMC is a parallelogram.

4 [a] In the opposite figure :



m (∠ ACB) = 90° , AB = 13 cm. , AD = 3 cm. , CD = 4 cm. , BC = 12 cm.

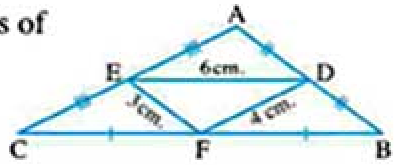
Find : 1 The length of $\overline{AC}$

2 The perimeter of the figure ABCD

[b] In the opposite figure :

ABC is a triangle in which : D , F and E are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively such that : DF = 4 cm. , FE = 3 cm. , DE = 6 cm.

Calculate : The perimeter of Δ ABC



5 [a] Using the lattice , find the images of the points :

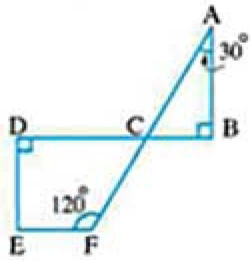
A (– 4 , 1) , B (0 , 4) and C (– 2 , 2) by reflection in X-axis.

[b] In the opposite figure :

$\overline{AB} \perp \overline{BC}$, $\overline{ED} \perp \overline{CD}$, $\overline{BD} \cap \overline{AF} = \{C\}$

, m ($\angle$ A) = 30° , m ($\angle$ F) = 120°

Find with proof : 1 m ($\angle$ ACB) 2 m ($\angle$ E)



3 Giza Governorate

El-haram Directorate
Fadl language School

Answer the following questions :

1 **Complete :**

1 The image of the point (2 , – 1) by reflection in X-axis is ....

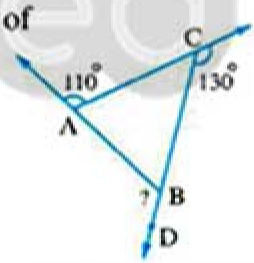
2 ABCD is a parallelogram in which m ($\angle$ A) = 60° , then m ($\angle$ B) = ...°

3 The measure of each angle of the regular hexagon equals ....°

4 The length of the line segment that joins two midpoints of two sides of a triangle equals .... the length of the third side.

5 In the opposite figure :

m ($\angle$ ABD) = ...°



2 **Choose the correct answer :**

1 The number of the diagonals of a pentagon is ....

(a) 3 (b) 5 (c) 7 (d) 9

2 The image of the point (2 , 5) by rotation about the origin point with an angle of measure 90° is ....

(a) (– 5 , 2) (b) (5 , – 2) (c) (– 2 , – 5) (d) (– 2 , 5)

3 ABC is a right-angled triangle at A , then $(AC)^2 = (BC)^2$ .... $(AB)^2$

(a) + (b) × (c) ÷ (d) –

4 The number of axes of symmetry of the rectangle is ....

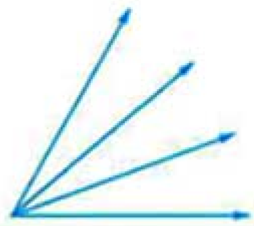
(a) 1 (b) 2 (c) 3 (d) 4

5 If the total surface area of a cuboid = 148 cm^2 and its lateral surface area = 118 cm^2 , then its base area = ... cm^2

(a) 30 (b) 15 (c) 45 (d) 266

6 Number of acute angles in the opposite figure is ....

(a) 3 (b) 4 (c) 5 (d) 6



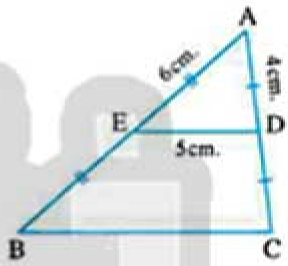
3 **[a] In the opposite figure :**

E and D are midpoints of $\overline{AB}$ and $\overline{AC}$

, AE = 6 cm. , ED = 5 cm.

, AD = 4 cm.

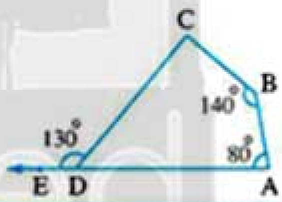
, Calculate the perimeter of Δ ABC



[b] In the opposite figure :

$D \in \overrightarrow{AE}$

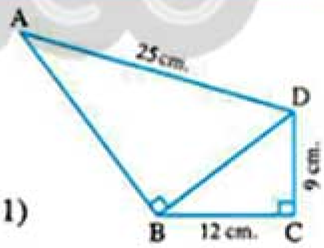
Find : m ($\angle$ C)



4 **[a] In the opposite figure :**

m ($\angle$ C) = m ($\angle$ ABD) = 90°

Find : The length of each of $\overline{BD}$ and $\overline{AB}$

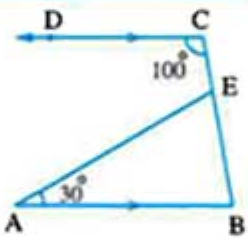


[b] On the square lattice , draw Δ ABC where A (0 , 5) , B (2 , 1) , C (4 , 7) , then draw the image of Δ ABC by reflection in y-axis.

5 **[a] In the opposite figure :**

$\overrightarrow{BA} \mathbin{//} \overrightarrow{CD}$, m ($\angle$ C) = 100° , m ($\angle$ A) = 30°

Find : m ($\angle$ AEC)



[b] On the square lattice , draw $\overline{AB}$ where A (4 , 3) , B (– 1 , 1) , then draw the image of $\overline{AB}$ by translation : $(x , y) \longrightarrow (x + 2 , y - 1)$

4 Giza Governorate

Experimental Schools Math inspector

Answer the following questions :

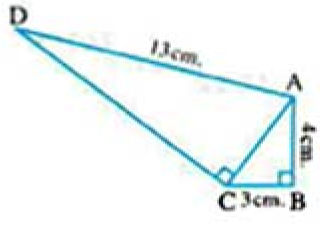
1 Complete the following :

1. The image of the point (5 , – 3) under rotation of an angle of measure 90° about the origin point is ....
2. The ray which is drawn from the midpoint of a side in a triangle parallel to another side .... the third side.
3. In the rhombus the two diagonals are ....
4. The sum of measures of the exterior angles of any convex polygon equals ....°
5. In Δ ABC , if m ($\angle$ A) + m ($\angle$ B) = m ($\angle$ C) , then Δ ABC is ....

2 Choose the correct answer :

1. If the measure of an interior angle of a regular polygon is 135° , then the number of its sides is ....
   (a) 6 (b) 4 (c) 7 (d) 8
2. The image of the point (– 1 , 4) by the translation $(x , y) \longrightarrow (x + 3 , y – 2)$ followed by reflection in the x-axis is ....
   (a) (2 , 2) (b) (– 2 , 2) (c) (– 2 , –2) (d) (2 , – 2)
3. ABCD is a parallelogram in which m ($\angle$ A) + m ($\angle$ C) = 110° , then m ($\angle$ B) = ...°
   (a) 125 (b) 80 (c) 100 (d) 110
4. The concave polygon should have .... angle.
   (a) an acute (b) a right (c) an obtuse (d) a reflex
5. The image of the point .... is itself by reflection in y-axis.
   (a) (0 , 3) (b) (3 , 0) (c) (3 , 3) (d) (– 3 , 3)
6. The identity rotation about the origin point with an angle of measure ....°
   (a) 90 (b) 180 (c) 360 (d) – 90

3 [a] In the opposite figure :



ABCD is a quadrilateral in which

m ($\angle$ B) = m ($\angle$ ACD) = 90°

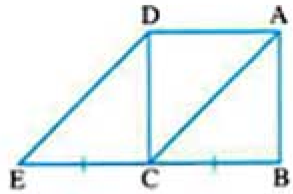
, AB = 4 cm. , BC = 3 cm. and AD = 13 cm.

Find : AC and DC

[b] In the opposite figure :

ABCD is a square , $E \in \overrightarrow{BC}$

such that BC = CE



1 **Prove that :** The figure ACED is a parallelogram.

2 **Find :** m (∠ E)

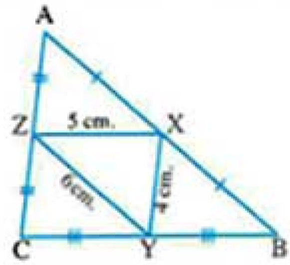
4 **[a] In the opposite figure :**

Δ ABC in which X , Y and Z are the midpoints of

$\overline{AB}$, $\overline{BC}$, $\overline{CA}$ respectively

, XZ = 5 cm. , XY = 4 cm. , YZ = 6 cm.

Find with proof : The perimeter of Δ ABC

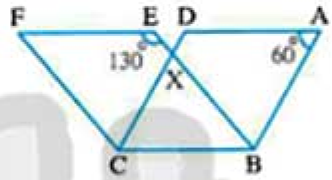


[b] In the opposite figure :

ABCD and EBCF are two parallelograms

, m (∠ BAD) = 60° , m (∠ BEF) = 130°

Find with proof : m (∠ BXC)

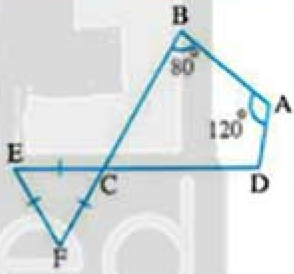


5 **[a] In the opposite figure :**

ABCD is a quadrilateral in which m (∠ A) = 120°

, m (∠ B) = 80° , Δ CEF is an equilateral triangle

Find : m (∠ D)



[b] Using the square lattice draw Δ ABC where :

A (2 , 4) , B (4 , 2) and C (3 , 1) , then map its image by the translation

$(x , y) \longrightarrow (x + 2 , y + 3)$

5 Alexandria Governorate

East Educational Zone
Mathematics Directing

Answer the following questions :

1 **Choose the correct answer :**

1 The measure of the exterior angle of the equilateral triangle = ...

(a) 60° (b) 120° (c) 90° (d) 180°

2 The diagonals are equal in length and not perpendicular in ....

(a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

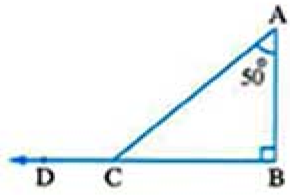
3 The image of the point $(-1 , 3)$ by translation $(4 , -2)$ is ....

(a) $(3 , 1)$ (b) $(3 , -1)$ (c) $(5 , 1)$ (d) $(5 , -5)$

4 The measure of the interior angle of the regular hexagon is ....

(a) 60° (b) 108° (c) 120° (d) 135°

5 **In the opposite figure :**



m (∠ ACD) = ...

(a) 40° (b) 140°

(c) 90° (d) 50°

6 If two adjacent sides in a parallelogram are equal in length , then it is a ....

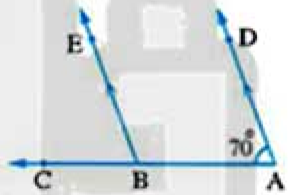
(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

2 Complete :

1 The length of the line segment joining two midpoints of two sides of a triangle ....

2 The angle of measure 89° is .... angle.

3 **In the opposite figure :**



m (∠ EBC) = ...°

4 The image of $(-3 , 2)$ by reflection in origin point is ....

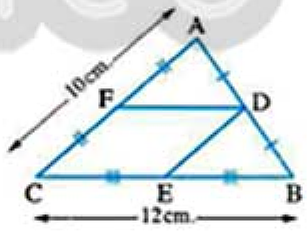
5 A rhombus of perimeter 24 cm. its side length equals ....

3

[a] Draw $\overline{AB}$ such that A $(2 , 1)$, B $(2 , 4)$

, then find its image by translation $(x , y) \longrightarrow (x + 5 , y + 2)$

[b] In the opposite figure :



D , E , F are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

, BC = 12 cm. , AC = 10 cm.

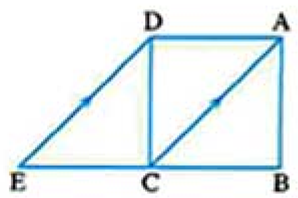
Find : The perimeter of DECF

4

[a] Draw Δ LMN such that L $(-4 , -1)$, M $(-1 , -3)$, N $(0 , -1)$

, then find its image by reflection in x-axis.

[b] In the opposite figure :



ABCD is a square , E ∈ $\overrightarrow{BC}$, $\overline{AC}$ // $\overline{DE}$

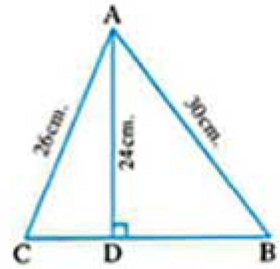
Prove that : ACED is a parallelogram.

5 [a] Find the number of sides of a regular polygon if the measure of its interior angle is 140°

[b] **In the opposite figure :**

Find the length of $\overline{BC}$

and the area of Δ ABC



6 Alexandria Governorate

Middle Educational Zone
Math's Supervision

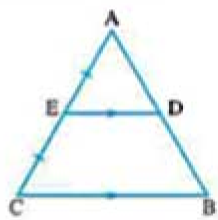
Answer the following questions :

1 **Complete each of the following :**

1. The measure of the straight angle equals ....°
2. The image of the point (4 , – 2) by reflection in X-axis is ....
3. The ray drawn from the midpoint of a side parallel to another side in a triangle ....
4. The image of the point (– 2 , 1) by rotation with an angle of measure 180° about origin point is ....
5. The length of line segment joining the midpoints of two sides of a triangle equals ....
6. A rhombus is a .... with two adjacent equal sides in length.

2 **Choose the correct answer :**

1. The perimeter of a square with side length 1 cm. equals .... cm.
   (a) 5 (b) 1 (c) 4 (d) 2
2. The number of diagonals in a pentagon is ....
   (a) 0 (b) 3 (c) 4 (d) 5
3. The edge length of a cube whose volume is 27 cm^3 is .... cm.
   (a) 9 (b) 3 (c) 27 (d) 6
4. In Δ ABC , if m ($\angle$ B) = 90° , AB = 3 cm. , BC = 4 cm. , then AC = ... cm.
   (a) 7 (b) 12 (c) 1 (d) 5
5. In the opposite figure : CB : ED = ...
   (a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4

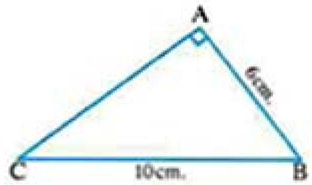


3 [a] **In the opposite figure :**

ABC is a right-angled triangle at A

, BC = 10 cm. , AB = 6 cm.

Find : The length of $\overline{AC}$

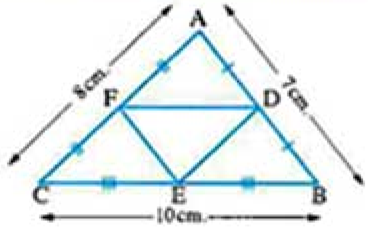


[b] Draw the image of the rectangle XYZL where X (– 5 , 1) , Y (– 5 , 4) , Z (– 1 , 4) , L (– 1 , 1) by reflection in the X-axis.

4 [a] **In the opposite figure :**

BC = 10 cm. , AB = 7 cm. , AC = 8 cm.

Calculate : The perimeter of Δ FDE



[b] The ratio between the measures of the angles of a quadrilateral is 2 : 2 : 3 : 5

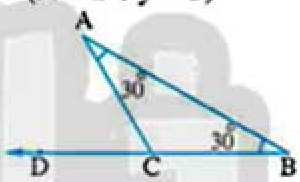
Calculate : The measure of the biggest angle.

5 [a] Find the image of the point A (5 , – 5) by translation $(X , y) \longrightarrow (X - 2 , y - 3)$

[b] **In the opposite figure :**

m (∠ A) = m (∠ B) = 30°

Find showing steps : m (∠ ACD)



7 El-Kalyoubia Governorate

Al-Obour Directorate of Education
Memphis Language School

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The sum of the measures of the interior angles of a triangle equals ……………

(a) 90° (b) 180° (c) 270° (d) 360°

2 The image of the point (– 1 , 3) by translation (4 , – 2) is ……………

(a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)

3 The measure of the exterior angle of the equilateral triangle is ……………

(a) 30° (b) 45° (c) 60° (d) 120°

4 In a parallelogram if the adjacent sides are equal in length , then the shape is a ……………

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

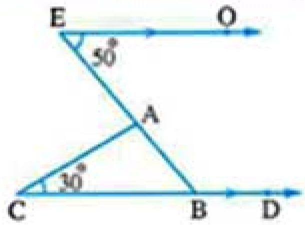
5 The number of the diagonals of a pentagon is ……………

(a) 3 (b) 5 (c) 7 (d) 9

2 **Complete the following :**

1. The line segment joining the midpoints of two sides of a triangle is .... the third side.
2. If Δ XYZ is a right-angled triangle at X , XY = 12 cm. and XZ = 9 cm.
   , then YZ = ...
3. If ABCD is a rhombus , then .... $\perp$ ....
4. The image of the point (– 1 , 2) by rotation about the origin point with an angle of measure 90° is ....
5. The sum of the measures of the accumulative angles about a point = ...

3 **[a] In the opposite figure :**

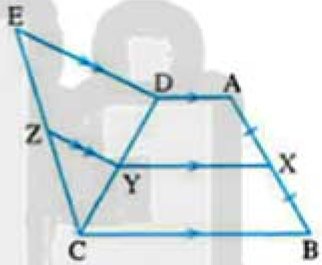


$\overrightarrow{EO} \parallel \overrightarrow{CD}$, m ($\angle$ E) = 50°

, m ($\angle$ C) = 30°

Find : The measures of the angles of Δ ABC , m ($\angle$ ABD)

[b] In the opposite figure :



X is the midpoint of $\overline{AB}$

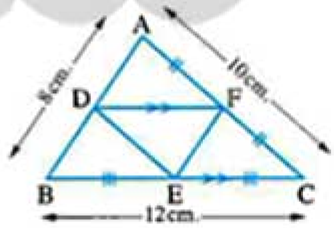
, Y $\in \overline{CD}$, Z $\in \overline{CE}$

, $\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, $\overline{YZ} \parallel \overline{DE}$

Is CZ = ZE ? Giving reason.

4 **[a]** Using the square lattice , draw Δ ABC , where A (– 2 , 3) , B (2 , 3) , C (2 , 6) , then find its image by translation $(x , y) \longrightarrow (x + 2 , y - 1)$

[b] In the opposite figure :



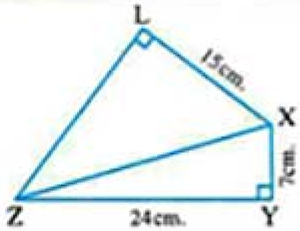
D , E , F are midpoints of

$\overline{AB}$, $\overline{BC}$, $\overline{CA}$ respectively

, AB = 8 cm. , BC = 12 cm. , AC = 10 cm.

Find : The perimeter of Δ DEF

5 **[a] In the opposite figure :**



XYZL is a quadrilateral in which m ($\angle$ Y) = m ($\angle$ L) = 90°

, XY = 7 cm. , YZ = 24 cm. , XL = 15 cm.

Find : The length of each of $\overline{XZ}$ and $\overline{LZ}$

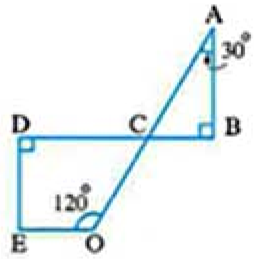
[b] In the opposite figure :

$\overline{AB}$ and $\overline{ED}$ are perpendicular to $\overline{BD}$

, $\overline{BD} \cap \overline{AO} = \{C\}$

, m ($\angle$ A) = 30 , m ($\angle$ EOC) = 120°

Find : m ($\angle$ E)



8 El-Gharbia Governorate

Official Language Schools
The Central Maths Supervision

Answer the following questions :

1 Choose the correct answer :

1 The measure of the exterior angle of the equilateral triangle equals ....°

(a) 30 (b) 45 (c) 60 (d) 120

2 If the image of the point (5 , – 3) by rotation around the origin point is itself , then the measure of the rotation angle equals ....°

(a) 90 (b) 270 (c) 180 (d) 360

3 The image of a square by rotation around the origin point with an angle of measure 90° is a ....

(a) rectangle. (b) square. (c) rhombus. (d) trapezium.

4 Any triangle has at least .... acute angles.

(a) 0 (b) 1 (c) 2 (d) 3

5 The measure of the interior angle of the regular hexagon equals ....°

(a) 60 (b) 108 (c) 120 (d) 135

6 The image of the point (– 5 , 0) by reflection in X-axis is ....

(a) (5 , 0) (b) (0 , 5) (c) (– 5 , 0) (d) (0 , – 5)

2 Complete :

1 The line segment joining the midpoints of two sides of a triangle is ....

2 If ABCD is a parallelogram in which m ($\angle$ A) = 60° , then m ($\angle$ B) = ...°

3 The image of the point (3 , 2) by reflection in the origin point is ....

4 Each two opposite angles in a parallelogram are ....

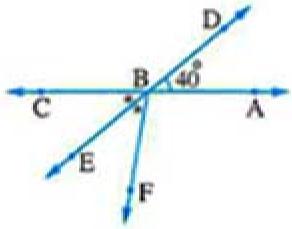
5 The rectangle is a parallelogram in which one of its angles is ....

3 [a] **In the opposite figure :**

$\overleftrightarrow{AC} \cap \overleftrightarrow{DE} = \{B\}$, m (∠ ABD) = 40°

and $\overrightarrow{BE}$ bisects ∠ CBF

Find with proof : m (∠ ABF)

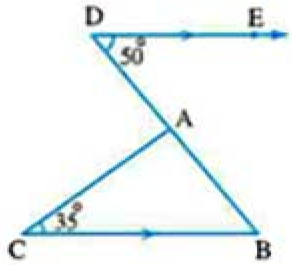


[b] **In the opposite figure :**

$\overrightarrow{DE} \mathbin{/\!/} \overrightarrow{CB}$, m (∠ D) = 50° , m (∠ C) = 35°

Find with proof :

m (∠ B) , m (∠ BAC)

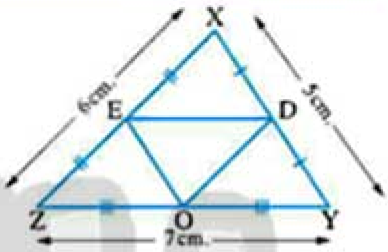


4 [a] **In the opposite figure :**

XYZ is a triangle in which XY = 5 cm.

, XZ = 6 cm. , YZ = 7 cm.

Find with proof : The perimeter of Δ DOE



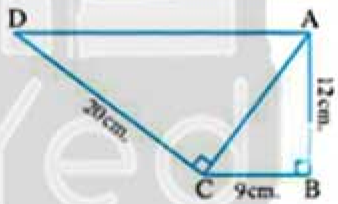
[b] On the square lattice , draw the triangle whose vertices are A (4 , 4) , B (4 , 2) , C (1 , 2) , then find its image by the translation (– 4 , – 2)

5 [a] **In the opposite figure :**

m (∠ B) = m (∠ ACD) = 90°

, AB = 12 cm. , BC = 9 cm. , CD = 20 cm.

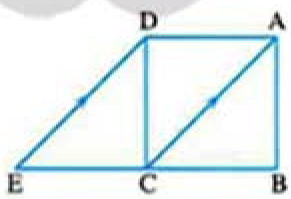
Find : The length of $\overline{AD}$



[b] **In the opposite figure :**

ABCD is a square , E ∈ $\overrightarrow{BC}$, $\overline{AC} \mathbin{/\!/} \overline{DE}$

Prove that : ACED is a parallelogram.



9 El-Dakahlia Governorate

Dakahlia Directorate of Education
Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The sum of measures of the interior angles of the hexagon equals ....

(a) 270° (b) 180° (c) 720° (d) 360°

2 The side length of a rhombus whose perimeter 8 cm. = ...

(a) 32 cm. (b) 2 cm. (c) 16 cm. (d) 12 cm.

3 The measure of each angle of a regular triangle equals ....

(a) 60° (b) 108° (c) 120° (d) 135°

4 The parallelogram whose diagonals are perpendicular and equal in length is called ....

(a) rhombus. (b) square. (c) rectangle. (d) trapezium.

5 The image of the point (– 3 , 5) by rotation about the origin point with an angle of measure 270° is ....

(a) (5 , 3) (b) (– 5 , 3) (c) (3 , 5) (d) (– 5 , – 3)

6 Any triangle has at least .... acute angles.

(a) zero (b) 1 (c) 2 (d) 3

2 Complete each of the following :

1 The measure of the exterior angle of an equilateral triangle equals ....

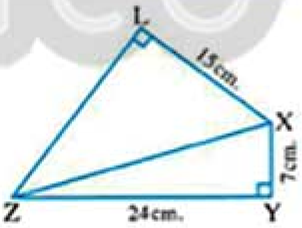
2 If Δ ABC is right-angled triangle at B , m ($\angle$ A) = 55° , then m ($\angle$ C) = ...

3 The parallelogram whose perimeter 28 cm. and the length of one of its sides is 10 cm. , then the length of its adjacent side equals ....

4 If the image of the point A (x , y) by a translation (1 , – 4) is the point (3 , – 2) , then point A is (.......... , ...)

5 The rectangle is a parallelogram in which one of its angles is ....

3 [a] In the opposite figure :



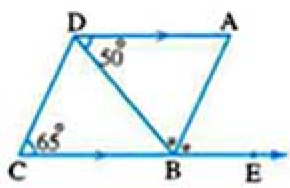
XYZL is quadrilateral in which m ($\angle$ Y) = m ($\angle$ L) = 90°

, XY = 7 cm. , YZ = 24 cm. , XL = 15 cm.

Find : The length of each of $\overline{XZ}$, $\overline{LZ}$

[b] Draw on square lattice the image of triangle ABC where A (0 , 1) , B (3 , 4) , C (5 , 1) by reflection in y-axis.

4 [a] In the opposite figure :

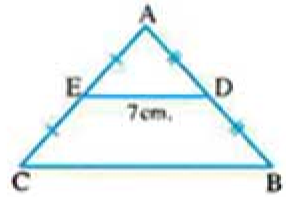


$\overrightarrow{DA}$ // $\overrightarrow{CE}$, $\overrightarrow{BA}$ bisects $\angle$ DBE

, m ($\angle$ ADB) = 50° , m ($\angle$ C) = 65°

Prove that : ABCD is a parallelogram.

[b] In the opposite figure :



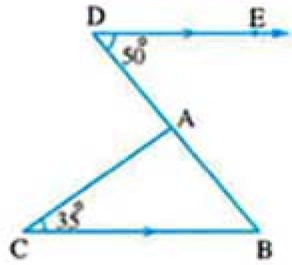
ABC is a triangle in which

D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, $\overline{DE} = 7$ cm.

Find : The length of $\overline{BC}$

5 **[a] In the opposite figure :**



$\overrightarrow{DE} \mathbin{/\mkern-6mu/} \overrightarrow{CB}$

, m (∠ D) = 50°

, m (∠ C) = 35°

Find : m (∠ B) and m (∠ BAC)

[b] Draw Δ OBC on a square lattice where O (0 , 0) , B (3 , 0) , C (0 , 4) , then find its image by rotation about the origin point with an angle of measure (– 180°)

10 Suez Governorate

Directorate of Education Math Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The two diagonals are equal in length and not perpendicular in a ....

(a) parallelogram. (b) rectangle. (c) square. (d) rhombus.

2 The image of (1 , 3) by translation (4 , 2) is ....

(a) (3 , 1) (b) (5 , 5) (c) (5 , 1) (d) (5 , – 5)

3 The measure of an exterior angle in an equilateral triangle is ....°

(a) 30 (b) 45 (c) 60 (d) 120

4 In a parallelogram if the two adjacent sides are equal in length , then the shape is a ....

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

5 The number of axes of symmetry of the equilateral triangle is ....

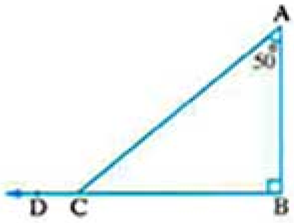
(a) 0 (b) 1 (c) 2 (d) 3

6 The sum of measures of the interior angles of a triangle is ....°

(a) 90 (b) 360 (c) 180 (d) 540

2 Complete :

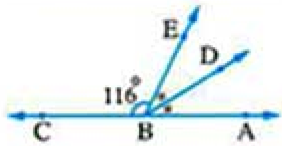
1. In the opposite figure : m (∠ ACD) = ……………°



2. The ray drawn from the midpoint of one side of a triangle parallel to another side ……………
3. The image of the point (1 , – 2) by reflection in X-axis is ……………
4. The sum of measures of the interior angles of the pentagon is ……………°
5. If ABCD is a rhombus , then …………… ⊥ ……………

3

[a] In the opposite figure :

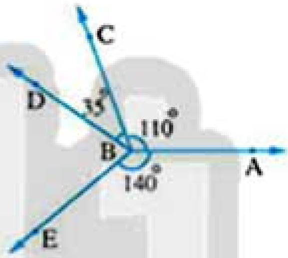


B ∈ $\overleftrightarrow{AC}$, m (∠ CBE) = 116°

, $\overrightarrow{BD}$ bisects ∠ ABE

Find : m (∠ ABD)

[b] In the opposite figure :



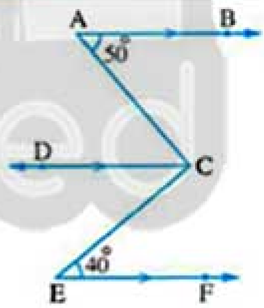
m (∠ ABC) = 110° , m (∠ CBD) = 35°

, m (∠ ABE) = 140°

Find : m (∠ DBE)

4

[a] In the opposite figure :

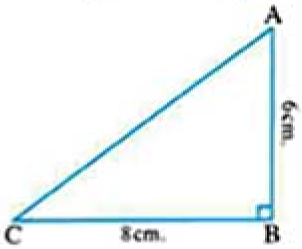


$\overrightarrow{AB}$ // $\overrightarrow{CD}$ // $\overrightarrow{EF}$

, m (∠ A) = 50° , m (∠ E) = 40°

Find with proof : m (∠ ACE)

[b] In the opposite figure :



Δ ABC is right-angled at B

, AB = 6 cm. , BC = 8 cm.

Find : The length of $\overline{AC}$

5

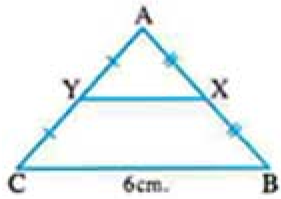
[a] Complete :

1. The image of (3 , 2) by rotation about the origin point with an angle of measure 90° is ……………
2. The image of (3 , 2) by reflection in the origin point is ……………

3 The image of (3 , 2) by rotation about the origin point with an angle of measure 180° is ....

4 In the opposite figure :

XY = ... cm.

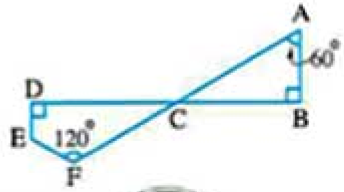


[b] In the opposite figure :

$\overline{AB}$, $\overline{DE}$ are perpendicular to $\overline{BD}$

, $\overline{BD} \cap \overline{AF} = \{C\}$, m (∠ A) = 60° , m (∠ CFE) = 120°

Find with proof : m (∠ E)



11 Damietta Governorate

Damietta inspection of Mathematic official language schools

Answer the following questions :

1 Choose the correct answer :

1 The measure of each angle of the regular pentagon equals ....°

(a) 60 (b) 108 (c) 120 (d) 135

2 The image of the point (2 , – 7) by reflection in the origin point is ....

(a) (2 , 7) (b) (– 2 , 7) (c) (– 2 , – 7) (d) (2 , – 7)

3 Any triangle has at least two .... angles.

(a) reflex (b) obtuse (c) acute (d) right

4 ABCD is parallelogram in which : m (∠ A) + m (∠ C) = 100 , then m (∠ B) = ...°

(a) 50 (b) 150 (c) 130 (d) 80

5 The ratio between the perimeter of an equilateral triangle and its side length = ...

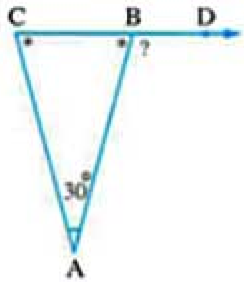
(a) 3 : 1 (b) 3 : 2 (c) 1 : 3 (d) 2 : 3

6 **In the opposite figure :**

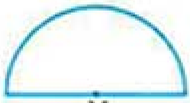
m (∠ ABD) = ...°

(a) 150 (b) 95

(c) 105 (d) 115



2 **Complete each of the following :**

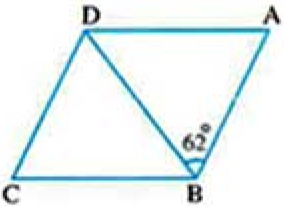
1. The ray drawn from the midpoint of a side of a triangle parallel to another side ....
2. The image of the point (3 , – 2) by rotation with an angle of measure 90° about the origin is ....
3. If m (∠ B) = 90° , then m (reflex ∠ B) = ...°
4. If ABC is right-angled triangle at B , AB = 3 cm. and BC = 4 cm. , then AC = ... cm.
5. This figure  has .... line (s) of symmetry.

3 [a] **In the opposite figure :**

ABCD is a rhombus in which

, $\overline{BD}$ is a diagonal and m (∠ ABD) = 62°

Find with proof : m (∠ A)

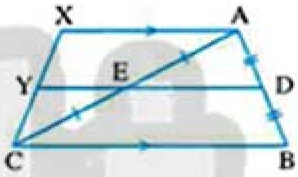


[b] **In the opposite figure :**

AD = BD , AE = EC , $\overline{AX}$ // $\overline{BC}$

, $\overrightarrow{DE} \cap \overline{XC} = \{Y\}$

Prove that : Y is the midpoint of $\overline{XC}$



4 [a] **In the opposite figure :**

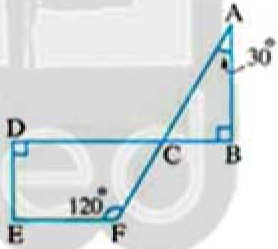
$\overline{AB}$ and $\overline{ED}$ are perpendicular to $\overline{BD}$

, $\overline{BD} \cap \overline{AF} = \{C\}$

, m (∠ A) = 30°

, m (∠ EFC) = 120°

Find : m (∠ E)



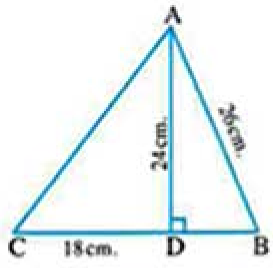
[b] **In the opposite figure :**

ABC is a triangle and $\overline{AD} \perp \overline{BC}$

If AD = 24 cm. , AB = 26 cm.

and CD = 18 cm.

Find : The length of each of $\overline{AC}$, $\overline{DB}$



5 [a] On the square lattice find the image of the triangle LMN where :

L (– 4 , – 1) , M (– 1 , – 3) , N (0 , – 1) by reflection in the X-axis.

[b] Using the square lattice , draw $\overline{AB}$ where A (4 , 3) , B (– 1 , 1) then find the image of $\overline{AB}$ by translation $(X , y) \longrightarrow (X + 2 , y - 1)$

12 Kafr El-Sheikh Governorate

General Maths Supervision

Answer the following questions :

1 Choose the correct answer :

1 The sum of measures of the interior angles of a hexagon equals ……………°

(a) 180 (b) 540 (c) 720 (d) 360

2 The two diagonals are equal in length and perpendicular in the ……………

(a) trapezium. (b) square. (c) rectangle. (d) parallelogram.

3 In Δ XYZ , if m ($\angle$ X) > m ($\angle$ Y) + m ($\angle$ Z) , then $\angle$ X is …………… angle.

(a) an acute (b) an obtuse (c) a right (d) a straight

4 The image of (3 , – 5) by rotation R (O , 90°) is ……………

(a) (– 3 , 5) (b) (– 3 , – 5) (c) (5 , 3) (d) (5 , – 3)

5 The sum of the measures of the exterior angles of a triangle equals ……………°

(a) 60 (b) 120 (c) 270 (d) 360

6 The measure of the exterior angle of the equilateral triangle is ……………°

(a) 90 (b) 120 (c) 360 (d) 60

2 Complete :

1 The image of A (– 2 , 3) by translation (x + 3 , y – 2) is ……………

2 The rhombus with a right angle is ……………

3 The image of (– 1 , 2) by reflection in the origin point is ……………

4 The length of the line segment joining two midpoints of two sides of a triangle is equal to …………… the length of the third side.

5 If ABCD is a parallelogram in which BC = 8 cm. and CD = 6 cm. , then its perimeter = …………… cm.

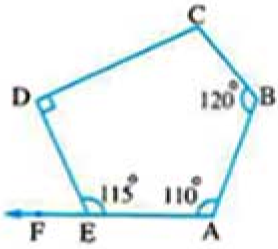
3 [a] In the opposite figure :

ABCDE is a pentagon in which F $\in \overrightarrow{AE}$

, m ($\angle$ A) = 110° , m ($\angle$ B) = 120° , m ($\angle$ DEA) = 115°

, m ($\angle$ D) = 90°

Find with proof : m ($\angle$ C)

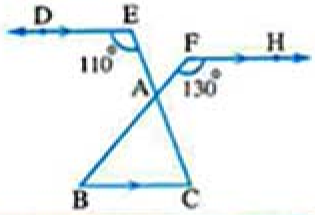


[b] In the opposite figure :

$\overleftrightarrow{ED} \parallel \overrightarrow{BC} \parallel \overrightarrow{FH}$

, m ($\angle$ E) = 110° , m ($\angle$ F) = 130°

Find with proof : m ($\angle$ BAC)



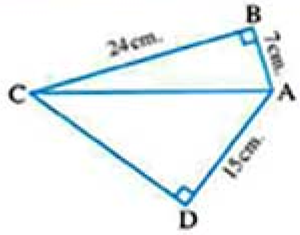
4 **[a] In the opposite figure :**

ABCD is a quadrilateral in which

m ($\angle$ B) = m ($\angle$ D) = 90°

, AB = 7 cm. , BC = 24 cm. and AD = 15 cm.

Find : The length of each of $\overline{AC}$ and $\overline{DC}$

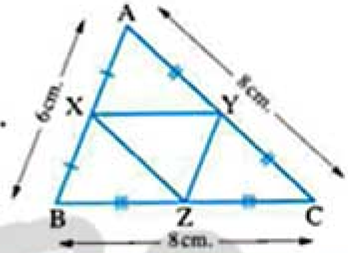


[b] In the opposite figure :

AB = 6 cm. , BC = 8 cm. , AC = 8 cm.

, X , Y , Z are the midpoints of $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ respectively.

Find with proof : The perimeter of Δ XYZ



5 **[a]** On a square lattice , draw Δ ABC , where A (2 , 4) , B (2 , 1) and C (6 , 1)

, then draw its image by :

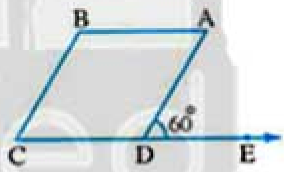
1 reflection in the y-axis. 2 rotation (O , 180°)

[b] In the opposite figure :

ABCD is a parallelogram , E $\in \overrightarrow{CD}$

, m ($\angle$ ADE) = 60°

Find with proof : m ($\angle$ B)



13 El-Beheira Governorate

Delengat Educational Directorate
Bani El-Delengat G.L.S

Answer the following questions :

1 **Complete :**

1 In the right-angled triangle , area of the square drawn on the hypotenuse equals ....

2 The reflection in a line reserves .... , ...

3 The measure of the exterior angle of the triangle is equal to the sum of ....

4 (– 3 , 2) is the image of the point (3 , 2) by reflection in .... axis.

5 The line segment joining the midpoints of two sides of the triangle is .... the third side.

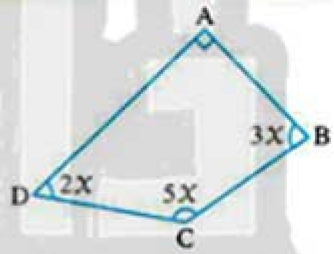
2 **Choose the correct answer :**

1. The image of the point (– 1 , 3) by translation (4 , – 2) is ....
   (a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)
2. The measure of the exterior angle of the equilateral triangle equals ....
   (a) 60° (b) 100° (c) 120° (d) 150°
3. The image of the point (– 3 , 5) by rotation about the origin point by an angle of measure 90° is ....
   (a) (5 , 3) (b) (5 , – 3) (c) (– 5 , – 3) (d) (– 5 , 3)
4. The sum of measures of the exterior angles of any polygon = ...
   (a) 180° (b) 360° (c) 120° (d) 540°
5. The number of lines of symmetry of the square = ...
   (a) 1 (b) 2 (c) 3 (d) 4
6. The parallelogram whose diagonals are .... is a rectangle.
   (a) parallel (b) perpendicular (c) equal in length (d) bisect each other

3 **[a] In the opposite figure :**

m (∠ A) = 90°

Find the value of : X

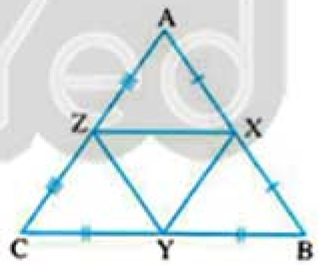


[b] In the opposite figure :

AC = AB = 7 cm.

, BC = 8 cm.

Find : The perimeter of Δ XYZ

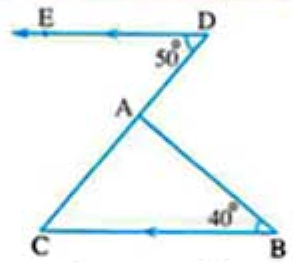


4 **[a] In the opposite figure :**

$\overrightarrow{DE} \parallel \overline{BC}$, m (∠ D) = 50°

, m (∠ B) = 40°

Find : m (∠ BAC)

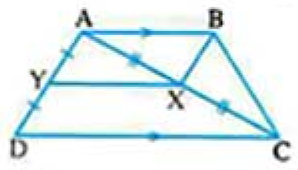


[b] In the opposite figure :

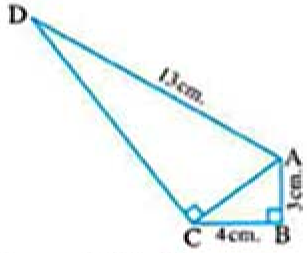
$\overline{AB} \parallel \overline{DC}$, AB = $\frac{1}{2}$ DC

, X is the midpoint of $\overline{AC}$, Y is the midpoint of $\overline{AD}$

Prove that : ABXY is a parallelogram.



5 [a] **In the opposite figure :**



m (∠ B) = m (∠ ACD) = 90°

, AB = 3 cm. , BC = 4 cm. , AD = 13 cm.

Find : The length of each of $\overline{AC}$ and $\overline{DC}$

[b] On the square lattice , draw Δ ABC where A (1 , 1) , B (4 , 1) , C (4 , 4) , then find the image of Δ ABC by reflection in X-axis.

14 Souhag Governorate

Math Supervision

Answer the following questions :

1 Choose the correct answer from those given :

1 The sum of the measures of the interior angles of a pentagon equals ....°

(a) 108 (b) 180 (c) 540 (d) 720

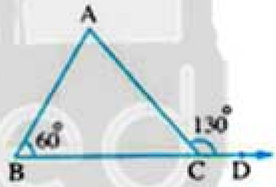
2 In the .... the two diagonals are perpendicular and not equal in length.

(a) square (b) rectangle (c) rhombus (d) parallelogram

3 The image of the point (– 2 , 3) by reflection in the y-axis is the point ....

(a) (3 , 2) (b) (– 3 , 2) (c) (2 , 3) (d) (– 3 , – 2)

4 **In the opposite figure :**



m (∠ A) = ...

(a) 40° (b) 50°

(c) 60° (d) 70°

5 The image of the point (– 1 , 3) by translation (4 , – 2) is ....

(a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)

6 In the triangle ABC , if m (∠ A) = 50° , m (∠ B) = 100° , then m (∠ C) = ...

(a) 30° (b) 180° (c) 32° (d) 23°

2 Complete each of the following :

1 The length of the line segment joining the midpoints of two sides of a triangle is ....

2 If Δ XYZ is a right-angled triangle at X , XY = 12 cm. and XZ = 9 cm. , then YZ = ... cm.

3 The image of the point (– 1 , 2) by rotation about the origin point with an angle of measure 90° is ....

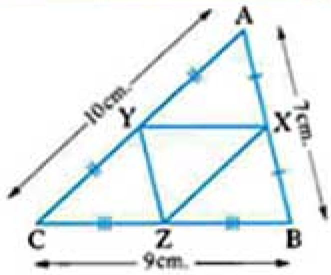
4 If $\Delta ABC \equiv \Delta XYZ$, then m ($\angle$ B) = m ($\angle$ ....)

5 The ray drawn from the midpoint of a side of a triangle parallel to another side ....

3 [a] **In the opposite figure :**

X , Y , Z are the midpoints of $\overline{AB}$, $\overline{AC}$, $\overline{BC}$ respectively , AB = 7 cm. , BC = 9 cm. , AC = 10 cm.

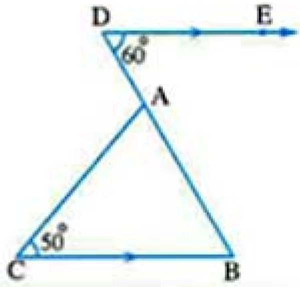
Find : The perimeter of Δ XYZ



[b] **In the opposite figure :**

$\overrightarrow{DE} \parallel \overrightarrow{CB}$, m ($\angle$ D) = 60° , m ($\angle$ C) = 50°

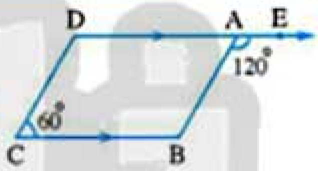
Find : m ($\angle$ DAC)



4 [a] **In the opposite figure :**

$\overrightarrow{DA} \parallel \overrightarrow{BC}$, m ($\angle$ DCB) = 60° , m ($\angle$ EAB) = 120°

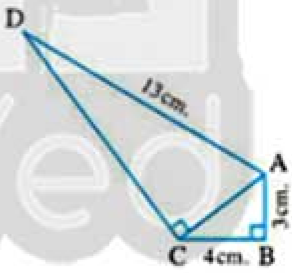
Prove that : ABCD is a parallelogram.



[b] **In the opposite figure :**

m ($\angle$ B) = m ($\angle$ ACD) = 90° , AB = 3 cm. , BC = 4 cm. and AD = 13 cm.

Find : The length of each of $\overline{AC}$, $\overline{CD}$



5 [a] On the square lattice draw Δ ABC , where A (1 , 2) , B (4 , 2) , C (4 , −1) , then find its image by rotation about the origin point with an angle of measure 180°

[b] **In the opposite figure :**

ABCDEF is a regular hexagon

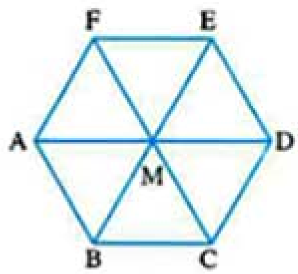
Find the image of Δ ABM by :

1 reflection in $\overleftrightarrow{EB}$

2 translation FE in direction of $\overrightarrow{FE}$

3 rotation (M , 120°)

4 reflection in M



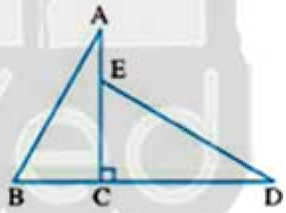
15 Qena Governorate

Qena Directorate of Education
Math's supervision

Answer the following questions :

1 Choose the correct answer :

1. The image of the point $(-3, 4)$ by reflection in y-axis is ....
   (a) $(2, -4)$ (b) $(-3, -5)$ (c) $(-3, -4)$ (d) $(3, 4)$
2. ABCD is a parallelogram , $m(\angle A) = 60°$, then $m(\angle B) =$ ....°
   (a) 120 (b) 60 (c) 90 (d) 100
3. ABCD is a parallelogram $m(\angle A) = m(\angle B)$, then ABCD is a ....
   (a) rectangle. (b) square. (c) rhombus. (d) trapezium.
4. Measure of each angle in the regular pentagon = ...
   (a) 108° (b) 100° (c) 90° (d) 60°
5. The image of the point $(-1, -2)$ by translation $(3, 1)$ is ....
   (a) $(1, 2)$ (b) $(2, -1)$ (c) $(0, 2)$ (d) $(3, -2)$
6. **In the opposite figure :**
   Δ ABC is the image of Δ DEC which is right-angled at C by rotation about C with an angle of measure ....

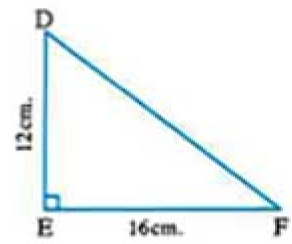


   (a) 90° (b) $-90°$
   (c) 180° (d) 360°

2 Complete :

1. The ray drawn parallel to one side of a triangle and passing through the midpoint of another side ....
2. $(-5, 1)$ is the image of the point $(5, 1)$ by reflection in .... axis.
3. If two adjacent sides are equal in length in a parallelogram , then it becomes ....
4. Sum of measures of the exterior angles of any convex polygon = ...
5. In Δ LMN if $m(\angle M) = 90°$, then $(LM)^2 =$ ....

3 [a] **In the opposite figure :**

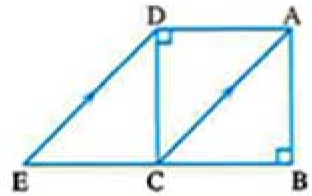


EDF is a right-angled triangle at E , DE = 12 cm.

, EF = 16 cm.

Find : DF

[b] **In the opposite figure :**



ABCD is a square , E $\in \overrightarrow{BC}$

, $\overline{AC} \parallel \overline{DE}$

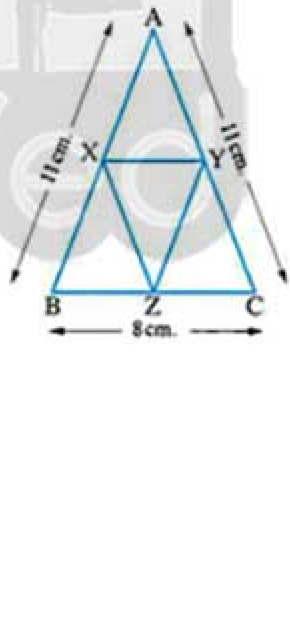
Prove that : ACED is a parallelogram.

4 [a] Using the square lattice , draw Δ ABC where A (– 4 , 2) , B (0 , 1) and C (– 1 , – 2) , then find the image of Δ ABC by rotation R (O , 180°)

[b] Find the number of sides of the regular polygon if the measure of its angle = 144°

5 [a] Using the square lattice , draw Δ ABC where A (– 1 , 3) , B (2 , 1) and C (4 , 5) , then find the image of Δ ABC by reflection in X-axis.

[b] **In the opposite figure :**



AB = 11 cm. , BC = 8 cm. , CA = 11 cm.

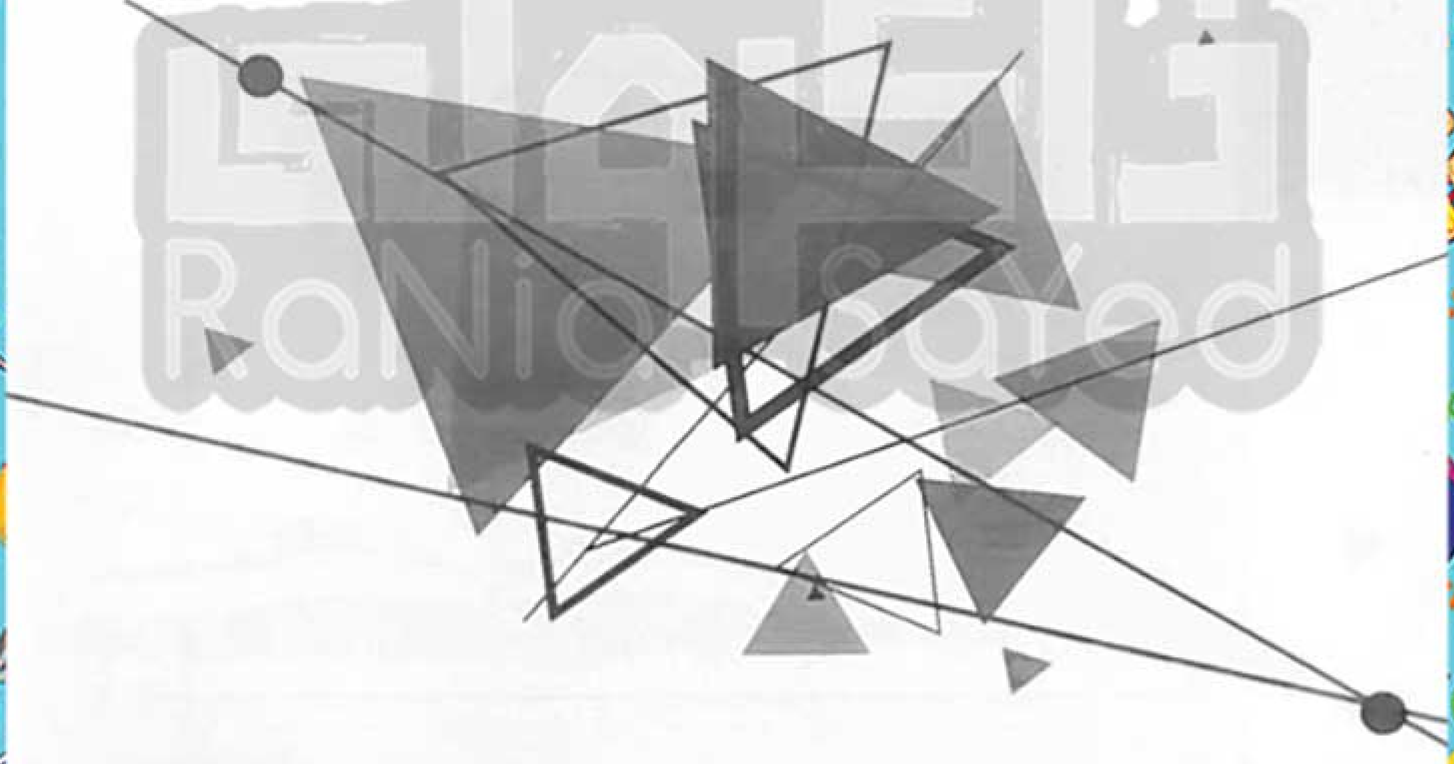
X , Z , Y are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

Find perimeter of : Δ XYZ

EL-MOASSER

In Mathematics
Guide Answers

For 1st Prep.
Second Term



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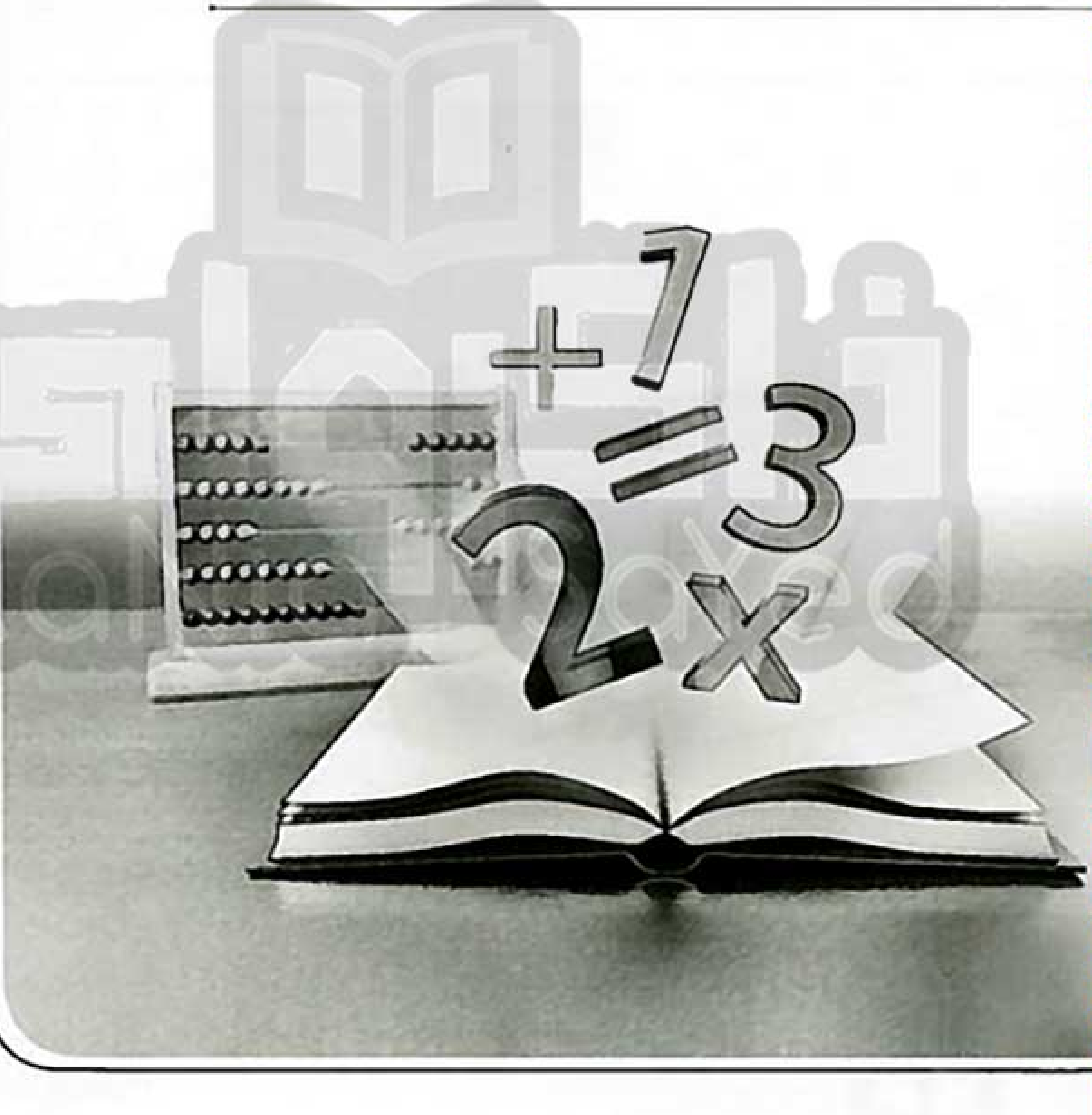
Guide Answers

of Algebra and Statistics Exercises



Answers of Unit One

Answers of Exercise 1

1

1. $\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$
2. $\left(\frac{1}{3}\right)^4 = \frac{1^4}{3^4} = \frac{1}{81}$
3. $\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2} = \frac{9}{25}$
4. $\left(-\frac{1}{7}\right)^3 = -\frac{1^3}{7^3} = -\frac{1}{343}$
5. $\left(-\frac{3}{4}\right)^4 = \frac{3^4}{4^4} = \frac{81}{256}$
6. $\left(\frac{5}{9}\right)^0 = 1$
7. $\left(1\frac{1}{5}\right)^2 = \left(\frac{6}{5}\right)^2 = \frac{36}{25}$
8. $\left(-2\frac{1}{2}\right)^3 = \left(-\frac{5}{2}\right)^3 = -\frac{125}{8}$
9. $(0.04)^2 = \left(\frac{4}{100}\right)^2 = \left(\frac{1}{25}\right)^2 = \frac{1}{625}$
10. $(1.5)^3 = \left(\frac{15}{10}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$
11. $(-3.2)^2 = \left(-\frac{16}{5}\right)^2 = \frac{256}{25}$
12. $\left(1 - 1\frac{2}{3}\right)^2 = \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$

2

1. $8 \times \frac{1}{8} = 1$
2. $\frac{9}{16} \times \frac{8}{27} = \frac{1}{6}$
3. $-\frac{27}{125} \times \left(-\frac{25}{27}\right) = \frac{1}{5}$
4. $\frac{9}{25} \div \left(-\frac{9}{125}\right) = \frac{9}{25} \times \left(-\frac{125}{9}\right) = -5$
5. $\frac{16}{9} \times \frac{27}{8} = 6$
6. $\frac{25}{36} \div \frac{15}{4} = \frac{25}{36} \times \frac{4}{15} = \frac{5}{27}$
7. $\left(\frac{5}{2}\right)^2 \times \frac{4}{25} = \frac{25}{4} \times \frac{4}{25} = 1$
8. $\frac{25}{9} \div \left(-\frac{5}{3}\right)^2 = \frac{25}{9} \div \frac{25}{9} = 1$

3

1. $\frac{16}{25} \times \frac{5}{16} \times 1 = \frac{1}{5}$
2. $\frac{3}{4} \times \left(-\frac{8}{27}\right) \times \frac{9}{4} = -\frac{1}{2}$
3. $\frac{625}{81} \times \left(-\frac{27}{125}\right) \times (-1) = \frac{5}{3}$
4. $-\frac{8}{27} \times \frac{1}{27} \div \frac{4}{81} = -\frac{8}{27} \times \frac{1}{27} \times \frac{81}{4} = -\frac{2}{9}$
5. $\left(\frac{125}{8} \div \frac{81}{16}\right) \times \frac{27}{125} = \frac{125}{8} \times \frac{16}{81} \times \frac{27}{125} = \frac{2}{3}$
6. $-\frac{1}{8} \div (-3) = \frac{1}{8} \times \frac{1}{3} = \frac{1}{24}$

4

1. (c)
2. (d)
3. (a)
4. (b)
5. (c)
6. (b)
7. (c)
8. (c)
9. (b)
10. (b)

5

1. 3
2. 2
3. 3
4. 2
5. 3
6. 2
7. $-\frac{8}{125}$
8. $\frac{9}{25}$
9. $\frac{1}{125}$
10. $\frac{1}{9}$
11. $-\frac{3}{8}$
12. 3
13. $\frac{81}{256}, \frac{243}{1024}$
14. $\left(\frac{1}{4}\right)^2$

6

$x^2 + y^3 = \left(-\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^3 = \frac{4}{9} - \frac{1}{27} = \frac{12}{27} - \frac{1}{27} = \frac{11}{27}$

7

$a^3 \div b^3 = \left(\frac{2}{3}\right)^3 \div \left(-\frac{4}{3}\right)^3 = \frac{8}{27} \div \left(-\frac{64}{27}\right)$

$= \frac{8}{27} \times -\frac{27}{64} = -\frac{1}{8}$

$|a^3 \div b^3| = \left|-\frac{1}{8}\right| = \frac{1}{8}$

8

$9xy^2 - z^3 = 9 \times \frac{1}{2} \times \left(-\frac{2}{3}\right)^2 - (-3)^3$

$= 9 \times \frac{1}{2} \times \frac{4}{9} + 27 = 2 + 27 = 29$

9

$a^3 b^2 + b^2 c - 8\,abc$

$= \left(-\frac{1}{2}\right)^3 \times 2^2 + 2^2 \times \frac{3}{4} - 8 \times \left(-\frac{1}{2}\right) \times 2 \times \frac{3}{4}$

$= -\frac{1}{8} \times 4 + 4 \times \frac{3}{4} + 6 = -\frac{1}{2} + 3 + 6 = 8\frac{1}{2}$

10

1. $x^2 y^2 z^2 = \left(-\frac{3}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 \times \left(-\frac{4}{3}\right)^2$

   $= \frac{9}{4} \times \frac{1}{4} \times \frac{16}{9} = 1$
2. $x^2 \div z^2 = \left(-\frac{3}{2}\right)^2 \div \left(-\frac{4}{3}\right)^2$

   $= \frac{9}{4} \div \frac{16}{9} = \frac{9}{4} \times \frac{9}{16} = \frac{81}{64}$
3. $x^2 - yz^2 = \left(-\frac{3}{2}\right)^2 - \frac{1}{2} \times \left(-\frac{4}{3}\right)^2 = \frac{9}{4} - \frac{1}{2} \times \frac{16}{9}$

   $= \frac{9}{4} - \frac{16}{18} = \frac{81}{36} - \frac{32}{36} = \frac{49}{36}$

4 $x^2 y^2 z^2 = 1$ from (1)

$, x + y = -\frac{3}{2} + \frac{1}{2} = -1$

i.e. $\frac{x^2 y^2 z^2}{x + y} = \frac{1}{-1} = -1$

11 The volume of the cube $= (1\frac{1}{2})^3 = (\frac{3}{2})^3 = \frac{27}{8}$ cm.³

12 1 (a) 2 (b)

13 The order is : $(-\frac{2}{3})^3$, $(-\frac{1}{3})^3$, $(-\frac{1}{3})^2$, $(\frac{2}{3})^2$

Answers of Exercise 2

1

1 $(\frac{2}{3})^{3+2} = (\frac{2}{3})^5 = \frac{32}{243}$

2 $-(\frac{2}{3})^{3+2} = -(\frac{2}{3})^5 = -\frac{32}{243}$

3 $(\frac{1}{5})^{1+4} = (\frac{1}{5})^5 = \frac{1}{3125}$

4 $(\frac{1}{6})^{9-8} = \frac{1}{6}$ 5 $(\frac{2}{7})^{5-3} = (\frac{2}{7})^2 = \frac{4}{49}$

6 $-(\frac{3}{5})^{7-5} = -(\frac{3}{5})^2 = -\frac{9}{25}$

7 $(\frac{5}{2})^2 \div \frac{5}{2} = (\frac{5}{2})^{2-1} = \frac{5}{2}$

8 $(\frac{1}{2})^{2+1+3} = (\frac{1}{2})^6 = \frac{1}{64}$

9 $(\frac{4}{5})^{8-6+1} = (\frac{4}{5})^3 = \frac{64}{125}$

2

1 $3^{7+3-6} = 3^4 = 81$ 2 $2^{6+1-3-4} = 2^0 = 1$

3 $5^{4+2-3} = 5^3 = 125$ 4 $2^{5+4-3-2} = 2^4 = 16$

5 $x^{2+3+4-7-1} = x$

6 $(-3)^{5-3} \times (-2)^{7-5} = (-3)^2 \times (-2)^2$

$= ((-3) \times (-2))^2 = 6^2 = 36$

7 $x^{4+5-6} \times y^{3-2} = x^3 y$ 8 $\frac{x^3 y^3}{x^2 y^2} = \frac{(x y)^3}{(x y)^2} = x y$

9 $\frac{(-\frac{3}{2})^5 \times (-\frac{5}{4})^8}{(-\frac{3}{2})^4 \times (-\frac{5}{4})^6} = (-\frac{3}{2})^{5-4} \times (-\frac{5}{4})^{8-6}$

$= (-\frac{3}{2}) \times (\frac{5}{4})^2$

$= (-\frac{3}{2}) \times \frac{25}{16} = -\frac{75}{32}$

3 1 $\frac{a^5 b^5}{c^5}$ 2 $\frac{5^2 x^2}{3^2 y^2} = \frac{25 x^2}{9 y^2}$

3 $\frac{2^4 a^4 b^4}{3^4 c^4} = \frac{16 a^4 b^4}{81 c^4}$ 4 $\frac{x^{2 \times 2}}{y^{3 \times 2}} = \frac{x^4}{y^6}$

5 $\frac{a^{3 \times 3} \times b^{2 \times 3}}{c^{5 \times 3}} = \frac{a^9 b^6}{c^{15}}$ 6 $-\frac{c^{2 \times 3}}{d^3} = -\frac{c^6}{d^3}$

7 $\frac{x^{3 \times 2}}{y^{2 \times 2}} = \frac{x^6}{y^4}$

8 $(\frac{4 x^3 y^2}{2 x^2 y})^7 = (2 x y)^7 = 2^7 x^7 y^7 = 128 x^7 y^7$

9 $\frac{2^3 a^3 \times 2^4 a^4}{2^6 a^6 \times a} = 2^{3+4-6} \times a^{3+4-6-1}$

$= 2 \times a^0 = 2 \times 1 = 2$

4 1 $(\frac{1}{2})^{2 \times 2} = (\frac{1}{2})^4 = \frac{1}{16}$

2 $(\frac{3}{2})^{2 \times 5} = (\frac{3}{2})^{10} = \frac{59049}{1024}$

3 $[(\frac{5}{2})^3]^2 = (\frac{5}{2})^{3 \times 2} = (\frac{5}{2})^6 = \frac{15625}{64}$

4 $[(-\frac{4}{3})^2]^3 = (\frac{4}{3})^{2 \times 3} = (\frac{4}{3})^6 = \frac{4096}{729}$

5 $(\frac{3}{5} \times \frac{5}{3})^{10} = 1^{10} = 1$

6 $(\frac{2}{7})^6 \times (\frac{7}{2})^6 = (\frac{2}{7} \times \frac{7}{2})^6 = 1^6 = 1$

7 $(\frac{5}{2})^2 \times (\frac{2}{5})^2 = (\frac{5}{2} \times \frac{2}{5})^2 = 1^2 = 1$

8 $(\frac{x^3}{y^2} \div \frac{x^2}{y^2})^{16} = (\frac{x^3}{y^2} \times \frac{y^2}{x^2})^{16} = x^{16}$

5 1 (e) 2 (a) 3 (h) 4 (i)
5 (f) 6 (c) 7 (b) 8 (d)

6 1 (a) 2 (d) 3 (c) 4 (d)
5 (c) 6 (a) 7 (d) 8 (c)
9 (d) 10 (c) 11 (c) 12 (d)
13 (c)

7 $\frac{2^4 y^4 \times 3^2 y^2}{12 y^5} = \frac{16 y^4 \times 9 y^2}{12 y^5} = \frac{144 y^6}{12 y^5} = 12 y$

At : $y = -\frac{1}{6}$

The result $= 12 \times (-\frac{1}{6}) = -2$

8 1 $(c^2 b)^3 = ((-\frac{2}{3})^2 \times \frac{3}{4})^3 = (\frac{4}{9} \times \frac{3}{4})^3 = (\frac{1}{3})^3 = \frac{1}{27}$

[2] $(4a^3c)^2 = 16a^6c^2 = 16 \times \left(\frac{1}{2}\right)^6 \times \left(-\frac{2}{3}\right)^2$

$= 16 \times \frac{1}{64} \times \frac{4}{9} = \frac{1}{9}$

[3] $(a^2bc^2)^2 = a^4b^2c^4 = \left(\frac{1}{2}\right)^4 \times \left(\frac{3}{4}\right)^2 \times \left(-\frac{2}{3}\right)^4$

$= \frac{1}{16} \times \frac{9}{16} \times \frac{16}{81} = \frac{1}{144}$

9

[1] $\frac{(a^2c^2)^2}{b} = \frac{a^4c^4}{b} = \left(\frac{5}{3}\right)^4 \times \left(\frac{2}{5}\right)^4 \div \left(-\frac{3}{2}\right)$

$= \left(\frac{5}{3} \times \frac{2}{5}\right)^4 \div \left(-\frac{3}{2}\right)$

$= \left(\frac{2}{3}\right)^4 \times \left(-\frac{2}{3}\right) = -\left(\frac{2}{3}\right)^5 = -\frac{32}{243}$

[2] $\left(\frac{2ab}{5c}\right)^3 = \left(\frac{2 \times \frac{5}{3} \times \left(-\frac{3}{2}\right)}{5 \times \frac{2}{5}}\right)^3 = \left(-\frac{5}{2}\right)^3 = -\frac{125}{8}$

10 [1] $x^3y^2 = \left(-\frac{1}{2}\right)^3 \times \left(\frac{3}{4}\right)^2 = -\frac{1}{8} \times \frac{9}{16} = -\frac{9}{128}$

[2] $y^3x^2 = \left(\frac{3}{4}\right)^3 \times \left(-\frac{1}{2}\right)^2 = \frac{27}{64} \times \frac{1}{4} = \frac{27}{256}$

[3] $\frac{x^3}{y^2z^2} = \frac{\left(-\frac{1}{2}\right)^3}{\left(\frac{3}{4}\right)^2 \times \left(-\frac{3}{2}\right)^2} = \frac{-\frac{1}{8}}{\frac{9}{16} \times \frac{9}{4}}$

$= \frac{-\frac{1}{8}}{\frac{81}{64}} = -\frac{1}{8} \times \frac{64}{81} = -\frac{8}{81}$

11 [1] 24 [2] $\frac{9}{16}$ [3] $((-3)^2)^4$

[4] zero [5] 4 [6] $2x$

12

The area of the square $= \left(\frac{2x}{5}\right)^2 = \frac{2^2x^2}{5^2} = \frac{4x^2}{25}$ cm².

13 The volume of the cube $= \left(\frac{3a^3}{7}\right)^3 = \frac{3^3(a^3)^3}{7^3}$

$= \frac{27a^9}{343}$ cm³.

14

The area of the shaded part

= the area of the great square – the area of the small square

$= \left(\frac{3x}{2}\right)^2 - \left(\frac{x}{2}\right)^2 = \frac{9x^2}{4} - \frac{x^2}{4} = \frac{8x^2}{4} = 2x^2$ cm².

15 The number of bytes in one terabyte

$= 2^{10} \times 2^{10} \times 2^{10} \times 2^{10} = 2^{40}$ bytes

16 The number is $4^3 \div 4 = 4^2$

Then $\frac{3}{4}$ the number $= \frac{3}{4} \times 4^2 = 12$

17

$x = \left(\frac{2}{3}\right)^{10}$

$, y = \left(\frac{3}{2}\right)^9 \times \left(3 - \frac{3}{2}\right) = \left(\frac{3}{2}\right)^9 \times \left(\frac{6}{2} - \frac{3}{2}\right)$

$= \left(\frac{3}{2}\right)^9 \times \frac{3}{2} = \left(\frac{3}{2}\right)^{10}$

$xy = \left(\frac{2}{3}\right)^{10} \times \left(\frac{3}{2}\right)^{10} = \left(\frac{2}{3} \times \frac{3}{2}\right)^{10} = 1^{10} = 1$

= the multiplicative neutral element.

i.e. x is the multiplicative inverse of y

18 $x^{15}y^{14} = x \times x^{14} \times y^{14} = x(xy)^{14}$

$= \frac{1}{5}\left(\frac{1}{5} \times 5\right)^{14} = \frac{1}{5} \times (1)^{14} = \frac{1}{5}$

19

[1] L.H.S. $= 5^x(5^2 - 5) = 5^x(25 - 5) = 20 \times 5^x =$ R.H.S.

[2] $3^{15} + 3^{14} = 3^{14}(3 + 1) = 4 \times 3^{14}$

i.e. $3^{15} + 3^{14}$ is divisible by 4

Answers of Exercise 3

1 [1] $4^{-1} = \frac{1}{4}$ [2] $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

[3] $\left(\frac{1}{2}\right)^{-1} = 2$ [4] $\left(-\frac{2}{3}\right)^{-2} = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

[5] $(0.2)^{-2} = \left(\frac{2}{10}\right)^{-2} = \left(\frac{10}{2}\right)^2 = 25$

[6] $(1.2)^{-1} = \left(\frac{6}{5}\right)^{-1} = \frac{5}{6}$

2 [1] $3^7 \times 3^{-3} = 3^{7+(-3)} = 3^4 = 81$

[2] $2^{-2} \times 2^{-3} = 2^{-2+(-3)} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

[3] $\frac{3}{3^{-2}} = 3^{1-(-2)} = 3^3 = 27$

[4] $\frac{6^{-2}}{6^{-3}} = 6^{-2-(-3)} = 6$

3 [1] $(5^{-1})^{-3} = 5^3 = 125$ [2] $(3^{-2})^2 = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

[3] $(0.25)^{-2} = \left(\frac{1}{4}\right)^{-2} = 4^2 = 16$

4 $(2^{-1} \times 2^{-2})^3 = (2^{-3})^3 = 2^{-9} = \frac{1}{2^9} = \frac{1}{512}$

5 $\left(\frac{3^{-1}}{3}\right)^2 = \frac{3^{-2}}{3^2} = 3^{-2-2} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

6 $\left(\frac{8^4}{8^{-4}}\right)^0 = 1$

4

1 $8^{1-2-(-3)} = 8^2 = 64$

2 $7^{-2+5-3} = 7^0 = 1$

3 $2^{5-2-(-4)-3} = 2^4 = 16$

4 $\frac{2^3 \times 2^{-3}}{2^4} = 2^{3-3-4} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

5 $\frac{3^{-6}}{3^{-2} \times 3^{-6}} = \frac{1}{3^{-2}} = 3^2 = 9$

6 $(9^{3+1-5})^{-3} = (9^{-1})^{-3} = 9^3 = 729$

7 $(4^{-2-(-3)} \times 3)^{-3} = (4 \times 3)^{-3} = \frac{1}{12^3} = \frac{1}{1728}$

8 $(2^{5-3} \times 3^{2-4})^{-1} = (2^2 \times 3^{-2})^{-1} = 2^{-2} \times 3^2 = \frac{9}{4}$

9 $1 \times 2^4 = 1 \times 16 = 16$

10 $\left(1 - \frac{1}{2^2}\right)^{-2} = \left(1 - \frac{1}{4}\right)^{-2} = \left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

11 $\frac{(10)^2 \times (10^{-2})^3}{(10)^{-3}} = \frac{(10)^2 \times (10)^{-6}}{(10)^{-3}} = (10)^{2-6+3}$

$= (10)^{-1} = \frac{1}{10}$

5

1 $\frac{7}{x}$

2 $\frac{y^2}{x}$

3 $\frac{1}{a^2 b^3}$

4 $x^{3-5} = x^{-2} = \frac{1}{x^2}$

5 $x^{3-2-1} = x^0 = 1$

6 $c^{-5-2} = c^{-7} = \frac{1}{c^7}$

7 $x^{7-(-5)} = x^{7+5} = x^{12}$

8 $a^{-6} = \frac{1}{a^6}$

9 b^3

10 $(a^{2-5})^2 = (a^{-3})^2 = a^{-6} = \frac{1}{a^6}$

11 $x^{-6} \times x^6 = x^0 = 1$

12 $(y^{5-(-2)})^{-3} = (y^7)^{-3} = y^{-21} = \frac{1}{y^{21}}$

13 $x^{2-3-(-4)-1} = x^2$

14 $\frac{x^{-6} \times x^{-2}}{x^{-3} \times x^{-4}} = x^{-6-2-(-3)-(-4)} = x^{-1} = \frac{1}{x}$

15 $(x^{-2-1} \times y^{-1-(-3)})^{-1} = (x^{-3} y^2)^{-1} = \frac{x^3}{y^2}$

16 $\frac{a^{-1}}{b^2} \times \frac{a^2}{2^{-2} b^{-4}} = \frac{2^2 \times a^{-1+2}}{b^{2-4}} = \frac{4a}{b^{-2}} = 4ab^2$

17 $x^2 + 2\,x\,x^{-1} + (x^{-1})^2 = x^2 + 2 + x^{-2}$

$= x^2 + \frac{1}{x^2} + 2$

6

1 $\frac{1}{8}$

2 3

3 x^3

4 -2 , x^2

5 $\frac{y^4}{9}$

6 $3a^2$

7 $x^2 y^3$

8 $\frac{y}{x}$

9 1

10 -2

11 0

12 1 , a^5

13 4

7

1 (b) 2 (c) 3 (c) 4 (c)

5 (b) 6 (b) 7 (d) 8 (d)

9 (d) 10 (d) 11 (b) 12 (a)

8

1 $>$ 2 $<$ 3 $<$ 4 $>$ 5 $>$ 6 $=$

9

$b^{-3} = \frac{1}{b^3}$ substituting $b = 0$, then $\frac{1}{b^3} = \frac{1}{0}$ (undefined).

10

1 At $x = -2$, $y = 2$

Then : $\left(-\frac{3}{5}\right)^{-2} \times \left(\frac{3}{5}\right)^2 = \left(\frac{5}{3}\right)^2 \times \left(\frac{3}{5}\right)^2$

$= \left(\frac{5}{3} \times \frac{3}{5}\right)^2 = 1^2 = 1$

2 At $x = -1$, $y = 2$

Then : $\left(-\frac{3}{5}\right)^{-1} \times \left(\frac{3}{5}\right)^2 = -\frac{5}{3} \times \frac{9}{25} = -\frac{3}{5}$

11

$\left(\frac{y}{x^2}\right)^{-2} = \left(\frac{2}{3} \div \left(-\frac{1}{3}\right)^2\right)^{-2} = \left(\frac{2}{3} \div \frac{1}{9}\right)^{-2}$

$= \left(\frac{2}{3} \times \frac{9}{1}\right)^{-2} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

12

The height of the jump $= 2^3 \div 2^{-4}$

$= 2^{3-(-4)} = 2^7$

$= 128$ times of its length

13

1 In 2 years the population will be $2\,(1.03)^2 = 2.1218$ millions.

2 The population now $= 2\,(1.03)^0 = 2$ millions.

3 The population last year was $2\,(1.03)^{-1} = \frac{2}{1.03} = 1.94$ millions.

14

$\frac{2^{10} \times 3^4}{(2 \times 2 \times 3)^5} = \frac{2^{10} \times 3^4}{(2^2 \times 3)^5} = \frac{2^{10} \times 3^4}{2^{10} \times 3^5}$

$= 2^{10-10} \times 3^{4-5} = 3^{-1} = \frac{1}{3}$

15

$$\frac{(2\times3)^{2n+1}\times(2^2)^{-n}}{2^n\times3^{2n+1}}=\frac{2^{2n+1}\times3^{2n+1}\times2^{-2n}}{2^n\times3^{2n+1}}$$
$$=2^{2n+1-2n-n}\times3^{2n+1-2n-1}$$
$$=2^{1-n}\times3^0=2^{1-n}$$

When n = 3 , then $2^{1-n}=2^{1-3}=2^{-2}=\frac{1}{4}$

16
1. $2^{n+1}=2^n\times2=3\times2=6$
2. $4^n=(2\times2)^n=2^n\times2^n=3\times3=9$
3. $4^{-n}=(2\times2)^{-n}=2^{-n}\times2^{-n}=\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
4. $2^{n-1}=2^n\times2^{-1}=3\times\frac{1}{2}=\frac{3}{2}$

17 $a^{51}b^{50}=a\times a^{50}b^{50}=a(ab)^{50}$
$$=5\left(5\times\frac{1}{5}\right)^{50}=5\times1=5$$

18 The order is : $(-5)^{15}$, $(-2)^{15}$, $(-2)^{-15}$, 2^{-20} , $(-2)^{20}$, $(-5)^{20}$

Answers of Exercise 4

1 The numbers which are in the standard form are (1) , (4) , (6) and (8)

2
1. 6×10^5
2. -2×10^4
3. 7×10^6
4. 1.9×10^7
5. 4.687×10^7
6. 5.8×10

3
1. 6×10^{-4}
2. 5.3×10^{-5}
3. 8.64×10^{-4}
4. 4.21×10^{-1}
5. 2.50003×10
6. -3.00501×10^2

4 (5.1×10^8) km^2.

5 (1.67×10^{-24}) gm.

6 The velocity of light = 300 000 × 1000
= 300 000 000
= 3×10^8 m./sec.

7 1×10^{-15} seconds

8 18 zeroes

9
1. 6.8×10^6
2. 6.8×10^{-4}
3. 7.2×10^8
4. 7.5×10^{-7}
5. -3.24×10^5
6. -7.025×10^{-6}
7. 4×10^{-11}
8. 5×10^{11}
9. 3.6×10^{-7}
10. 2.0205×10^9

10
1. >
2. <
3. <
4. <
5. >
6. >
7. >
8. >

11 The order is
8.35×10^{-2} , 1×10^{-2} , 3.6×10^{-3} , 5.2×10^{-5} , 6.08×10^{-8}

12
1. (d)
2. (b)
3. (c)
4. (b)
5. (c)
6. (b)
7. (a)
8. (b)
9. (c)
10. (c)
11. (d)

13
1. 9.6×10^{13}
2. $17.22\times10^3=1.722\times10^4$
3. $0.502\times10^{-7}=5.02\times10^{-8}$
4. $(4.4\times10^3)\times(32\times10^5)=140.8\times10^8=1.408\times10^{10}$
5. 2×10^2
6. $25.1\times10^{-7}=2.51\times10^{-6}$
7. 1×10^3
8. $(625\times10^4)\div(2.5\times10^{-3})=250\times10^7=2.5\times10^9$

14
1. $10^4(3.8\times10+4.6)=10^4(38+4.6)$
   $=42.6\times10^4=4.26\times10^5$
2. $10^3(4.54\times10+3.76)=10^3\times49.16=4.916\times10^4$
3. $10^7(5.3\times10-0.8)=10^7\times52.2=5.22\times10^8$
4. $10^{-3}(2.65\times10-6.34)=10^{-3}\times20.16=2.016\times10^{-2}$

15
1. $5\times10^3\times3\times10^3=15\times10^6=1.5\times10^7$
2. $4\times10^2\times7\times10^{-5}=28\times10^{-3}=2.8\times10^{-2}$
3. $8\times10^3\div4\times10^{-3}=2\times10^6$
4. $3.3\times10^{-5}\div5\times10^2=0.66\times10^{-7}=6.6\times10^{-8}$
5. $(2\times10^4)^3=8\times10^{12}$
6. $(2\times10^{-3})^2=4\times10^{-6}$
7. $(1\times10^{-1})^{-8}=10^8$

16
1. $800\,000=8\times10^5$ $\therefore n=5$
2. $0.00000006=6\times10^{-8}$ $\therefore n=-8$
3. $0.00052=5.2\times10^{-4}$ $\therefore n=-4$
4. $0.000357=3.57\times10^{-4}$ $\therefore n=-4$

5 $(0.004)^2 = (4 \times 10^{-3})^2$

$= 16 \times 10^{-6}$

$= 1.6 \times 10^{-5}$ $\therefore n = -5$

6 $76293 = 7.6293 \times 10^4$ $\therefore n = 7.6293$

17

The earth is the greater

and the difference between the two diameter lengths

$= (1.27 \times 10^4) - (6.79 \times 10^3)$

$= 10^3 (1.27 \times 10 - 6.79) = 5.91 \times 10^3$ km.

18

[a] The distance from the Sun to the Earth

$= v \times t = 3 \times 10^8 \times 8 \times 60$

$= 1440 \times 10^8 = 1.44 \times 10^{11}$ m.

[b] The time elapsed $= \frac{d}{v} = \frac{108 \times 10^6 \times 10^3}{3 \times 10^8}$

$= 36 \times 10 = 360$ seconds $= 6$ minutes

19 $\frac{10^3 (9.02 + 4.98 \times 10)}{2.5 \times 10^{-5}} = \frac{58.82 \times 10^3}{2.5 \times 10^{-5}}$

$= \frac{5.882 \times 10^4}{2.5 \times 10^{-5}} = 2.3528 \times 10^9$

20

1 $10^{29} - 10^{28} = 10^{28} (10 - 1) = 9 \times 10^{28}$

2 $2^{19} \times 5^{15} = 2^{15} \times 2^4 \times 5^{15} = (2 \times 5)^{15} \times 2^4$

$= 10^{15} \times 16 = 1.6 \times 10^{16}$

21

$X = 5 + 30 + 400 + 6000 + 90\,000 + 400\,000 + 2\,000\,000$

$\therefore X = 2496435 = 2.496435 \times 10^6$

Answers of exams on the first part of unit one

Model 1

1 1 (b) 2 (d) 3 (c) 4 (b) 5 (c) 6 (b)

2 1 8 2 1.3×10^{-5} 3 3 4 10^{-4} 5 y^3

3 [a] 1 [b] 1 8.8×10^8 2 5.22×10^8

4 [a] 81 [b] $-\frac{8}{81}$

5 [a] 1 $n = -4$ 2 $n = 6$

[b] $8\frac{1}{2}$

Model 2

1

1 (c) 2 (b) 3 (a) 4 (d) 5 (b) 6 (b)

2

1 7 2 -2 3 2 4 $\frac{1}{4}$ 5 x^3

3 [a] a^4 [b] 12

4 [a] 1 2×10^5 2 6.58×10^5

[b] 64

5 [a] 1 5.4×10^{-4} 2 7.94×10^7

3 4.65×10^4

[b] The volume of the cube $= \frac{27\,a^6}{125}$ cm^3

Answers of Exercise 5

1 1 5 2 1 3 10 4 19 5 12 6 45 7 143

2 1 49 2 18 3 9 4 36 5 3 6 40 7 378 8 11 9 108 10 97 11 30 12 31

3 1 zero 2 zero 3 12 4 13 5 9 6 10 7 9 8 10 9 86 10 22 11 5 12 19

4 1 20 2 7 3 -14 4 -7 5 -18 6 -55

5 1 2 2 6 3 -20 4 2 5 2 6 zero 7 5 8 23 9 1

6 1 $26\frac{1}{4}$ 2 -678 3 17.97 4 21

7 $2\left(\frac{5\times3+3}{4\times3-3}\right)=4$

8 (1) $(2+5)^2=49$ (2) $(5-2)^3=27$
(3) $\left(\frac{5}{2}\right)^3=\frac{125}{8}$ (4) $\frac{6^2}{5-1}=9$
(5) $\frac{5-2}{5^3}=\frac{3}{125}$ (6) $\frac{12}{4\times25}=\frac{3}{25}$

9 $16\times9\div(4\times6)+3\times6\times9=168$

10 $X=38$, $y=9$
$\therefore 2X+4y=2\times38+4\times9=112$

11 $X=32$, $y=8$ $\therefore X-4y=32-4\times8=\text{zero}$

12 $X=15$, $y=30$ $\therefore \left(\frac{y}{X}\right)^{-3}=\left(\frac{30}{15}\right)^{-3}=\frac{1}{8}$

13 (1) $T=6s^2=6\times3^2=54\ m^2$
(2) $T=6s^2=6\times(0.8)^2=3.84\ cm^2$

14 (1) $T=2(Xy+yz+zX)$
$=2(2\times3+3\times5+5\times2)=62\ cm^2$
(2) $T=2(Xy+yz+zX)$
$=2\left(\frac{3}{5}\times0.4+0.4\times\frac{1}{5}+\frac{3}{5}\times\frac{1}{5}\right)=\frac{22}{25}\ m^2$

15 (1) $A=\frac{1}{2}h(a+b)=\frac{1}{2}\times2\left(\frac{3}{4}+\frac{1}{4}\right)=1\ m^2$
(2) $A=\frac{1}{2}h(a+b)=\frac{1}{2}\times4\left(\frac{1}{2}+\frac{1}{2}\right)=2\ m^2$

16 (1) $3+96\div(12\times4)=5$
(2) $3+(96\div12)\times4=35$
(3) $(3+96)\div12\times4=33$

Answers of Exercise 6

1 (1) 4 (2) -5 (3) ±50
(4) ±200 (5) $\frac{3}{7}$ (6) $-\frac{8}{5}$
(7) $\sqrt{\frac{81}{100}}=\frac{9}{10}$ (8) $\pm\sqrt{\frac{144}{100}}=\pm\frac{12}{10}=\pm\frac{6}{5}$
(9) $\sqrt{\frac{25}{4}}=\frac{5}{2}$ (10) $-\sqrt{\frac{36}{25}}=-\frac{6}{5}$ (11) -4
(12) ±8 (13) $\frac{81}{100}$ (14) $\left|-\frac{3}{4}\right|=\frac{3}{4}$
(15) $\pm\frac{24}{35}$ (16) $-\sqrt{\frac{25}{400}}=-\sqrt{\frac{1}{16}}=-\frac{1}{4}$
(17) $-\left|\frac{7a^2}{5b^3}\right|$ (18) $\pm\left|\frac{4b^4}{11h}\right|$ (19) $\left|\frac{7a^2b}{3}\right|$
(20) $\left|\frac{5Xy}{6}\right|$

2 (1) ±8 (2) ±12 (3) $\pm\frac{3}{5}$
(4) $\pm\sqrt{\frac{25}{4}}=\pm\frac{5}{2}$ (5) ±0.5 (6) ±0.07

3 (1) $3+4=7$ (2) $\sqrt{100}=10$ (3) $\sqrt{16}=4$
(4) $-\sqrt{144}=-12$ (5) $\sqrt{9+16}=\sqrt{25}=5$
(6) $-\sqrt{100-64}=-\sqrt{36}=-6$
(7) $\sqrt{\frac{9}{16}+\frac{16}{16}}=\sqrt{\frac{25}{16}}=\frac{5}{4}$
(8) $-\sqrt{\frac{1}{4}\left(\frac{4}{4}-\frac{3}{4}\right)}=-\sqrt{\frac{1}{4}\times\frac{1}{4}}=-\frac{1}{4}$
(9) $\sqrt{5^{4+3-5}}=\sqrt{5^2}=5$
(10) $\sqrt{\left(\frac{1}{2}\div\frac{1}{5}\right)^2}=\sqrt{\left(\frac{1}{2}\times5\right)^2}=\sqrt{\left(\frac{5}{2}\right)^2}=\frac{5}{2}$
(11) $\sqrt{\left(\frac{1}{2}\times\frac{1}{3}\right)^4}=\sqrt{\left(\frac{1}{6}\right)^4}=\left(\frac{1}{6}\right)^2=\frac{1}{36}$
(12) $\sqrt{\left(\frac{1}{4}\right)^5}=\sqrt{\left(\left(\frac{1}{2}\right)^2\right)^5}=\sqrt{\left(\frac{1}{2}\right)^{10}}=\left(\frac{1}{2}\right)^5=\frac{1}{32}$

4 (1) 1 (2) $\frac{2}{3}$ (3) 1 (4) 100
(5) 80 (6) $\frac{5}{2}$ (7) $\frac{10}{7}$ (8) $\frac{1}{2}$
(9) $\frac{3}{4}$ (10) $\left(\frac{5}{2}\right)^2$ or $\left(-\frac{5}{2}\right)^2$
(11) $\sqrt{\left(\frac{5}{8}\right)^2}=\frac{5}{8}$ (12) 3 (13) a^2b^4
(14) 1 (15) $\frac{3}{4}$ (16) 3
(17) -2 (18) 2222

5 (1) (c) (2) (b) (3) (b) (4) (d) (5) (a)
(6) (c) (7) (b) (8) (a) (9) (d) (10) (b)

6 (1) $-\frac{1^3}{2^3}\times\frac{8}{3}=-\frac{1}{8}\times\frac{8}{3}=-\frac{1}{3}$
(2) $\frac{7}{2}\times1\times\frac{2^2}{7^2}=\frac{7}{2}\times\frac{4}{49}=\frac{2}{7}$
(3) $\frac{2}{5}\times\frac{3}{4}\div\left(-\frac{1^3}{2^3}\right)=\frac{2}{5}\times\frac{3}{4}\times\left(-\frac{8}{1}\right)=-\frac{12}{5}$
(4) $\frac{1^2}{3^2}+\frac{8}{9}-1=\frac{1}{9}+\frac{8}{9}-\frac{9}{9}=\text{zero}$
(5) $\frac{3}{4}\times\left(-\frac{2^3}{3^3}\right)\times\frac{9}{4}=-\frac{3}{4}\times\frac{8}{27}\times\frac{9}{4}=-\frac{1}{2}$
(6) $\frac{25}{4}\times\frac{4}{25}=1$

Another solution :

$\frac{25}{4}\times\left(\frac{2}{5}\right)^2=\left(\frac{5}{2}\right)^2\times\left(\frac{2}{5}\right)^2$
$=\left(\frac{5}{2}\times\frac{2}{5}\right)^2=1^2=1$

7 (1) $4+5=9$ (2) $\sqrt{4+5}=\sqrt{9}=3$

(3) $\sqrt{(4+5)^2}=\sqrt{(9)^2}=9$

8 $\sqrt{\frac{4}{9}}=\frac{2}{3}$

$\because$ The L.C.M. of 3 and 4 is 12

$\therefore \frac{2}{3}=\frac{8}{12}$, $\frac{3}{4}=\frac{9}{12}$, $\frac{8}{12}=\frac{24}{36}$, $\frac{9}{12}=\frac{27}{36}$

$\therefore$ The two numbers are $\frac{25}{36}$ and $\frac{26}{36}$

9 $\sqrt{\frac{4}{9}}=\frac{2}{3}$

$\because$ The L.C.M. of 3 and 5 is 15

$\therefore \frac{3}{5}=\frac{9}{15}$, $\frac{2}{3}=\frac{10}{15}$ $\therefore \sqrt{\frac{4}{9}}>\frac{3}{5}$

$\therefore$ The difference $=\frac{10}{15}-\frac{9}{15}=\frac{1}{15}$

10 $\sqrt{2\frac{1}{4}}=\sqrt{\frac{9}{4}}=\frac{3}{2}$, $(-\frac{2}{3})^2=\frac{4}{9}$

$\because$ L.C.M. of 2 and 9 is 18

$\therefore \frac{3}{2}=\frac{27}{18}$, $\frac{4}{9}=\frac{8}{18}$ $\therefore \sqrt{\frac{9}{4}}>(-\frac{2}{3})^2$

$\therefore$ The difference $=\frac{27}{18}-\frac{8}{18}=\frac{19}{18}$

11 (1) $\sqrt{25-10+1}=\sqrt{16}=4$

Another solution : $\sqrt{(5-1)^2}=\sqrt{4^2}=4$

(2) $\sqrt{\frac{1}{16}-\frac{1}{2}+1}=\sqrt{\frac{1}{16}-\frac{8}{16}+\frac{16}{16}}=\sqrt{\frac{9}{16}}=\frac{3}{4}$

Another solution :

$\sqrt{(\frac{1}{4}-1)^2}=\sqrt{(-\frac{3}{4})^2}=|-\frac{3}{4}|=\frac{3}{4}$

(3) $\sqrt{9}=3$ (4) $\sqrt{16}=4$

(5) $\sqrt{121}=11$ (6) $\sqrt{6+3\times10-11}=\sqrt{25}=5$

12

(1) $\because (XY)^2=25$ $\therefore XY=\sqrt{25}=5$ cm.

, $\because$ E is the midpoint of $\overline{XY}$

$\therefore XE=2.5$ cm.

(2) $\because (AB)^2=144$ $\therefore AB=\sqrt{144}=12$ cm.

, $\because (BC)^2=625$ $\therefore BC=\sqrt{625}=25$ cm.

$\therefore AC=37$ cm.

(3) The side length of the square $=\sqrt{0.49}=0.7$ cm.

$\therefore$ Its perimeter = the side length $\times 4$

$=0.7\times4=2.8$ cm.

(4) The area of the square = the area of the triangle

$=\frac{1}{2}\times9\times8=36$ cm².

$\therefore$ The side length of the square $=\sqrt{36}=6$ cm.

(5) $\because$ The area of the circle $=\pi r^2$

$\therefore 154=\frac{22}{7}\times r^2$ $\therefore r^2=154\div\frac{22}{7}=49$

$\therefore r=\sqrt{49}=7$ cm.

(6) $\because$ The area of the circle $=\pi r^2$

$\therefore 78.5=3.14\times r^2$ $\therefore r^2=78.5\div3.14=25$

$\therefore r=\sqrt{25}=5$ cm.

$\therefore$ Its diameter length $=2\times5=10$ cm.

(7) $\because$ The area of the circle $=\pi r^2$

$\therefore 616=\frac{22}{7}\times r^2$ $\therefore r^2=616\div\frac{22}{7}=196$

$\therefore r=\sqrt{196}=14$ cm.

$\therefore$ Its circumference $=2\pi r=2\times\frac{22}{7}\times14=88$ cm.

(8) $\because$ The area of the square $=1\frac{11}{64}\times\frac{4}{3}=\frac{25}{16}$ m².

$\therefore$ The side length $=\sqrt{\frac{25}{16}}=\frac{5}{4}=1\frac{1}{4}$ m.

(9) $2\times$ the width of the rectangle $\times$ the width of the rectangle $=24.5$

$\therefore$ The width of the rectangle $\times$ the width of the rectangle $=12.25$

$\therefore$ The width of the rectangle $=\sqrt{12.25}=3.5$ cm.

$\therefore$ The length of the rectangle $=2\times3.5=7$ cm.

13 $\frac{m^2}{n^2}=\frac{16}{100}$ $\therefore \frac{m}{n}=\pm\sqrt{\frac{16}{100}}=\pm\frac{4}{10}$

$\therefore (\frac{m}{n})^3=(\pm\frac{4}{10})^3=\pm\frac{64}{1000}=\pm0.064$

Answers of Exercise 7

1

(1) $\because X-7=3$ $\therefore X-7+7=3+7$

$\therefore X=10$ $\therefore$ The S.S. $=\{10\}$

(2) $\because X+17=13$ $\therefore X+17-17=13-17$

$\therefore (13-17)\notin \mathbb{N}$ $\therefore$ The S.S. $=\varnothing$

(3) $\because 5X=20$ $\therefore 5X\times\frac{1}{5}=20\times\frac{1}{5}$

$\therefore X=4$ $\therefore$ The S.S. $=\{4\}$

(4) $\because \frac{2}{5}X=\frac{1}{5}$ $\therefore \frac{2}{5}X\times\frac{5}{2}=\frac{1}{5}\times\frac{5}{2}$

$\therefore X=\frac{1}{2}$ $\therefore$ The S.S. $=\{\frac{1}{2}\}$

(5) $\because -4+y+4=13+4$

$\therefore y=17$ $\therefore$ The S.S. $=\{17\}$

(6) $\because m-(-3)=1$ $\therefore m+3-3=1-3$

$\therefore m=-2$ $\therefore$ The S.S. $=\{-2\}$

7 $\because x-7=0$ $\therefore x-7+7=0+7$
$\therefore x=7$ $\therefore$ The S.S. $=\{7\}$

8 $\because y-(-5)=-3$ $\therefore y+5-5=-3-5$
$\therefore y=-8$ $\therefore$ The S.S. $=\{-8\}$

9 $\because x-6\frac{1}{4}=12\frac{1}{2}$
$\therefore x-6\frac{1}{4}+6\frac{1}{4}=12\frac{1}{2}+6\frac{1}{4}$
$\therefore x=18\frac{3}{4}$ $\therefore$ The S.S. $=\{18\frac{3}{4}\}$

10 $\because 8.91+x=11.09$
$\therefore 8.91+x-8.91=11.09-8.91$
$\therefore x=2.18$ $\therefore$ The S.S. $=\{2.18\}$

2

1 $\because 2x-1=5$ $\therefore 2x-1+1=5+1$
$\therefore 2x=6$ $\therefore 2x\times\frac{1}{2}=6\times\frac{1}{2}$
$\therefore x=3$

2 $\because 8x+4=12$ $\therefore 8x+4-4=12-4$
$\therefore 8x=8$ $\therefore 8x\times\frac{1}{8}=8\times\frac{1}{8}$
$\therefore x=1$

3 $\because 3x-13+13=26+13$ $\therefore 3x=39$
$\therefore \frac{3x}{3}=\frac{39}{3}$ $\therefore x=13$

4 $\because 2x+14-14=14-14$ $\therefore 2x=\text{zero}$
$\therefore \frac{2x}{2}=\frac{0}{2}$ $\therefore x=\text{zero}$

5 $\because 8+2x-8=14-8$ $\therefore 2x=6$
$\therefore \frac{2x}{2}=\frac{6}{2}$ $\therefore x=3$

6 $\because \frac{5}{6}x-4+4=11+4$ $\therefore \frac{5}{6}x=15$
$\therefore \frac{5}{6}x\times\frac{6}{5}=15\times\frac{6}{5}$ $\therefore x=18$

7 $\because 8-2x-8=-2-8$ $\therefore -2x=-10$
$\therefore \frac{-2x}{-2}=\frac{-10}{-2}$ $\therefore x=5$

8 $\because 2-5x-2=0-2$ $\therefore -5x=-2$
$\therefore -5x\times-\frac{1}{5}=-2\times-\frac{1}{5}$ $\therefore x=\frac{2}{5}$

9 $\because 2x+3x+25=5$
$\therefore 5x+25-25=5-25$
$\therefore 5x=-20$ $\therefore \frac{5x}{5}=\frac{-20}{5}$
$\therefore x=-4$

10 $\because 6x-2x+7=4$ $\therefore 4x+7-7=4-7$
$\therefore 4x=-3$ $\therefore \frac{4x}{4}=\frac{-3}{4}$
where $-\frac{3}{4}$ is impossible in $\mathbb{Z}$
$\therefore$ The equation has no solution in $\mathbb{Z}$

3

1 $\because 2x-6=4$ $\therefore 2x=10$
$\therefore x=5$

2 $\because 3x+10x-6=7$ $\therefore 13x=13$ $\therefore x=1$

3 $\because 7x-14-3x-3=3$ $\therefore 4x-17=3$
$\therefore 4x=20$ $\therefore x=5$

4 $\because 3x+6+7x-7=12$ $\therefore 10x-1=12$
$\therefore 10x=13$ $\therefore x=1.3$

5 $\because 4x-4-x-3=0$ $\therefore 3x-7=0$
$\therefore 3x=7$ $\therefore x=\frac{7}{3}$

6 $\because 5x-10+2x+8=-16$ $\therefore 7x-2=-16$
$\therefore 7x=-14$ $\therefore x=-2$

7 $\because 2x-6+3x-6-4x=-3$
$\therefore x-12=-3$ $\therefore x=9$

8 $\because 3y+6y+18-8y+16=60$
$\therefore y+34=60$ $\therefore y=26$

4

1 $\because 2x-x=9-5$ $\therefore x=4$
$\therefore$ The S.S. $=\{4\}$

2 $\because 5x-2x=11+4$ $\therefore 3x=15$
$\therefore x=5$ $\therefore$ The S.S. $=\{5\}$

3 $\because x+3x=18-3$ $\therefore 4x=15$
$\therefore x=\frac{15}{4}$ $\therefore$ The S.S. $=\{\frac{15}{4}\}$

4 $\because 3x+5x=30-6$ $\therefore 8x=24$
$\therefore x=3$ $\therefore$ The S.S. $=\{3\}$

5 $\because 4x+4=2x-2$ $\therefore 4x-2x=-2-4$
$\therefore 2x=-6$ $\therefore x=-3$
$\therefore$ The S.S. $=\{-3\}$

6 $\because 3x-6=5x-10$ $\therefore -6+10=5x-3x$
$\therefore 4=2x$ $\therefore x=2$
$\therefore$ The S.S. $=\{2\}$

7 $\because 6a-2=6-2a$ $\therefore 6a+2a=6+2$
$\therefore 8a=8$ $\therefore a=1$
$\therefore$ The S.S. $=\{1\}$

8 $\because 6x-24-2x-2=x-3$
$\therefore 4x-26=x-3$ $\therefore 4x-x=-3+26$
$\therefore 3x=23$ $\therefore x=\frac{23}{3}$
$\therefore$ The S.S. $=\{\frac{23}{3}\}$

9 $\because 4(x+1)=3(x-1)$ $\therefore 4x+4=3x-3$
$\therefore x=-7$ $\therefore$ The S.S. $=\{-7\}$

10 $\because 5(1-2X)=3(4+4X)$

$\therefore 5-10X=12+12X$

$\therefore 5-12=12X+10X \quad \therefore -7=22X$

$\therefore X=-\frac{7}{22} \quad \therefore$ The S.S. $=\{-\frac{7}{22}\}$

5

1 2   2 12   3 10   4 14

5 2   6 9   7 $1\frac{1}{3}$ 8 $(X-5)$ years

9 $(y+4)$ years   10 $(X-5)$ years

11 $(X-6)$ years   12 $\frac{1}{3}X$ 13 $6X$

14 $5-X$ 15 $X+2$

6

1 (a)   2 (d)   3 (c)   4 (c)

5 (a)   6 (a)   7 (c)   8 (b)

7

Since the sum of measures of the angles of the triangle = 180°

1 $X+2X+2X+5°=180°$

$\therefore 5X+5°=180° \quad \therefore 5X=175°$

$\therefore X=35°$

$\therefore$ The measures of the angles of the triangle are 35° , 70° , 75°

2 $y+3y+8y=180° \quad \therefore 12y=180°$

$\therefore y=15° \quad \therefore 3y=45° , 8y=120°$

$\therefore$ The measures of the angles of the triangle are 15° , 45° , 120°

3 $a+\frac{1}{5}a+90°=180° \quad \therefore \frac{5}{5}a+\frac{1}{5}a=90°$

$\therefore \frac{6}{5}a=90°$

$\therefore a=90°\times\frac{5}{6}=75° , \frac{1}{5}a=15°$

$\therefore$ The measures of the angles of the triangle are 75° , 15° , 90°

8

$\because C\in\overleftrightarrow{AB}$ then $X+4X+3X+20°=180°$

$\therefore 8X+20°=180° \quad \therefore 8X=160° \quad \therefore X=20°$

$\therefore m(\angle DCE)=4X=4\times20°=80°$

9

1 $\because \overrightarrow{DE} \parallel \overrightarrow{BC}$, $\overrightarrow{DB}$ is a transversal to them

$\therefore m(\angle DBC)+m(\angle D)=180°$

(Two interior angles in the same side of the transversal)

$\therefore m(\angle DBC)=180°-110°=70°$

$, \because \overrightarrow{AG} \parallel \overrightarrow{BD}$, $\overrightarrow{AB}$ is the transversal

$\therefore m(\angle A)=m(\angle DBC)$ (corresponding angles)

$\therefore 3X+10°=70° \quad \therefore 3X=60°$

$\therefore X=\frac{60°}{3}=20°$

2 $\because \overrightarrow{DE} \parallel \overrightarrow{AB}$, $\overrightarrow{DB}$ is a transversal to them

$\therefore m(\angle D)+m(\angle DBA)=180°$

By the same way : $m(\angle A)+m(\angle DBA)=180°$

$\therefore m(\angle A)=m(\angle D) \quad \therefore X+20°=2X-30°$

$\therefore 2X-X=20°+30° \quad \therefore X=50°$

10

Let the length of the rectangle = X metres

$\therefore$ the width of the rectangle = $(X-4)$ metres

$, \because$ the perimeter = (length + width) × 2

$\therefore 68=(X+X-4)\times 2 \quad \therefore 34=2X-4$

$\therefore 2X=38 \quad \therefore X=38\div2=19$ metres

$\therefore$ the dimensions of the rectangle are 19 metres , 15 metres

11

Let the width of the rectangle = X cm.

$\therefore$ the length of the rectangle = $(2X-4)$ cm.

$, \because$ the perimeter of the rectangle = the perimeter of the square

$\therefore$ (length + width) × 2 = side length × 4

$\therefore (X+2X-4)\times2=7\times4$

$\therefore (3X-4)\times2=28 \quad \therefore 3X-4=14$

$\therefore 3X=18 \quad \therefore X=18\div3=6$ cm.

$\therefore$ the width of the rectangle = 6 cm.

, the length of the rectangle = $2\times6-4=8$ cm.

12 Let the width of the rectangle = X cm.

$\therefore$ the length of the rectangle = $2X$ cm.

$\therefore 2X-5=X+6 \quad \therefore 2X-X=5+6$

$\therefore X=11$

$\therefore$ the width of the rectangle = 11cm.

and the length of the rectangle = 22 cm.

$\therefore$ the area of the rectangle = $11\times22=242$ cm^2

13 $\because 7X-2X=25 \quad \therefore 5X=25$

$\therefore X=\frac{25}{5}=5$

$\therefore$ The two numbers are : 10 , 35

14 Let one of the two numbers be x

$\therefore$ the other number $= 2x$

$\therefore x + 2x = 108$ $\quad \therefore 3x = 108$

$\therefore x = \frac{108}{3} = 36$

$\therefore$ the two numbers are 36 , 72

15 Let the great number be x

$\therefore$ the small number $= x - 5$

$\therefore x + x - 5 = 21$ $\quad \therefore 2x - 5 = 21$

$\therefore 2x = 26$ $\quad \therefore x = \frac{26}{2} = 13$

$\therefore$ The two numbers are 13 , 8

16 Let the number be x $\quad \therefore$ its triple $= 3x$

$\therefore 3x + x = 32$ $\quad \therefore 4x = 32$

$\therefore x = \frac{32}{4} = 8$ $\quad \therefore$ The number is 8

17 Let the number be x $\quad \therefore$ its triple $= 3x$

$\therefore 3x - 9 = 6$ $\quad \therefore 3x = 15$

$\therefore x = \frac{15}{3} = 5$ $\quad \therefore$ The number is 5

18 Let the small number be x

$\therefore$ the middle number $= x + 1$

, the great number $= x + 2$

$\therefore x + x + 1 + x + 2 = 213$ $\quad \therefore 3x + 3 = 213$

$\therefore 3x = 210$ $\quad \therefore x = \frac{210}{3} = 70$

$\therefore$ the numbers are 70 , 71 , 72

19 Let the three numbers be x , $x + 2$, $x + 4$

$\therefore x + x + 2 + x + 4 = 966$ $\quad \therefore 3x + 6 = 966$

$\therefore 3x = 960$ $\quad \therefore x = 320$

$\therefore$ the numbers are 320 , 322 , 324

20 Let the three numbers be x , $x + 2$, $x + 4$

$\therefore x + x + 2 + x + 4 = 357$ $\quad \therefore 3x + 6 = 357$

$\therefore 3x = 351$ $\quad \therefore x = \frac{351}{3} = 117$

$\therefore$ the numbers are 117 , 119 , 121

21 Let the age of the son be x years

$\therefore$ the age of the father $= 3x$

after 2 years the age of the son $= (x + 2)$ years

the age of the father $= (3x + 2)$ years

$\therefore x + 2 + 3x + 2 = 52$ $\quad \therefore 4x + 4 = 52$

$\therefore 4x = 48$ $\quad \therefore x = \frac{48}{4} = 12$

$\therefore$ the age of the son $= 12$ years

, the age of the father $= 36$ years

22 Let the age of Bassim be x years

$\therefore$ Amgad's age $= x + 2$ years ,

Ayman's age $= x - 6$ years

$\therefore x + x + 2 + x - 6 = 89$ $\quad \therefore 3x - 4 = 89$

$\therefore 3x = 93$ $\quad \therefore x = \frac{93}{3} = 31$ years

$\therefore$ the age of Bassim $= 31$ years ,

the age of Amgad $= 31 + 2 = 33$ years ,

the age of Ayman $= 31 - 6 = 25$ years ,

23 Let the price of one metre of silk $= x$ pounds

$\therefore$ the price of one metre of wool $= (x + 2)$ pounds

$\therefore 3(x + 2) + 4x = 671$ $\therefore 3x + 6 + 4x = 671$

$\therefore 7x = 665$ $\quad \therefore x = 95$

$\therefore$ the price of one metre of silk is 95 pounds and the price of one metre of wool is 97 pounds.

24 (1) $\because 5 + 1 = \frac{6}{x}$ $\quad \therefore 6 = \frac{6}{x}$

$\therefore x = 1$ $\quad \therefore$ The S.S. $= \{1\}$

(2) $\because \frac{x}{10} + \frac{x}{5} = -\frac{1}{5} + \frac{3}{5}$ $\quad \therefore \frac{x}{10} + \frac{2x}{10} = \frac{2}{5}$

$\therefore \frac{3x}{10} = \frac{2}{5}$ $\quad \therefore 3x = \frac{2 \times 10}{5}$

$\therefore 3x = 4$ $\quad \therefore x = \frac{4}{3}$

$\therefore$ The S.S. $= \{\frac{4}{3}\}$

25 (1) $\because x^2 + 6x + 9 - (x^2 - 4x + 4) = 15$

$\therefore x^2 + 6x + 9 - x^2 + 4x - 4 = 15$

$\therefore 10x + 5 = 15$ $\quad \therefore 10x = 10$

$\therefore x = 1$ $\quad \therefore$ The S.S. $= \{1\}$

(2) $\because 4x^2 + 4x - 3 - (4x^2 - 4x + 1) = 14$

$\therefore 4x^2 + 4x - 3 - 4x^2 + 4x - 1 = 14$

$\therefore 8x - 4 = 14$ $\quad \therefore 8x = 18$

$\therefore x = \frac{18}{8} = \frac{9}{4}$ $\quad \therefore$ The S.S. $= \{\frac{9}{4}\}$

26 $\because 12x + 3 = 39$ $\quad \therefore 12x = 36$ $\quad \therefore x = 3$

$\therefore$ The S.S. $= \{3\}$

$\therefore x = 3$ satisfies the equation $a x - 12 = a$

$\therefore 3a - 12 = a$ $\therefore 3a - a = 12$ $\quad \therefore 2a = 12$

$\therefore a = 6$

27 $\because$ a + 1 is a solution for the equation

$\therefore x = a + 1$ satisfies the equation.

$\therefore (a + 1 + a)(a + 1 - a) = (a + 1)^2 - a(a + 1) + 3$

$\therefore 2a + 1 = a^2 + 2a + 1 - a^2 - a + 3$

$\therefore 2a + 1 = a + 4 \quad \therefore 2a - a = 4 - 1 \quad \therefore a = 3$

28 Let the age of the boy who was born in 1980 be x

$\therefore$ the ages of the two other boys are $x - 4$, $x - 6$

$\therefore x + x - 4 + x - 6 = 41$

$\therefore 3x - 10 = 41 \quad \therefore 3x = 51 \quad \therefore x = \frac{51}{3} = 17$

$\therefore$ the year in which the sum of their ages became 41 years = 1980 + 17 = 1997

Answers of Exercise 8

1

1) -5 2) 4 3) 7 4) -9
5) 1.5 6) -0.6 7) $2\frac{1}{2}$ 8) $-\frac{1}{3}$

2 $x + 3 - 3 \le 6 - 3 \qquad \therefore x \le 3$

1) The S.S. = $\{3, 2, 1, 0, ...\}$

2) The S.S. = $\{3, 2, 1, 0\}$

3

1) $x + 2 - 2 > 5 - 2 \qquad \therefore x > 3$

$\therefore$ The S.S. = $\{x : x \in \mathbb{Q}, x > 3\}$

2) $x + 4 - 4 > 1 - 4 \qquad \therefore x > -3$

$\therefore$ The S.S. = $\{x : x \in \mathbb{Q}, x > -3\}$

3) $y - 5 + 5 > 7 + 5 \qquad \therefore y > 12$

$\therefore$ The S.S. = $\{y : y \in \mathbb{Q}, y > 12\}$

4) $19 - 14 < y + 14 - 14 \qquad \therefore 5 < y$

$\therefore$ The S.S. = $\{y : y \in \mathbb{Q}, y > 5\}$

5) $-1 + 3 \ge x - 3 + 3 \qquad \therefore 2 \ge x$

$\therefore$ The S.S. = $\{x : x \in \mathbb{Q}, x \le 2\}$

6) $-5\frac{1}{2} - 1\frac{1}{4} > a + 1\frac{1}{4} - 1\frac{1}{4} \qquad \therefore -6\frac{3}{4} > a$

$\therefore$ The S.S. = $\{a : a \in \mathbb{Q}, a < -6\frac{3}{4}\}$

7) $-2x \times -\frac{1}{2} > 12 \times -\frac{1}{2} \qquad \therefore x > -6$

$\therefore$ The S.S. = $\{x : x \in \mathbb{Q}, x > -6\}$

8) $\frac{2}{3}x \times \frac{3}{2} \ge 1 \times \frac{3}{2} \qquad \therefore x \ge \frac{3}{2}$

$\therefore$ The S.S. = $\{x : x \in \mathbb{Q}, x \ge \frac{3}{2}\}$

9) $-\frac{1}{4}x \times -4 \ge \frac{1}{4} \times -4 \qquad \therefore x \ge -1$

$\therefore$ The S.S. = $\{x : x \in \mathbb{Q}, x \ge -1\}$

4

1) $3x - 2 + 2 < 1 + 2 \qquad \therefore 3x < 3$

$\therefore 3x \times \frac{1}{3} < 3 \times \frac{1}{3} \qquad \therefore x < 1$

2) $2x + 3 - 3 < 9 - 3 \qquad \therefore 2x < 6$

$\therefore 2x \times \frac{1}{2} < 6 \times \frac{1}{2} \qquad \therefore x < 3$

3) $4x + 2 - 2 \ge -10 - 2 \qquad \therefore 4x \ge -12$

$\therefore 4x \times \frac{1}{4} \ge -12 \times \frac{1}{4} \qquad \therefore x \ge -3$

4) $3x - 2 + 2 \ge 5 + 2 \qquad \therefore 3x \ge 7$

$\therefore 3x \times \frac{1}{3} \ge 7 \times \frac{1}{3} \qquad \therefore x \ge \frac{7}{3}$

5) $3x - 9 + 9 < 0 + 9 \qquad \therefore 3x < 9$

$\therefore 3x \times \frac{1}{3} < 9 \times \frac{1}{3} \qquad \therefore x < 3$

6) $1 + 2x - 1 \le -3 - 1 \qquad \therefore 2x \le -4$

$\therefore 2x \times \frac{1}{2} \le -4 \times \frac{1}{2} \qquad \therefore x \le -2$

7) $9 - 6x - 9 < 15 - 9 \qquad \therefore -6x < 6$

$\therefore -6x \times -\frac{1}{6} > 6 \times -\frac{1}{6} \qquad \therefore x > -1$

8) $2 - 3x - 2 \le 4 - 2 \qquad \therefore -3x \le 2$

$\therefore -3x \times -\frac{1}{3} \ge 2 \times -\frac{1}{3} \qquad \therefore x \ge -\frac{2}{3}$

9) $\frac{3x - 2}{5} \times 5 \ge \frac{1}{2} \times 5 \qquad \therefore 3x - 2 \ge \frac{5}{2}$

$\therefore 3x - 2 + 2 \ge \frac{5}{2} + 2 \qquad \therefore 3x \ge \frac{9}{2}$

$\therefore 3x \times \frac{1}{3} \ge \frac{9}{2} \times \frac{1}{3} \qquad \therefore x \ge \frac{3}{2}$

10) $5x + 1 - 1 \le 29 - 1 \qquad \therefore 5x \le 28$

$\therefore 5x \times \frac{1}{5} \le 28 \times \frac{1}{5} \qquad \therefore x \le \frac{28}{5}$

11) $4n - 2n + 2 \ge 0 \qquad \therefore 2n + 2 - 2 \ge 0 - 2$

$\therefore 2n \ge -2 \qquad \therefore 2n \times \frac{1}{2} \ge -2 \times \frac{1}{2}$

$\therefore n \ge -1$

12) $-3m + 6m - 24 > 9$

$\therefore 3m - 24 + 24 > 9 + 24 \qquad \therefore 3m > 33$

$\therefore 3m \times \frac{1}{3} > 33 \times \frac{1}{3} \qquad \therefore m > 11$

5

1. $6d - 5d \le -3 - 1$ $\therefore d \le -4$
2. $6X - 5X \ge 14 - 2$ $\therefore X \ge 12$
3. $-2 + 8 < 5X - 3X$ $\therefore 6 < 2X$
   $\therefore 3 < X$ $\therefore X > 3$
4. $8 \le 5X + 2X$ $\therefore 8 \le 7X$
   $\therefore X \ge \frac{8}{7}$
5. $5X + 1 \ge 2X + 4$ $\therefore 5X - 2X \ge 4 - 1$
   $\therefore 3X \ge 3$ $\therefore X \ge 1$
6. $3X + 6 < -X + 4$ $\therefore 3X + X < 4 - 6$
   $\therefore 4X < -2$ $\therefore X < -\frac{2}{4}$
   $\therefore X < -\frac{1}{2}$
7. $3X + 6 \ge -2X - 2$ $\therefore 3X + 2X \ge -2 - 6$
   $\therefore 5X \ge -8$ $\therefore X \ge -\frac{8}{5}$
8. $2 - 3X + 15 \ge X + 7$ $\therefore 17 - 7 \ge X + 3X$
   $\therefore 10 \ge 4X$ $\therefore X \le \frac{10}{4}$
   $\therefore X \le \frac{5}{2}$
9. $21y - 1 \le 20y - 1$
   $\therefore 21y - 20y \le -1 + 1$ $\therefore y \le 0$
10. $3 - 1 \le 2X - \frac{X}{2}$ $\therefore 2 \le \frac{3}{2}X$
   $\therefore X \ge \frac{4}{3}$

6

1. $9 - 1 \le 4X + 1 - 1 \le 17 - 1$ $\therefore 8 \le 4X \le 16$
   $\therefore 8 \times \frac{1}{4} \le 4X \times \frac{1}{4} \le 16 \times \frac{1}{4}$ $\therefore 2 \le X \le 4$
   $\therefore$ The S.S. $= \{2, 3, 4\}$
2. $9 - 2 \le 3X + 2 - 2 < 12 - 2$
   $\therefore 7 \times \frac{1}{3} \le 3X \times \frac{1}{3} < 10 \times \frac{1}{3}$ $\therefore \frac{7}{3} \le X < \frac{10}{3}$
   $\therefore$ The S.S. $= \{X : X \in \mathbb{Q}, \frac{7}{3} \le X < \frac{10}{3}\}$
3. $9 - 6 > X + 6 - 6 > 2 - 6$
   $\therefore 3 > X > -4$ $\therefore$ The S.S. $= \{0, 1, 2\}$

7 1. > 2. < 3. z 4. X 5. 3 > a 6. a > −5 7. < 8. > 9. >

8 1. (b) 2. (d) 3. (d) 4. (d) 5. (b) 6. (d) 7. (b) 8. (c) 9. (c)

9

When $a = 4$ and $b = 3$, then $a > b$
$c = 8$ and $d = 2$, then $c > d$ then $a - c = 4 - 8 = -4$
and $b - d = 3 - 2 = 1$
i.e. $a - c < b - d$
So we see that
If $a > b$ and $c > d$ it is not always correct that $a - c > b - d$

10

1. (✓) 2. (✗)
   For example :
   When $X = -3$ and $y = -5$, then $-3 > -5$ but $-3 < 0$
3. (✓)
4. (✗) For example :
   When $y = \frac{1}{2}$, then $y^2 = \frac{1}{4}$ but $\frac{1}{4} < \frac{1}{2}$
5. (✗) For example :
   When $X = 4$, $y = -2$, it is $4 \times -2 < 0$ $-8 < 0$
6. (✗) For example :
   When $X = -4$, $y = -9$, then
   $X + y = -4 + (-9) = -13$ $-13 < -9$
7. (✗) For example :
   When $y = -1$, $X = 5$, then $y^2 = 1$ but $1 < 5$
8. (✗) For example :
   When $X = 4$, $y = -2$, then $y^2 = 4$, $Xy = -8$
   but $4 > -8$
9. (✗) For example :
   When $X = -2$, $y = -4$, then $Xy = -2 \times -4 = 8$,
   $X^2 = 4$ but $8 > 4$
10. (✗) For example :
   When $X = 2$, $y = 1$, then $X^3 = 8$, $y^2 = 1$
   but $8 > 1$

11

Let the number of shirts be X
$\therefore 40X + 70 \le 200$ $\therefore 40X \le 200 - 70$
$\therefore 40X \le 130$ $\therefore X \le 3.25$
$\therefore$ The greatest number of shirts can be bought is 3 shirts.

12

$a \le 3X - 5 \le b$ $\therefore a + 5 \le 3X \le b + 5$

$\therefore \frac{a+5}{3} \le X \le \frac{b+5}{3}$, $\because 2 \le X \le 5$

$\therefore \frac{a+5}{3} = 2$ $\therefore a + 5 = 6$ $\therefore a = 1$

, $\frac{b+5}{3} = 5$ $\therefore b + 5 = 15$ $\therefore b = 10$

13

1 The greatest possible value of the expression $X + y$ is when $X = 5$, $y = 7 \longrightarrow$ the value $= 5 + 7 = 12$

2 The greatest possible value of the expression $y - X$ is when $y = 7$, $X = -4$

The value $= 7 - (-4) = 11$

3 The smallest possible value of the expression $X y$ is when $X = -4$, $y = 7$

The value $= -4 \times 7 = -28$

4 The smallest possible value of the expression $X^2 + y^2$ is when $X = 0$, $y = 0$

The value $= 0^2 + 0^2 = 0$

Answers of exams on the second part of unit one

Model 1

1

1 (b) 2 (c) 3 (d)

4 (b) 5 (a) 6 (d)

2

1 $X - 9$ 2 $\frac{7}{3} |ab|$ 3 143

4 $\{X : X \in \mathbb{Q} , X > 0\}$ 5 14

3

[a] The S.S. $= \{X : X \in \mathbb{Q} , X \le 1\}$

[b] $\frac{2}{7}$

4

[a] 1 23 2 40

[b] 4 cm. , 8 cm.

5

[a] The S.S. $= \{\frac{15}{4}\}$

[b] The S.S. $= \{X : X \in \mathbb{Q} , \frac{7}{3} \le X < \frac{10}{3}\}$

Model 2

1

1 (c) 2 (b) 3 (a)

4 (a) 5 (b) 6 (a)

2

1 zero 2 0.03 3 3 4 -5 5 $3X$

3

[a] 22

[b] $\frac{2}{5}$

4

[a] The S.S. $= \{\frac{5}{3}\}$

[b] The S.S. $= \{X : X \in \mathbb{Q} , X \le 5\}$

5

[a] The S.S. $= \{X : X \in \mathbb{Q} , X \ge 6\}$

[b] The number is : 7

Answers of Unit Two

Answers of Exercise 9

Answer by yourself.

Answers of Exercise 10

First : Problems on experimental probability

Answer by yourself.

Second : Problems on theoretical probability

1

1. $\frac{2}{3}$ 2. $\frac{1}{3}$ 3. $\frac{1}{2}$ 4. $\frac{1}{6}$ 5. zero
6. 1 7. $\frac{1}{2}$ 8. $\frac{1}{6}$ 9. $\frac{1}{6}$ 10. $\frac{1}{3}$

2

1. zero , 1 2. $\frac{1}{2}$ 3. $\frac{1}{2}$ 4. $\frac{1}{5}$ 5. zero
6. $\frac{5}{9}$ 7. $\frac{1}{12}$, $\frac{11}{12}$ 8. $\frac{3}{8}$ 9. $\frac{1}{3}$ 10. 600

3

1. (d) 2. (b) 3. (a) 4. (b) 5. (b)
6. (c) 7. (b) 8. (d) 9. (b) 10. (c)

4

1. $\because$ The numbers from 1 to 25 and divisible by 5 are 5 , 10 , 15 , 20 , 25 and their number = 5
$\therefore$ The probability = $\frac{5}{25} = \frac{1}{5}$
2. $\because$ The numbers from 1 to 25 and more than or equal to 20 are 20 , 21 , 22 , 23 , 24 , 25 its number is 6
$\therefore$ The probability = $\frac{6}{25}$
3. $\because$ The numbers from 1 to 25 and each of them is a perfect square are 1 , 4 , 9 , 16 , 25 and their number is 5
$\therefore$ The probability = $\frac{5}{25} = \frac{1}{5}$

5

S = {1 , 2 , 3 , 4 , 5 , 6 , 7 , 8}

1. The probability of getting an even number $= \frac{4}{8} = \frac{1}{2}$
2. The probability of getting an odd number $= \frac{4}{8} = \frac{1}{2}$
3. The probability of getting a number greater than or equal to 6 = $\frac{3}{8}$
4. The probability of getting a number divisible by 3 $= \frac{2}{8} = \frac{1}{4}$

6

1. The probability that the letter S = $\frac{1}{5}$
2. The probability that the letter E = $\frac{1}{5}$
3. The probability that the letter R = zero

7

The total number of balls = 5 + 3 + 2 = 10

1. The probability that the ball is yellow = $\frac{3}{10}$ = 0.3
2. The probability that the ball is yellow or red = $\frac{3+5}{10} = \frac{8}{10}$ = 0.8
3. The probability that the ball is not yellow = $\frac{5+2}{10} = \frac{7}{10}$ = 0.7

8

1. The probability that it shows an odd number $= \frac{5}{10} = \frac{1}{2}$
2. The probability that it shows a prime number $= \frac{4}{10} = \frac{2}{5}$
3. The probability that it shows an even number $= \frac{5}{10} = \frac{1}{2}$
4. The probability that it shows an odd number greater than 3 = $\frac{3}{10}$

9

1. The probability of appearing an even number less than or equal to 4 = $\frac{2}{6} = \frac{1}{3}$
2. The probability of appearing a number between 0 and 10 = $\frac{6}{6}$ = 1
3. The probability of appearing a number divisible by 7 = $\frac{0}{6}$ = zero
4. The probability of appearing a number not divisible by 2 = $\frac{3}{6} = \frac{1}{2}$

10

S = {1 , 2 , 3 , 4 , 5 , 6}

1. The probability of getting a number greater than 6 $= \frac{0}{6}$ = zero.

2 The probability of getting a number satisfying the inequality $1 \le X \le 6 = \frac{6}{6} = 1$

3 The probability of getting a number satisfying the inequality $2 < X < 4 = \frac{1}{6}$

11

1 The probability that the card carries a number whose tens digit is even $= \frac{2}{8} = \frac{1}{4}$

2 The probability that the card carries a number whose units digit is odd $= \frac{2}{8} = \frac{1}{4}$

3 The probability that the card carries a number multiple of $4 = \frac{4}{8} = \frac{1}{2}$

12

1 $S = \{1 , 2 , 3\}$

2 The probability that the appearant number on the upper face is $2 = \frac{1}{3}$

3 The probability that the appearant number is odd $= \frac{2}{3}$

13 The number of red marbles $= \frac{2}{5} \times 30 = 12$ marbles

14

$\because$ The probability of drawing a red ball $= \frac{1}{4}$

$\therefore$ The probability of drawing a blue ball $= \frac{3}{4}$

$\therefore$ The number of blue balls $= \frac{3}{4} \times 80 = 60$ balls

15

$S = \{22 , 32 , 52 , 33 , 23 , 53 , 55 , 25 , 35\}$

1 The probability that the tens digit is odd $= \frac{6}{9} = \frac{2}{3}$

2 The probability that the units digit is odd $= \frac{6}{9} = \frac{2}{3}$

3 The probability that the sum of the two digits $7 = \frac{2}{9}$

4 The probability that the product of the two digits $15 = \frac{2}{9}$

16

$\because$ The number of red marbles $= 22 - 12 = 10$ and after drawing two red marbles the rest marbles will be 20 and the red marbles is 8

$\therefore$ The probability that the drawn marble is black $= \frac{12}{20} = \frac{3}{5}$

17

The number of girls = 20 , The number of boys = 30

$\therefore$ The probability that the student is a boy $= \frac{30}{50} = \frac{3}{5}$

18

1 (c) 2 (c) 3 (b) 4 (d)
5 (c) 6 (b) 7 (b)

19

1 (a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{4}{8} = \frac{1}{2}$

2 The probability that the pointer does not stop at the red colour = The probability that the pointer stops at green or yellow $= \frac{7}{8}$

20

The probability that the pointer stops at yellow colour $= 1 - (\frac{1}{8} + \frac{1}{4} + \frac{3}{8}) = 1 - \frac{6}{8} = \frac{2}{8}$

$\therefore$ The probability that the pointer stops at the yellow or red colour $= \frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

21

1 The probability that the student succeeded in math $= \frac{30}{40} = \frac{3}{4}$

2 The probability that the student succeeded in science $= \frac{24}{40} = \frac{3}{5}$

3 The probability that the student failed in science $= 1 - \frac{3}{5} = \frac{2}{5}$

4 $\because$ The number of succeeded students in both math and science is 20 students.

$\therefore$ The number of students who succeeded in math only $= 30 - 20 = 10$ students

$\therefore$ The number of students who succeeded in science only $= 24 - 20 = 4$ students

$\therefore$ The number of students who failed in both math and science $= 40 - (20 + 10 + 4) = 6$ students

$\therefore$ The probability that the student failed in both math and science $= \frac{6}{40} = \frac{3}{20}$

22

$\because$ The probability that the first player scores a goal $= \frac{18}{21} \approx 0.86$

The probability that the second player scores a goal $= \frac{25}{32} \approx 0.78$

$\because 0.86 > 0.78$

$\therefore$ The best is choosing the first player because his probability is the greater.

23

1. No : because the product of an odd number × an even number = an even number.
   i.e. The probability of getting an even number is the greater.
2. Souad : because she wins when the result is an even number and its probability is the greater.

24

The probability of shooting the shaded part

$= \frac{\text{The area of the shaded rectangle}}{\text{The area of the external rectangle}} = \frac{5 \times 10}{20 \times 10} = \frac{1}{4}$

25

1. $\because$ The area of the part E $= \frac{1}{2} \times \frac{1}{4}$ the area of the rectangle $= \frac{1}{8}$ the area of the rectangle
   $\therefore$ The probability of shooting the part E $= \frac{1}{8}$
2. $\because$ The area of one of the parts A , B , C $= \frac{1}{4} \times \frac{1}{2}$ the area of the rectangle $= \frac{1}{8}$ the area of the rectangle
   $\therefore$ The area of the part formed from A , B , C together $= \frac{3}{8}$ the area of the rectangle.
   $\therefore$ The probability of shooting the part formed from A , B , C together $= \frac{3}{8}$

26

$\because$ The probability of drawing a red ball $= \frac{2}{3}$

$\therefore$ The probability of drawing a white ball $= 1 - \frac{2}{3} = \frac{1}{3}$

$\therefore$ The total number of balls $= 3 \times 5 = 15$ balls

27

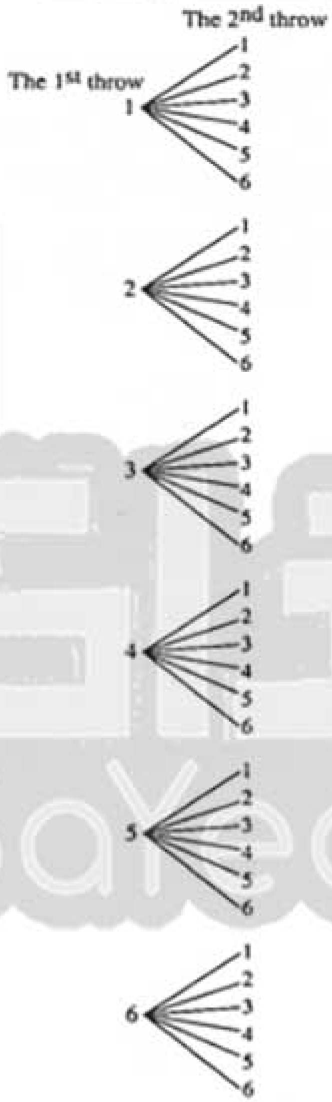
The probability that the drawn card carries a number less than or equal to 8

$= 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore$ The number of cards $= 8 \times \frac{3}{2} = 12$ cards

$\therefore n = 12$

28



$\therefore$ The probability of appearance of the number 3 in the two times $= \frac{1}{36}$

Answer of exam on unit two

1
1 (a) 2 (d) 3 (d)
4 (b) 5 (c) 6 (c)

2
1 zero 2 $\frac{1}{2}$ 3 $\frac{4}{7}$
4 $\frac{1}{5}$ 5 $\frac{1}{6}$

3
1 $\frac{1}{3}$ 2 $\frac{8}{15}$ 3 1 4 $\frac{2}{3}$

4
1 $\frac{1}{2}$ 2 $\frac{1}{6}$ 3 $\frac{5}{6}$
4 zero 5 1

5
1 $\frac{2}{3}$ 2 $\frac{1}{3}$ 3 $\frac{2}{9}$

Answers of accumulative basic skills

1
1 (c) 2 (b) 3 (a) 4 (c)
5 (b) 6 (a) 7 (b) 8 (a)
9 (c) 10 (c) 11 (d) 12 (d)
13 (c) 14 (a) 15 (c)

2
1 30.398 2 −2 3 $\frac{1}{9}$ 4 11
5 $\frac{9x}{8}$ 6 90 7 50 8 $\frac{5}{6}$
9 10 10 40 11 25 12 15
13 1 14 3 15 15

Guide Answers

of Geometry and Measurement Exercises

Answers of Unit Three

Answers of Exercise 1

1 1 Given 2 $A \in \overline{CE}$ 3 Given
4 Corresponding angles. 5 Alternate angles

2

Given :
$m(\angle AMB) = 50°$, $m(\angle EMD) = 80°$,
$m(\angle CMD) = 65°$, $\overrightarrow{MC}$ bisects $\angle BMD$
R.T.F. : $m(\angle AME)$
Proof :
$\because$ $\overrightarrow{MC}$ bisects $\angle BMD$ (given)
$\therefore m(\angle BMC) = m(\angle CMD) = 65°$
$\because m(\angle AMB) + m(\angle BMC) + m(\angle CMD) + m(\angle DME) + m(\angle AME) = 360°$
$\therefore m(\angle AME) = 360° - 260° = 100°$ (The req.)

3

Given : $\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$, $m(\angle BMC) = 120°$,
$\overrightarrow{ME}$ bisects $\angle AMD$
R.T.F. : $m(\angle EMC)$
Proof :
$\because \overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$
$\therefore m(\angle BMC) = m(\angle AMD)$ (V.O.A.)
$\therefore m(\angle AMD) = 120°$,
$\because$ $\overrightarrow{ME}$ bisects $\angle AMD$
$\therefore m(\angle AME) = m(\angle EMD)$
$\therefore m(\angle EMD) = \frac{120°}{2} = 60°$
$\because M \in \overleftrightarrow{BD}$
$\therefore m(\angle BMC) + m(\angle CMD) = 180°$
$\therefore m(\angle DMC) = 180° - 120° = 60°$
$\because m(\angle EMC) = m(\angle EMD) + m(\angle DMC)$
$\therefore m(\angle EMC) = 60° + 60° = 120°$ (The req.)

4

Given :
$AB = AC$, $BD = CD$
R.T.P. : $\overrightarrow{AD}$ bisects $\angle BAC$
Proof : $\because$ In $\Delta\Delta ADB$, ADC :
$\begin{cases} AB = AC \text{ (given)} \\ BD = CD \text{ (given)} \\ \overline{AD} \text{ is a common side} \end{cases}$
$\therefore \Delta ADB \equiv \Delta ADC$, then we deduce that :
$m(\angle BAD) = m(\angle CAD)$
$\therefore \overrightarrow{AD}$ bisects $\angle BAC$ (Q.E.D.)

5

$\because m(\angle EBD) + m(\angle CBD) + m(\angle ABC) + m(\angle ABE) = 360°$
$\therefore m(\angle EBD) + 35° + 110° + 140° = 360°$
$\therefore m(\angle EBD) = 360° - 285° = 75°$ (The req.)

6

$\because B \in \overleftrightarrow{FD}$
$\therefore m(\angle DBC) = 180° - (30° + 40°) = 110°$ (The req.)

7

$\because B \in \overleftrightarrow{AC}$
$\therefore m(\angle ABE) + m(\angle CBE) = 180°$
$\therefore m(\angle ABE) = 180° - 116° = 64°$
, $\because$ $\overrightarrow{BD}$ bisects $\angle ABE$
$\therefore m(\angle ABD) = m(\angle DBE) = 64° \div 2 = 32°$ (The req.)

8

$\because \overleftrightarrow{AC} \cap \overleftrightarrow{DE} = \{B\}$
$\therefore m(\angle CBE) = m(\angle ABD) = 40°$ (V.O.A.)
$\because$ $\overrightarrow{BE}$ bisects $\angle CBF$
$\therefore m(\angle FBE) + m(\angle EBC) = 80°$
$\therefore m(\angle FBC) = 80°$
$\because B \in \overleftrightarrow{AC}$
$\therefore m(\angle ABF) = 180° - 80° = 100°$ (The req.)

9

$\because m(\angle BEC) + m(\angle AED) + m(\angle AEB) + m(\angle CED) = 360°$
$\therefore m(\angle BEC) + m(\angle AED) + 65° + 85° = 360°$
$\therefore m(\angle BEC) + m(\angle AED) = 360° - (65° + 85°) = 210°$
, $\because m(\angle BEC) = m(\angle AED)$
$\therefore m(\angle BEC) = \frac{210°}{2} = 105°$ (First req.)
, $\because m(\angle AEB) + m(\angle BEC) = 65° + 105° = 170° \neq 180°$
$\therefore$ A , E and C are not on the same straight line.
(Second req.)

10

1 $m(\angle ABC) = 180° - 72° = 108°$
(Two interior angles in the same side of the transversal)

2 m (∠ ABC) = m (∠ BAD) = 57° (Alternate angles)

3 m (∠ ABC) = m (∠ EAD) = 63°
(Corresponding angles)

11

∵ m (∠ EAC) + m (∠ BAC) + m (∠ EAB) = 360°

∴ m (∠ BAC) = 360° − (130° + 90°) = 140° (First req.)

∴ $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$, $\overleftrightarrow{AC}$ is a transversal to them

∵ m (∠ C) + m (∠ CAB) = 180°
(Two interior angles in the same side of the transversal)

∴ m (∠ C) = 180° − 140° = 40° (Second req.)

12

∵ $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$, $\overleftrightarrow{AC}$ is a transversal to them

∴ m (∠ ACD) + (∠ A) = 180°
(Two interior angles in the same side of the transversal)

∴ m (∠ ACD) = 180° − 60° = 120° (1)

∵ $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$, $\overleftrightarrow{AB}$ // $\overleftrightarrow{EF}$

∴ $\overleftrightarrow{CD}$ // $\overleftrightarrow{EF}$, $\overleftrightarrow{CE}$ is a transversal to them.

∴ m (∠ DCE) + m (∠ E) = 180°
(Two interior angles in the same side of the transversal)

∴ m (∠ DCE) = 180° − 35° = 145° (2)

∵ m (∠ ACD) + m (∠ DCE) + m (∠ ACE) = 360°

From (1) and (2)

∴ m (∠ ACE) = 360° − (120° + 145°) = 95° (The req.)

13 ∵ $\overleftrightarrow{DE}$ // $\overleftrightarrow{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

∴ m (∠ B) = m (∠ DAB) = 80° (Alternate angles)

∵ $\overleftrightarrow{DE}$ // $\overleftrightarrow{BC}$, $\overleftrightarrow{AC}$ is a transversal to them.

∴ m (∠ C) = m (∠ EAC) = 50° (Alternate angles)

, ∵ A ∈ $\overline{DE}$

∴ m (∠ BAC) = 180° − (50° + 80°) = 50° (The req.)

14 ∵ $\overrightarrow{OA}$ // $\overline{CE}$, $\overleftrightarrow{DC}$ is a transversal to them

∴ m (∠ DOA) = m (∠ C) = 60°
(Corresponding angles)

, ∵ $\overrightarrow{OD}$ // $\overrightarrow{AB}$, $\overleftrightarrow{AO}$ is a transversal to them.

∴ m (∠ BAX) = m (∠ DOA) = 60°
(Corresponding angles)

, ∵ $\overrightarrow{OA}$ // $\overline{CE}$, $\overleftrightarrow{AE}$ is a transversal to them.

∴ m (∠ XAE) = m (∠ E) = 70° (Alternate angels)

∴ m (∠ BAE) = 60° + 70° = 130° (The req.)

15

1 ∵ E ∈ $\overleftrightarrow{AB}$ ∴ m (∠ BEF) = 180° − 122° = 58°

∴ m (∠ BEF) = m (∠ EFC) (Alternate angles)

∴ $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$ (Q.E.D.)

2 ∵ E ∈ $\overleftrightarrow{AB}$

∴ m (∠ BEF) = 180° − 100° = 80°

∴ m (∠ BEF) = m (∠ DFN) (Corresponding angles)

∴ $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$ (Q.E.D.)

3 ∵ m (∠ BEF) = m (∠ AEM) = 132° (V.O.A.)

∴ m (∠ BEF) = m (∠ DFN) (Corresponding angles)

∴ $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$ (Q.E.D.)

16

∵ $\overleftrightarrow{AB}$ // $\overline{CD}$, $\overleftrightarrow{AC}$ is a transversal to them.

∴ m (∠ ACD) = m (∠ A) = 50° (Alternate angles)

, ∵ m (∠ ACE) = 90°

∴ m (∠ DCE) = 90° − 50° = 40°

∴ m (∠ DCE) = m (∠ E) (But they are alternate angles)

∴ $\overline{CD}$ // $\overline{EF}$

, ∵ $\overline{AB}$ // $\overline{CD}$ ∴ $\overline{AB}$ // $\overline{EF}$ (Q.E.D.)

17

∵ $\overline{EF}$ // $\overline{CD}$, $\overline{EC}$ is a transversal to them.

∴ m (∠ ECD) = m (∠ CEF) = 95° (Alternate angles)

∴ m (∠ ACD) = 95° − 30° = 65°

∴ m (∠ ACD) + m (∠ A) = 65° + 115° = 180°

(But they are interior angles in the same side of the transversal)

∴ $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$, ∵ $\overleftrightarrow{CD}$ // $\overleftrightarrow{EF}$

∴ $\overleftrightarrow{AB}$ // $\overleftrightarrow{EF}$ (Q.E.D.)

18 ∵ Δ ABC ≡ Δ DEF

∴ m (∠ ACB) = m (∠ DFE)
(But they are corresponding angles)

∴ $\overline{BC}$ // $\overline{EF}$ (Q.E.D.)

19

∵ $\overline{AD}$ ∩ $\overline{BC}$ = {M}

∴ m (∠ AMB) = m (∠ DMC) (V.O.A.)

∴ In Δ Δ AMB and DMC

$$\begin{cases} AM = DM \\ BM = CM \\ m(\angle AMB) = m(\angle DMC) \end{cases}$$

∴ The two triangles are congruent
and we deduce that AB = CD (Q.E.D.1)
, m (∠ A) = m (∠ D)
(But they are alternate angles)

∴ $\overline{AB}$ // $\overline{CD}$ (Q.E.D.2)

20

$\because R \in \overline{DX}$

$\therefore m(\angle FRC) = 180° - 110° = 70°$

$\therefore m(\angle FCR) = m(\angle FRC) = 70°$

$\therefore m(\angle FCR) = m(\angle A)$ (But they are alternate angles)

$\therefore \overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$ (First req.)

, $\because C \in \overrightarrow{DR}$ $\quad \therefore m(\angle DCF) = 180° - 70° = 110°$

$\therefore m(\angle AFE) = m(\angle DCF) = 110°$

(Correspodning angles)

, $\because \overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$, $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$ $\quad \therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$

, $\because \overleftrightarrow{AF}$ a transversal to them

$\therefore m(\angle AFE) = 180° - m(\angle A) = 180° - 70° = 110°$

, $\because m(\angle DCF) = 110°$

$\therefore m(\angle EFC) = 180° - 110° = 70°$

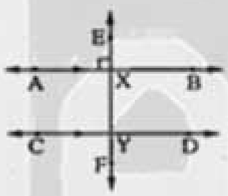
(interior angles in the same side of the transversal)

, $F \in \overline{AC}$

$\therefore m(\angle AFE) = 180° - 70° = 110°$ (Second req.)

21

1. Let $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{EF} \perp \overleftrightarrow{AB}$



$\because \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ is a transversal to them.

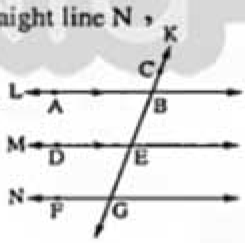
$\therefore m(\angle XYC) = m(\angle EXA)$

(Corresponding angles)

$\therefore m(\angle XYC) = 90°$

$\therefore \overleftrightarrow{EF} \perp \overleftrightarrow{CD}$ (Q.E.D.)

2. Let the straight line L // the straight line M , The straight line L // the straight line N , , the straight line K is a transversal to all of them.



$\because$ The straight line L // the straight line M , and the straight line K is a transversal to them.

$\therefore m(\angle ABC) = m(\angle DEB)$

(corresponding angles) ,

$\therefore$ The straight line L // the straight line N , and the straight line K is a transversal to them.

$\therefore m(\angle ABC) = m(\angle FGE)$

(Corresponding angles)

$\therefore m(\angle DEB) = m(\angle FGE)$

(But they are correspoding angles)

$\therefore$ The straight line M // the straight line N (Q.E.D.)

22 $\Delta\Delta$ ABD , CBD in them :

$$\begin{cases} AB = CB \\ AD = CD \\ \overline{BD} \text{ is a common side} \end{cases}$$

$\therefore \Delta ABD \equiv \Delta CBD$, then we deduce that :

$m(\angle ADB) = m(\angle CDB)$

$\therefore \overrightarrow{DB}$ bisects $\angle ADC$ (Q.E.D. 1)

$\because \Delta\Delta$ ADF , CDF in them :

$$\begin{cases} AD = CD \\ \overline{DF} \text{ is a common side} \\ m(\angle ADF) = m(\angle CDF) \end{cases}$$

$\therefore \Delta ADF \equiv \Delta CDF$, Then we deduce that :

$m(\angle AFD) = m(\angle CFD)$

, $\because F \in \overline{AC}$

$\therefore m(\angle AFD) + m(\angle CFD) = 180°$

$\therefore m(\angle AFD) = m(\angle CFD) = \frac{180°}{2} = 90°$

$\therefore \overline{AC} \perp \overline{DB}$ (Q.E.D. 2)

23

$\because \overline{AE} \cap \overline{DC} = \{F\}$

$\therefore m(\angle AFD) = m(\angle EFC)$ (V.O.A.)

$\therefore \Delta\Delta$ AFD and EFC in them :

$$\begin{cases} m(\angle D) = m(\angle FCE) \\ m(\angle AFD) = m(\angle EFC) \\ FD = FC \end{cases}$$

$\therefore \Delta AFD \equiv \Delta EFC$

Then we deduce that CE = AD

, $\because$ ABCD is a square

$\therefore BC = AD$ $\quad \therefore CE = CB$ (Q.E.D.)

24

$\Delta\Delta$ ABD , CDB in them :

$$\begin{cases} AD = CB \\ \overline{BD} \text{ is a common side} \\ m(\angle ADB) = m(\angle CBD) \end{cases}$$

$\therefore \Delta ABD \equiv \Delta CDB$, then we deduce that :

AB = CD (Q.E.D. 1)

$m(\angle ABD) = m(\angle CDB)$

(And they are two alternate angles)

$\therefore \overline{AB} \parallel \overline{CD}$ (Q.E.D. 2)

25

ΔΔ DXB , EYC in them :

$\begin{cases} DB = EC \\ DX = EY \\ m(\angle DXB) = m(\angle EYC) = 90^\circ \end{cases}$

$\therefore \Delta DXB \equiv \Delta EYC$, Then we deduce that :

$m(\angle B) = m(\angle C)$ (1)

$\because \overline{BC} \,/\!/\, \overline{DE}$ and $\overleftrightarrow{AB}$ is a transversal to them

$\therefore m(\angle ADE) = m(\angle B)$ (2)

(Corresponding angles)

Similarly : $m(\angle AED) = m(\angle C)$ (3)

(Corresponding angles)

From (1) and (2) and (3) :

$\therefore m(\angle ADE) = m(\angle AED)$ (Q.E.D.)

26

ΔΔ ABD , ACE in them :

$\begin{cases} AD = AE \\ \angle A \text{ is a common angle} \\ m(\angle ADB) = m(\angle AEC) = 90^\circ \end{cases}$

$\therefore \Delta ABD \equiv \Delta ACE$

$\therefore BD = CE$ (Q.E.D.)

27

ΔΔ ABE , ACD in them :

$\begin{cases} AE = AD \\ m(\angle AEB) = m(\angle ADC) \\ \angle A \text{ is a common angle} \end{cases}$

$\therefore \Delta ABE \equiv \Delta ACD$

Then we deduce that : BE = CD (Q.E.D. 1)

, AB = AC $\therefore AD + DB = AE + EC$

$\because AD = AE$

$\therefore DB = EC$ (Q.E.D. 2)

28

1 $\because 3x - 5^\circ = 70^\circ$ (V.O.A.)

$\therefore 3x = 70^\circ + 5^\circ = 75^\circ$

$\therefore x = \frac{75^\circ}{3} = 25^\circ$

2 $\because x^2 = 36^\circ + 64^\circ = 100^\circ$ (V.O.A.)

$\therefore x = \pm 10^\circ$

3 $\because (3x - 8^\circ) + x = 180^\circ$ $\therefore 4x - 8^\circ = 180^\circ$

$\therefore 4x = 180^\circ + 8^\circ = 188^\circ$

$\therefore x = \frac{188^\circ}{4} = 47^\circ$

4 $x = 60^\circ$ (Alternate angles) ,

$y = 61^\circ$ (Corresponding angles)

5 $\because 3x + 90^\circ = 180^\circ$

$\therefore 3x = 180^\circ - 90^\circ = 90^\circ$

$\therefore x = \frac{90^\circ}{3} = 30^\circ$, $\because 4y + 5y = 180^\circ$

$\therefore 9y = 180^\circ$ $\therefore y = \frac{180^\circ}{9} = 20^\circ$

6 $\because m(\angle ABC) = 180^\circ - 120^\circ = 60^\circ$

$\therefore x = 60^\circ$ (Corresponding angles)

$\therefore 3y + 6^\circ = x$ (Corresponding angles)

$\therefore 3y = 60^\circ - 6^\circ = 54^\circ$

$\therefore y = \frac{54^\circ}{3} = 18^\circ$

29

$\because m(\angle AMB) + m(\angle AMD) = 115^\circ + 45^\circ = 160^\circ$

$\therefore m(\angle BMD) = 160^\circ$

$\because m(\angle BMD) + m(\angle BMC) + m(\angle CMD) = 360^\circ$

$\therefore m(\angle BMC) + m(\angle CMD) = 360^\circ - 160^\circ = 200^\circ$

$\therefore 3x + 5x = 200$ $\therefore 8x = 200$

$\therefore x = 25^\circ$ (First req.) $\therefore m(\angle BMC) = 25 \times 3 = 75^\circ$

$\therefore m(\angle BMC) + m(\angle AMB) = 75^\circ + 115^\circ = 190^\circ$

$\therefore$ A , M and C are not collinear (Second req.)

30

Yes $\Delta ADE \equiv \Delta CBF$ (Two angles and side) (Q.E.D. 1)

Then we deduce that DE = BF

, AE = FC

, $m(\angle AED) = m(\angle CFB)$

$\therefore m(\angle DEF) = m(\angle BFE)$

Δ Δ DEF , BFE in them :

$\begin{cases} DE = BF \\ \overline{FE} \text{ is a common side} \\ m(\angle DEF) = m(\angle BFE) \end{cases}$

$\therefore \Delta DEF \equiv \Delta BFE$ (Q.E.D. 2)

Then we deduce that : DF = BE

$m(\angle BEF) = m(\angle DFE)$

$\therefore m(\angle BEA) = m(\angle DFC)$

$\therefore$ ΔΔ ABE , CDF in them :

$\begin{cases} AE = FC \\ DF = BE \\ m(\angle BEA) = m(\angle DFC) \end{cases}$

$\therefore \Delta ABE \equiv \Delta CDF$ (Q.E.D. 3)

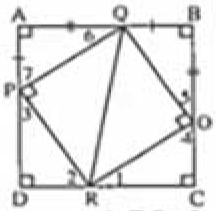
31

Yes Δ PAQ $\equiv$ Δ QBO

"two sides and included angle" (Q.E.D. 1)

, then we deduce that : PQ = QO

$\therefore$ $\Delta\Delta$ PQR , OQR in them :



$\left\{\begin{array}{l} \overline{PQ} = QO \\ \overline{QR} \text{ is a common side} \\ m(\angle QPR) = m(\angle ROQ) = 90^\circ \end{array}\right.$

$\therefore$ Δ PQR $\equiv$ Δ OQR (Q.E.D. 2)

, then we deduce that : PR = OR

, $\because$ Δ PAQ $\equiv$ Δ QBO

$\therefore m(\angle 5) = m(\angle 6)$

, $\because m(\angle 5) + m(\angle 4) = 90^\circ$

$\therefore m(\angle 6) + m(\angle 4) = 90^\circ$

, $\because m(\angle 4) + m(\angle 1) = 90^\circ$

$\therefore m(\angle 6) = m(\angle 1)$ (1)

, $\because m(\angle 6) + m(\angle 7) = 90^\circ$

, $m(\angle 3) + m(\angle 7) = 90^\circ$

$\therefore m(\angle 6) = m(\angle 3)$ (2)

From (1) and (2) :

$\therefore m(\angle 1) = m(\angle 3)$

$\therefore$ From $\Delta\Delta$ PDR , RCO :

$m(\angle 1) = m(\angle 3)$, $m(\angle C) = m(\angle D)$,

$m(\angle 4) = m(\angle 2)$

$\therefore$ In $\Delta\Delta$ PDR , RCO :

$\left\{\begin{array}{l} m(\angle 3) = m(\angle 1) \\ m(\angle 2) = m(\angle 4) \\ PR = RO \end{array}\right.$

$\therefore$ Δ PDR $\equiv$ Δ RCO (Q.E.D. 3)

Answers of Exercise 2

1

1 (a) All its sides are equal in length.

(b) All its angles are equal in measure.

2 360°	3 540°	4 720°	5 900°
6 108° , 128 $\frac{4}{7}$°		7 360°	
8 5 , 120°		9 135°	

2

1 (b)	2 (c)	3 (c)	4 (d)
5 (d)	6 (c)	7 (c)	

3

$\because$ The number of diagonals of the polygon of n sides $= \frac{n(n-3)}{2}$

1 The number of diagonals of the triangle $= \frac{3(3-3)}{2}$ = zero

2 The number of diagonals of the quadrilateral $= \frac{4(4-3)}{2} = 2$

3 The number of diagonals of the pentagon $= \frac{5(5-3)}{2} = 5$

4

Fig. (1) : $m(\angle D) = 360^\circ - (90^\circ + 100^\circ + 80^\circ) = 90^\circ$

Fig. (2) : $m(\angle D) = 540^\circ - (90^\circ + 90^\circ + 112^\circ + 130^\circ)$
$= 118^\circ$

5

Given : $\overline{AE} // \overline{BC}$, $m(\angle B) = 70^\circ$,
$m(\angle C) = 150^\circ$, $m(\angle D) = 80^\circ$

R.T.F. : $m(\angle E)$

Proof :

$\because \overline{AE} // \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore m(\angle A) + m(\angle B) = 180^\circ$

(Two interior angles in the same side of the transversal)

$\because m(\angle B) = 70^\circ$

$\therefore m(\angle A) = 180^\circ - 70^\circ = 110^\circ$

$\because$ ABCDE is a pentagon

$\therefore$ The sum of measures of its interior angles
$= (5-2) \times 180^\circ = 540^\circ$

$\therefore m(\angle E) = 540^\circ - (70^\circ + 150^\circ + 80^\circ + 110^\circ)$
$= 130^\circ$ (The req.)

6

Given : $\overline{CE} \cap \overline{AD} = \{B\}$, $m(\angle A) = 50^\circ$,
$m(\angle C) = 70^\circ$, $m(\angle D) = 130^\circ$,
$m(\angle F) = 90^\circ$

R.T.F. : $m(\angle E)$

Proof :

$\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore$ In Δ ABC :

$m(\angle ABC) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ$

$\because \overline{CE} \cap \overline{AD} = \{B\}$

$\therefore m(\angle EBD) = m(\angle ABC) = 60°$ (V.O.A.)

$\because$ The sum of measures of the interior angles of the quadrilateral $= 360°$

$\therefore m(\angle E) = 360° - (90° + 130° + 60°) = 80°$ (The req.)

7

$\because Z \in \overrightarrow{YF}$

$\therefore m(\angle LZY) = 180° - 120° = 60°$

From the quadrilateral XYZL

$\therefore m(\angle X) = 360° - (70° + 60° + 90°) = 140°$ (The req.)

8

From the hexagon ABCDEF :

$m(\angle A) + m(\angle C) + m(\angle B) + m(\angle D) + m(\angle E) + m(\angle F) = 720°$

$\therefore m(\angle A) + m(\angle C) = 720° - (90° + 115° + 165° + 110°) = 240°$

$\because m(\angle FAB) = m(\angle DCB) = \frac{240°}{2} = 120°$

$\therefore X = 120°$ (The req.)

9

From the quadrilateral ABCD :

$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360°$

$\therefore 90° + 3X + 5X + 2X = 360°$

$\therefore 10X = 360° - 90° = 270°$

$\therefore X = \frac{270°}{10} = 27°$ (The req.)

10

From the figure (The quadrilateral ABCD) :

$m(\angle C) = 360° - (120° + 85° + 90°) = 65°$

$\because \overrightarrow{BE} \parallel \overline{CD}$, $\overrightarrow{BC}$ is a transversal to them

$\therefore m(\angle C) + m(\angle EBC) = 180°$

(Two interior angles in the same side of the transversal)

$\therefore m(\angle EBC) = 180° - 65° = 115°$

$\therefore m(\angle ABE) = 115° - 85° = 30°$ (The req.)

11

$\because \Delta DFE$ is an equilateral triangle

$\therefore m(\angle FDE) = \frac{180°}{3} = 60°$

$\because \overline{AE} \cap \overline{CF} = \{D\}$

$\therefore m(\angle ADC) = m(\angle FDE) = 60°$ (V.O.A.)

$\therefore$ From the quadrilateral ABCD :

$\therefore m(\angle B) = 360° - (120° + 60° + 105°) = 75°$ (The req.)

12

From the quadrilateral ARFE :

$\therefore m(\angle RAE) = 360° - (120° + 45° + 105°) = 90°$

$, \because \overline{ED} \cap \overline{RB} = \{A\}$

$\therefore m(\angle DAB) = m(\angle RAE) = 90°$ (V.O.A.)

From the quadrilateral ABCD :

$m(\angle B) = 360° - (90° + 130° + 80°) = 60°$ (The req.)

13

From ΔABC :

$m(\angle B) = 180° - (30° + 58°) = 92°$

$\because \overline{AB} \parallel \overline{DE}$, $\overline{BD}$ is a transversal to them

$\therefore m(\angle D) = m(\angle B) = 92°$ (alternate angles)

$, \because \overline{AF} \cap \overline{BD} = \{C\}$

$\therefore m(\angle DCF) = m(\angle ACB) = 58°$ (V.O.A.)

$\therefore$ From the quadrilateral CFED

$m(\angle E) = 360° - (58° + 125° + 92°) = 85°$ (The req.)

14

From the quadrilateral EOCY

$m(\angle OCY) = 360° - (50° + 90° + 90°) = 130°$

$, \because \overleftrightarrow{DY} \cap \overleftrightarrow{OA} = \{C\}$

$\therefore m(\angle DCA) = m(\angle OCY) = 130°$ (V.O.A.)

$, \because \overline{CD} \parallel \overline{AB}$, $\overline{AC}$ is a transversal to them

$\therefore m(\angle DCA) + m(\angle BAC) = 180°$

(Two interior angles in the same side of the transversal)

$\therefore X = 180° - 130° = 50°$ (The req.)

15

From the quadrilateral ABCD :

$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360°$

$\therefore m(\angle B) + m(\angle D) = 360° - (95° + 85°) = 180°$

$\because m(\angle B) = \frac{1}{2} m(\angle D)$

$\therefore m(\angle D) + \frac{1}{2} m(\angle D) = 180°$

$\therefore \frac{3}{2} m(\angle D) = 180°$ $\therefore m(\angle D) = \frac{180° \times 2}{3} = 120°$

$\therefore m(\angle B) = 60°$ (The req.)

16

1. $X = \frac{(5-2) \times 180°}{5} = 108°$
2. $y = \frac{(8-2) \times 180°}{8} = 135°$, $X = 180° - 135° = 45°$

3 $2X = \frac{(6-2)\times 180°}{6} = 120°$

$\therefore X = \frac{120°}{2} = 60°$, $y = 180° - 120° = 60°$

, $z = \frac{180°}{3} = 60°$

4 $y = \frac{(8-2)\times 180°}{8} = 135°$ $\therefore X = \frac{135°}{2} = 67.5°$

, $z = 180° - 135° = 45°$, $a = 360° - 135° = 225°$

17

1 $\because$ The figure is a pentagon

$\therefore a + 2a + 2a + a + 2a = 540°$

$\therefore 8a = 540°$ $\therefore a = 67.5°$

2 $\because$ The figure is a pentagon

$\therefore a + a + a + 90° + 90° = 540°$

$\therefore 3a + 180° = 540°$

$\therefore 3a = 540° - 180° = 360°$ $\therefore a = 120°$

3 $b = 180° - (90° + 42°) = 48°$,

$a = 90°$ (alternate angle) ,

$c = 180° - (90° + 38°) = 52°$

4 $a = 180° - (2 \times 62°) = 56°$,

$b = \frac{180° - 56°}{2} = 62°$

18

From the quadrilateral ABCD :

$m(\angle DCB) = 360° - (110° + 130° + 60°) = 60°$

, $\because C \in \overline{BE}$

$\therefore m(\angle DCE) = 180° - 60° = 120°$

, $\because \overrightarrow{CF}$ bisects $\angle DCE$

$\therefore m(\angle FCE) = \frac{120°}{2} = 60°$

$\therefore m(\angle FCE) = m(\angle B) = 60°$

(And they are two corresponding angles)

$\therefore \overleftrightarrow{CF} \parallel \overleftrightarrow{AB}$ (Q.E.D.)

19

From Δ XYB :

$m(\angle XBY) = 180° - (60° + 90°) = 30°$

$\because$ The figure ABCDEF is a regular hexagon

$\therefore m(\angle ABC) = \frac{(6-2)\times 180°}{6} = 120°$

, $\because B \in \overleftrightarrow{YC}$

$\therefore m(\angle ABY) = 180° - 120° = 60°$

$\because m(\angle ABY) = m(\angle YBX) + m(\angle XBA)$

$\therefore 60° = 30° + m(\angle XBA)$

$\therefore m(\angle XBA) = 30°$

$\therefore \overrightarrow{BX}$ bisects $\angle ABY$ (Q.E.D.)

20

$\because \angle EBC$ is an exterior angle of Δ EFB

$\therefore m(\angle EBC) = 45° + 60° = 105°$

From Δ ABC :

, $m(\angle C) = 180° - (105° + 30°) = 45°$

, $\because E \in \overline{FD}$

$\therefore m(\angle BED) = 180° - 45° = 135°$

From the quadrilateral EBCD

$m(\angle EDC) = 360° - (135° + 105° + 45°)$

$= 75°$ (The req.)

21

Let the measures of the interior angles of the pentagon be $3X$, $3X$, $2X$, $3X$, $4X$

$\because$ The sum of the measures of the interior angles of the pentagon $= (5-2) \times 180° = 540°$

$\therefore 3X + 3X + 2X + 3X + 4X = 540°$

$\therefore 15X = 540°$ $\therefore X = \frac{540°}{15} = 36°$

$\therefore$ The greatest measure $= 4 \times 36° = 144°$ (The req.)

22

Let the number of the sides of the polygon be (n)

$\because$ The measure of the exterior angle of the polygon $= 30°$

$\therefore$ The measure of the interior angle of the polygon

$= 180° - 30° = 150°$

$\therefore \frac{(n-2)\times 180°}{n} = 150°$ $\therefore 150° n = 180° n - 360°$

$\therefore 360° = 180° n - 150° n$ $\therefore 360° = 30° n$

$\therefore n = \frac{360°}{30°} = 12$

The sum of the measures of the interior angles

$= (12-2) \times 180° = 1800°$ (The req.)

23

Let the number of sides of the polygon be (n)

$\therefore \frac{(n-2) \times 180^\circ}{n} = 100^\circ$

$\therefore 100^\circ n = 180^\circ n - 360^\circ$

$\therefore 360^\circ = 180^\circ n - 100^\circ n$ $\therefore 360^\circ = 80^\circ n$

$\therefore n = \frac{360^\circ}{80^\circ} = 4.5 \notin \mathbb{Z}$

∴ There is no regular polygon of interior angles each of them is of measure 100° (The req.)

24

∵ The sum of measures of the interior angles of the polygon of 9 sides
$= (9-2) \times 180^\circ = 1260^\circ$

∴ The measure of the remained angle
$= 1260^\circ - 1140^\circ = 120^\circ$ (First req.)

∵ The measure of the interior angle of a regular polygon of 9 sides $= \frac{(9-2) \times 180^\circ}{9} = 140^\circ$

∵ The polygon has an interior angle of measure 120°

, ∵ $120^\circ \neq 140^\circ$

∴ The polygon is not regular. (Second req.)

25

1. The sum of the measures of the interior angles of the polygon of 15 sides $= (15-2) \times 180^\circ = 2340^\circ$
2. The sum of the measures of the interior and exterior angles at five vertices $= 5 \times 180^\circ = 900^\circ$
   ∴ The sum of the measures of the interior angles at these vertices $= 900^\circ - 200^\circ = 700^\circ$
   ∴ The sum of the measures of the ten remaind interior angles $= 2340^\circ - 700^\circ = 1640^\circ$ (The req.)

26

In Δ AMD :

∵ m (∠ AMD) = 71°

∴ m (∠ MAD) + m (∠ MDA) = 180° – 71° = 109°

∵ $\overrightarrow{AM}$ bisects ∠ BAD , $\overrightarrow{DM}$ bisects ∠ ADC

∴ m (∠ BAD) + m (∠ ADC)
= 2 [m (∠ MAD) + m (∠ MDA)]
= 2 × 109° = 218°

∵ The sum of measures of the interior angles of the quadrilateral = 360°

∴ m (∠ B) + m (∠ C) = 360° – 218° = 142° (Q.E.D.)

27

∵ $\overline{AE} // \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

∴ m (∠ A) + m (∠ B) = 180° (1)
(Two interior angles in the same side of the transversal)

From the pentagon ABCDE :

∴ m (∠ A) + m (∠ B) + m (∠ C) + m (∠ D) + m (∠ E) = 540°

∵ m (∠ A) + m (∠ B) = 180° "from (1)"

∴ m (∠ C) + m (∠ D) + m (∠ E)
= 540° – 180° = 360°

∴ m (∠ C) = m (∠ D) = m (∠ E) = $\frac{360^\circ}{3}$ = 120°

∴ m (∠ A) = 120°

∴ m (∠ B) = 180° – 120° = 60° (The req.)

Answers of Exercise 3

1

1. parallel and equal in length
2. equal in measure
3. supplementary
4. bisect each other
5. a trapezium
6. each two opposite sides are parallel
7. 130°
8. 120°

2

1. 6 , 2
2. 105° , 75° , 75°
3. 16

3

Given : XYZL is a parallelogram , XY = 3 cm. , YZ = 4 cm. , m (∠ LXZ) = 31° and m (∠ LZX) = 43°

R.T.F. : (1) m (∠ Y)
(2) The perimeter of the parallelogram XYZL

Proof : ∵ Δ XLZ in which :
m (∠ LXZ) = 31° , m (∠ LZX) = 43°
∴ m (∠ L) = 180° – (31° + 43°) = 106°
, ∵ The figure XYZL is a parallelogram.
∴ m (∠ Y) = m (∠ L) = 106° (First req.)
, the perimeter of the parallelogram XYZL
= (XY + YZ) × 2 = (3 + 4) × 2 = 7 × 2 = 14 cm. (Second req.)

4

Given : ABCD is a quadrilateral , $\overline{AC} \cap \overline{BD}$ = {M} , MA = MC , MB = MD , m (∠ AMB) = 110° and m (∠ MBA) = 25°

R.T.P. : ABCD is a parallelogram

R.T.F. : m (∠ ACD)

Proof : In the figure ABCD :

∵ MA = MC (Given) , MB = MD (Given)

∴ Its diagonals **bisect** each other.

∴ The figure ABCD is a **parallelogram.** (First req.)

In Δ MBA :

∵ m (∠ AMB) = **110°** , m (∠ MBA) = **25°**

∴ m (∠ MAB) = 180° − **(110° + 25°) = 45°**

, ∵ The figure ABCD is a **parallelogram**.

∴ $\overline{AB}$ // $\overline{CD}$

, ∵ $\overleftrightarrow{CA}$ is a transversal to them.

∴ m (∠ ACD) = m (∠ **CAB**)

= **45° (alternate** angles) (Second req.)

5

∵ ABCD is a parallelogram

∴ The two diagonals bisect each other

∴ MB = MD = 1.8 cm.

, MA = MC = 2.5 cm. , AB = CD = 2 cm.

∴ The perimeter of Δ AMB

= 2 + 2.5 + 1.8 = 6.3 cm. (The req.)

6

From Δ XYZ :

m (∠ YXZ) = 180° − (118° + 27°) = 35° (First req.)

, ∵ XYZL is a parallelogram

∴ $\overline{XY}$ // $\overline{LZ}$, $\overline{XZ}$ is a transversal

∴ m (∠ LZX) = m (∠ YXZ) = 35° (alternate angles) (Second req.)

, ∵ $\overline{XL}$ // $\overline{YZ}$, $\overline{XZ}$ is a transversal

∴ m (∠ LXZ) = m (∠ XZY) = 27° (alternate angles) (Third req.)

, m (∠ L) = m (∠ Y) = 118° (Fourth req.)

7

∵ ABCD is a parallelogram

∴ $\overline{AD}$ // $\overline{BC}$, $\overline{AB}$ is a transversal

∴ m (∠ ABL) = m (∠ A) = 50° (alternate angles) (1)

, ∵ BHOC is a parallelogram

∴ $\overline{CO}$ // $\overline{BH}$, $\overline{CB}$ is a transversal

∴ m (∠ LBH) = m (∠ BCO)

= 60° (Corresponding angles) (2)

From (1) , (2) :

∴ m (∠ ABH) = m (∠ ABL) + m (∠ LBH)

= 50° + 60° = 110° (The req.)

8

∵ $\overleftrightarrow{AD}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ B) = m (∠ EAB)

= 110° (alternate angles)

∴ m (∠ B) + m (∠ C) = 110° + 70° = 180°

and they are interior angles on the same side of the transversal

∴ $\overline{AB}$ // $\overline{CD}$

, ∵ $\overline{AD}$ // $\overline{BC}$

∴ ABCD is a parallelogram (Q.E.D.)

9

∵ ABCD is a parallelogram

∴ m (∠ B) + m (∠ C) = 180°

∴ m (∠ C) = 180° − 120° = 60°

From Δ DHC :

m (∠ HDC) = 180° − (90° + 60°) = 30° (The req.)

10

∵ ABCD is a parallelogram

∴ m (∠ A) + m (∠ ABC) = 180°

∴ m (∠ ABC) = 180° − 60° = 120°

∴ m (∠ ABO) = m (∠ ABC) − m (∠ OBC)

= 120° − 40° = 80° (The req.)

11

∵ ABCD is a parallelogram.

∴ m (∠ A) + m (∠ B) = 180° (1)

, ∵ The sum of measures of the interior angles of the quadrilateral DEBF = 360°

∴ m (∠ EDF) + m (∠ B) = 360° − (90° + 90°) = 180° (2)

From (1) and (2) :

∴ m (∠ EDF) = m (∠ A) (Q.E.D.)

12

∵ M ∈ $\overline{AC}$

∴ m (∠ BMC) = 180° − 70° = 110°

∴ In Δ BMC :

m (∠ MCB) = 180° − (110° + 40°) = 30°

∴ m (∠ MCB) = m (∠ MAD)

and they are alternate angles

$\therefore \overline{AD} \parallel \overline{BC}$

, $\because \overline{AB} \parallel \overline{DC}$

$\therefore$ ABCD is a parallelogram. (Q.E.D.)

13

$\because$ ABCD is a parallelogram.

$\therefore m(\angle A) = m(\angle C)$ (1)

, $\because$ XYCL is a parallelogram.

$\therefore m(\angle X) = m(\angle C)$ (2)

From (1) and (2) :

$\therefore m(\angle A) = m(\angle X)$ (Q.E.D.)

14

$\because$ ABCD is a parallelogram

$\therefore BC = AD$ (1)

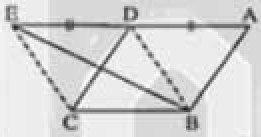
, $\because$ HBCO is a parallelogram

$\therefore BC = HO$ (2)

From (1) , (2) :

$\therefore AD = HO$ (Q.E.D.)

15



$\because$ ABCD is a parallelogram.

$\therefore AD = BC$

, $\because AD = DE$

$\therefore DE = BC$ (1)

, $\because \overrightarrow{AD} \parallel \overline{BC}$

(Because ABCD is a parallelogram)

$\therefore \overline{DE} \parallel \overline{BC}$ (2)

From (1) and (2) :

$\therefore$ DECB is a parallelogram.

$\therefore$ The two diagonals bisect each other.

$\therefore \overline{DC}$ and $\overline{BE}$ bisect each other. (Q.E.D.)

16

$\because m(\angle ABE) = m(\angle C)$

and they are corresponding angles

$\therefore \overline{AB} \parallel \overline{CD}$ (1)

, $\because m(\angle A) = m(\angle ABE)$ and they are alternate angles

$\therefore \overline{AD} \parallel \overleftrightarrow{BC}$ (2)

From (1) , (2) :

$\therefore$ ABCD is a parallelogram (Q.E.D.)

17

$\because$ ABCD is a parallelogram $\therefore AB = DC$

$\therefore \frac{1}{2} AB = \frac{1}{2} DC$

, $\because$ H is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{CD}$

$\therefore HB = DO$ (1)

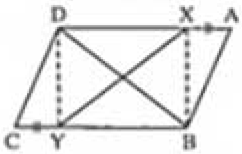
, $\because \overline{AB} \parallel \overline{CD}$

i.e. : $\overline{HB} \parallel \overline{DO}$ (2)

From (1) , (2) :

$\therefore$ HBOD is a parallelogram (Q.E.D.)

18



$\because$ ABCD is a parallelogram.

$\therefore AD = BC$

, $\because AX = CY$

$\therefore AD - AX = BC - CY$

$\therefore XD = BY$, $\because \overline{XD} \parallel \overline{BY}$

$\therefore$ The figure XBYD is a parallelogram.

$\therefore \overline{XY}$ and $\overline{BD}$ bisect each other. (Q.E.D.)

19

$\because$ ABCD is a parallelogram

$\therefore AB = DC$

, $\because \Delta$ DHC is equilateral

$\therefore DC = HC$ $\therefore AB = HC$ (First req.)

, $\because \Delta$ DHC is equilateral

$\therefore m(\angle C) = 60^\circ$

, $\because m(\angle B) + m(\angle C) = 180^\circ$

(Two consecutive angles in ▱ ABCD)

$\therefore m(\angle B) + 60^\circ = 180^\circ$

$\therefore m(\angle B) = 180^\circ - 60^\circ = 120^\circ$ (Second req.)

, $\because \overline{AD} \parallel \overline{BC}$, $\overline{DH}$ is a transversal

$\therefore m(\angle HDA) = m(\angle DHC)$ (alternate angles)

, $\because m(\angle DHC) = 60^\circ$

$\therefore m(\angle HDA) = 60^\circ$ (Third req.)

20

$\because \overrightarrow{DH}$ bisects $\angle ADC$ $\therefore m(\angle ADH) = 64^\circ$

, $\because$ ABCD is a parallelogram

$\therefore \overline{AD} \parallel \overline{BC}$, $\overline{DH}$ is a transversal

$\therefore m(\angle ADH) + m(\angle DHB) = 180^\circ$

$\therefore m(\angle DHB) = 180^\circ - 64^\circ = 116^\circ$ (First req.)

, $\because$ ABCD is a parallelogram

$\therefore m(\angle ABC) = m(\angle ADC) = 2\, m(\angle HDC)$

$= 2 \times 64^\circ = 128^\circ$ (Second req.)

21

$\because E \in \overline{BC}$

$\therefore m(\angle AEC) = 180^\circ - 70^\circ = 110^\circ$

, ∵ AECD is a quadrilateral.

∴ m (∠ EAD) = 360° − (110° + 115° + 65°) = 70°

∴ m (∠ BAD) = 70° + 45° = 115°

∴ m (∠ BAD) = m (∠ C)   (1)

, In Δ ABE :

∵ m (∠ B) = 180° − (45° + 70°) = 65°

∴ m (∠ B) = m (∠ D)   (2)

From (1) and (2) :

∴ ABCD is a parallelogram.   (Q.E.D.)

22

∵ ABCD is a parallelogram

∴ m (∠ ABC) = m (∠ ADC) = 115°

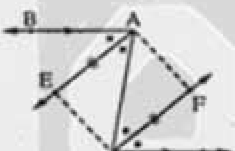
, ∵ HBCO is a parallelogram

∴ m (∠ HBC) = m (∠ O) = 50°

∴ m (∠ ABH) = m (∠ ABC) − m (∠ HBC)

= 115° − 50° = 65°   (The req.)

23



∵ $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

, $\overline{AC}$ is a transversal

∴ m (∠ BAC) = m (∠ ACD) (alternate angles)

∴ $\frac{1}{2}$ m (∠ BAC) = $\frac{1}{2}$ m (∠ ACD)

∴ m (∠ EAC) = m (∠ ACF) and they are alternate angles

∴ $\overleftrightarrow{AE} \parallel \overleftrightarrow{CF}$

, ∵ AE = CF

∴ AECF is a parallelogram   (Q.E.D.)

24

∵ ABCD is a parallelogram

∴ m (∠ DAB) + m (∠ ABC) = 180°

∴ $X° + 2X° + 5X° = 180°$

∴ $8X° = 180°$ ∴ $X° = 22.5°$

∴ m (∠ BCD) = m (∠ BAD)

= 3 X° = 3 × 22.5° = 67.5°   (First req.)

, m (∠ ADC) = m (∠ ABC) = 5 X° = 5 × 22.5° = 112.5°

(Second req.)

25 1 (a)   2 (c)   3 (c)   4 (d)   5 (b)

26

To decide the place of the building D ,
we can do this by two methods :

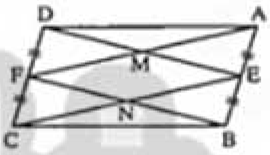
First :

- Draw $\overline{AC}$ and determine its midpoint M
- Draw $\overline{BM}$ and determine the point D on it where BM = MD
- Then , D is the required position.

Second :

- Draw $\overline{BC}$ and determine its midpoint M
- Draw $\overline{AM}$ and determine the point D on it where AM = MD
- Then , D is the required position.

27



∵ ABCD is a parallelogram.

∴ AB = CD

, ∵ E and F are the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively.

∴ BE = DF   (1)

, ∵ $\overline{AB} \parallel \overline{CD}$

∴ $\overline{BE} \parallel \overline{DF}$ (2)

From (1) and (2) :

∴ The figure BEDF is a parallelogram.

∴ $\overline{ED} \parallel \overline{BF}$ (3)   (Q.E.D. 1)

By the same way we can prove that :

AECF is a parallelogram.

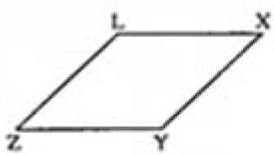
∴ $\overline{AF} \parallel \overline{EC}$ (4)

, from (3) and (4) :

∴ $\overline{EM} \parallel \overline{NF}$, $\overline{EN} \parallel \overline{MF}$

∴ FMEN is a parallelogram.   (Q.E.D. 2)

28



∵ XYZL is a parallelogram

∴ m (∠ X) + m (∠ Y) = 180°

, ∵ m (∠ Y) = 3 m (∠ X)

∴ m (∠ X) + 3 m (∠ X) = 180°

∴ 4 m (∠ X) = 180°

∴ m (∠ X) = $\frac{180°}{4}$ = 45°

∴ m (∠ Y) = 3 m (∠ X) = 3 × 45° = 135°

, m (∠ Z) = m (∠ X) = 45°

, m (∠ L) = m (∠ Y) = 135°   (The req.)

Answers of Exercise 4

1

1 rhombus.
2 equal in length
3 square.
4 rhombus
5 parallelogram
6 parallelogram
7 parallelogram
8 rhombus
9 square
10 square
11 square
12 parallelogram
13 $\overline{AC} \perp \overline{BD}$
14 side length × 4 , (length + width) × 2 , side length × 4
15 10.5

2 1 (b) 2 (a) 3 (c) 4 (b) 5 (b) 6 (b) 7 (a) 8 (c)

3 1 2 2 3 3 8

4 1 16 2 135° 3 110°

5

Fig. (1) : m (∠ ADB) = 40° , m (∠ CDB) = 80° , m (∠ A) = 60° , m (∠ C) = 60°

Fig. (2) : m (∠ MFE) = 45° , m (∠ L) = 90° , m (∠ LMF) = 45° , m (∠ FME) = 45° , m (∠ E) = 90°

Fig. (3) : m (∠ XZY) = 35° , m (∠ ZXH) = 35° , m (∠ HZX) = 35° , m (∠ H) = 110°

Fig. (4) : m (∠ NWK) = 60° , m (∠ WMK) = 30° , m (∠ KMN) = 60° , m (∠ MNW) = 60° , m (∠ MZN) = 60° , m (∠ WNK) = 30° , m (∠ NZK) = 120°

6

Given : ABCD is a square , AB = 5 cm. , E ∈ $\overline{AC}$, m (∠ EDC) = 30°

R.T.F. : (1) The perimeter of the square ABCD
(2) m (∠ AED)

Proof : ∵ The perimeter of the square = side length × 4
∴ The perimeter of the square ABCD = 5 × 4 = 20 cm. (First req.)
, ∵ ABCD is a square , $\overline{AC}$ is a diagonal
∴ m (∠ ACD) = 45°
In Δ DEC :
m (∠ DEC) = 180° – (45° + 30°) = 105°
, ∵ E ∈ $\overline{AC}$
∴ m (∠ AED) = 180° – 105° = 75° (Second req.)

7

Given : ABCD is a parallelogram , E ∈ $\overrightarrow{CB}$, BC = BE , DE = DC

R.T.P. : The figure AEBD is a rectangle.

Proof : ∵ ABCD is a parallelogram.
∴ AD = BC , $\overline{AD}$ // $\overline{BC}$
, ∵ EB = BC , E ∈ $\overrightarrow{CB}$ (Given)
∴ AD = EB , $\overline{AD}$ // $\overline{EB}$
∴ The figure AEBD is a parallelogram.
, ∵ DE = DC (Given)
, AB = DC (Properties of parallelogram)
∴ DE = AB
∴ The two diagonals of the parallelogram AEBD are equal in length.
∴ The figure AEBD is a rectangle. (Q.E.D.)

8

∵ ABCD is a rectangle ∴ m (∠ B) = 90°
From Δ ABH :
m (∠ AHB) = 180° – (44° + 90°) = 46°
, ∵ H ∈ $\overline{BC}$
∴ m (∠ AHB) + m (∠ AHD) + m (∠ DHC) = 180°
∴ 46° + m (∠ AHD) + 46° = 180°
∴ m (∠ AHD) = 88° (The req.)

9

From Δ AHM :
m (∠ MAH) = 180° – (90° + 32°) = 58°
, ∵ ABCD is a rhombus , $\overline{AC}$ is a diagonal
∴ m (∠ BAD) = 2 m (∠ BAM) = 2 × 58° = 116°
∴ m (∠ BCD) = 116°
, m (∠ ABC) = 180° – 116° = 64°
, m (∠ ADC) = 64° (The req.)

10

∵ ABCD is a rhombus and $\overline{BD}$ is a diagonal in it.
∴ m (∠ ABC) = 2 m (∠ ABD) = 2 × 62° = 124°
∴ m (∠ A) = 180° – 124° = 56° (The req.)

11

∵ XYZL is a rhombus
∴ m (∠ XYL) = m (∠ XLY)
, ∵ $\overline{DH}$ // $\overline{YX}$, $\overline{YL}$ is a transversal
∴ m (∠ HDL) = m (∠ XYL) (corresponding angles)
∴ m (∠ HDL) = m (∠ HLD) (Q.E.D.)

12

∵ ABCD is a rectangle ∴ AD = BC (1)

, ∵ OBCH is a parallelogram ∴ OH = BC (2)

From (1) , (2) : ∴ AD = OH

by subtracting AH from both sides

∴ AD – AH = OH – AH

∴ HD = AO (Q.E.D.)

13

∵ ABCD is a square. ∴ $\overline{AD} \parallel \overline{BC}$

, ∵ $E \in \overline{BC}$ ∴ $\overline{AD} \parallel \overline{CE}$

, ∵ $\overline{AC} \parallel \overline{DE}$

∴ ACED is a parallelogram. (First req.)

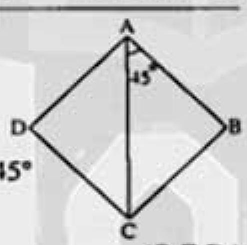
, ∵ ABCD is a square and $\overline{AC}$ is a diagonal in it.

∴ m (∠ DAC) = 45°

, ∵ ACED is a parallelogram.

∴ m (∠ ACE) = 180° – 45° = 135° (Second req.)

14



∵ ABCD is a rhombus

∴ $\overline{AC}$ bisects ∠ BAD

∴ m (∠ BAC) = m (∠ CAD) = 45°

∴ m (∠ BAD) = 90°

∴ ABCD is a square (Q.E.D.)

15

∵ ABCD is a rectangle ∴ $\overline{AB} \parallel \overline{DC}$

, ∵ ABYX is a square ∴ $\overline{AB} \parallel \overline{XY}$

∴ $\overline{XY} \parallel \overline{DC}$, $\overline{YD}$ is a transversal

∴ m (∠ XYD) = m (∠ YDC) = 52° (alternate angles)

, ∵ ABYX is a square , $\overline{AY}$ is a diagonal

∴ m (∠ AYX) = 45°

∴ m (∠ AYD) = m (∠ AYX) + m (∠ XYD)

= 45° + 52° = 97° (The req.)

16

∵ ABCD is a square and $\overline{BD}$ is a diagonal in it.

∴ m (∠ BDC) = 45° , m (∠ C) = 90°

∴ In the quadrilateral DEFC :

m (∠ EFC) = 360° – (45° + 90° + 102°) = 123°

∴ y = 123°

, ∵ $F \in \overline{BC}$

∴ m (∠ AFB) = 180° – 123° = 57°

∴ In the right-angled triangle ABF at B :

m (∠ BAF) = 180° – (90° + 57°) = 33°

∴ x = 33° (The req.)

17

∵ ABCD is a square. ∴ AM = CM

, ∵ AE = CF and by subtracting ∴ EM = FM

, ∵ BM = MD (properties of the square)

∴ The figure EBFD is a parallelogram.

, ∵ ABCD is a square. ∴ $\overline{AC} \perp \overline{BD}$

∴ $\overline{EF} \perp \overline{BD}$

∴ The parallelogram EBFD is a rhomubs. (Q.E.D.)

18

∵ ABCD is a rectangle. ∴ $\overline{AB} \parallel \overline{DC}$

, ∵ $E \in \overline{DC}$ ∴ $\overline{AB} \parallel \overline{CE}$

, ∵ $\overline{AC} \parallel \overline{BE}$

∴ The figure ABEC is a parallelogram.

∴ AB = CE but AB = DC

∴ DC = CE (Q.E.D. 1)

, ∵ The two diagonals of the parallelogram ABEC bisect each other.

∴ MC = $\frac{1}{2}$ BC

, ∵ AD = BC ∴ MC = $\frac{1}{2}$ AD (Q.E.D. 2)

19 1 square 2 parallelogram 3 rhombus 4 rectangle

20 1 all 2 some 3 all 4 some 5 all 6 some

21

∵ ABCD is a parallelogram.

∴ AB = CD but AB = GF

∴ CD = GF (1)

, ∵ AD = BC but AD = CG

∴ BC = CG (2)

, ∵ The figure CEFG is a rhombus.

∴ CE = EF = FG = GC (3)

From (1) , (2) and (3) :

∴ CD = BC = CG = CE

∴ The two diagonals are equal in length and bisect each other.

∴ BEGD is a rectangle. (Q.E.D.)

Answers of exams on the first part of unit three

Model 1

1 1 (b) 2 (b) 3 (d) 4 (c) 5 (b) 6 (d)

2 1 900° 2 360° 3 120° 4 rhombus 5 trapezium

3 [a] $a = 120°$ [b] Prove by yourself

4 [a] $m(\angle B) = 75°$ [b] $m(\angle A) = 72°$

5 [a] Prove by yourself
[b] 1 Prove by yourself
2 $m(\angle ACE) = 135°$

Model 2

1
1 (c) 2 (b) 3 (b) 4 (c) 5 (d) 6 (b)

2
1 144° 2 bisect each other
3 parallelogram 4 125° 5 118°

3 [a] $X = 30°$
[b] Prove by yourself.

4 [a] $X = 30°$, $y = 120°$
[b] $m(\angle E) = 150°$

5 [a] 1 MD = 4 cm , AB = 3 cm.
2 The perimeter of Δ ABM = 11 cm.
[b] Prove by yourself

Answers of Exercise 5

1
1 180°
2 the measures of its nonadjacent interior angles.
3 right-angled 4 obtuse-angled 5 90°
6 obtuse 7 60°

2 1 (a) 2 (b) 3 (a) 4 (c) 5 (c) 6 (b)

3
Fig. (1) : $m(\angle C) = 180° - (70° + 65°) = 45°$
Fig. (2) : $m(\angle A) = 180° - (90° + 50°) = 40°$
Fig. (3) : $m(\angle B) = 180° - (28° + 124°) = 28°$
Fig. (4) : $\because m(\angle ACB) = 52°$ (V.O.A.)
$\therefore m(\angle A) = 180° - (90° + 52°) = 38°$
Fig. (5) : $m(\angle ACD) = 50° + 60° = 110°$
Fig. (6) : $m(\angle ACD) = 67° + 42° = 109°$
Fig. (7) : $m(\angle B) = 120° - 60° = 60°$
Fig. (8) : $m(\angle A) = 125° - 35° = 90°$
Fig. (9) : $m(\angle A) = \frac{100°}{2} = 50°$
Fig. (10) : $m(\angle DAB) = 60° + 60° = 120°$
Fig. (11) : $m(\angle C) = \frac{180° - 30°}{2} = 75°$
$\therefore m(\angle ABD) = 30° + 75° = 105°$
Fig. (12) : $m(\angle XAC) = m(\angle ABC) + m(\angle ACB)$
$= (180° - 110°) + (180° - 130°)$
$= 70° + 50° = 120°$
Fig. (13) : $m(\angle ACB) = 180° - 120° = 60°$
$\therefore m(\angle A) = 100° - 60° = 40°$
Fig. (14) : $m(\angle ABC) = 180° - 150° = 30°$
$\therefore m(\angle ACD) = 25° + 30° = 55°$
Fig. (15) : $m(\angle BCE) = 72° + 48° = 120°$
$m(\angle ACB) = 180° - 120° = 60°$
$m(\angle BAC) = 120° - 53° = 67°$
Fig. (16) : $m(\angle ABC) = 115° - 50° = 65°$
$\therefore m(\angle EBG) = 65°$ (V.O.A.)
Fig. (17) : $\because m(\angle ACB) = 42°$ (alternate angles)
$\therefore m(\angle BAC) = 180° - (42° + 63°) = 75°$
Fig. (18) : $\because m(\angle ABC) = m(\angle ACB)$
$= \frac{180° - 70°}{2} = 55°$
$\therefore m(\angle ABD) = 180° - (73° + 55°) = 52°$
, $m(\angle ACE) = 180° - (82° + 55°) = 43°$

4
Given :
$\overrightarrow{BD} \cap \overrightarrow{AE} = \{C\}$, $\overrightarrow{AF} \parallel \overrightarrow{BC}$,
$m(\angle BAF) = 65°$, $m(\angle DCE) = 55°$
R.T.F. : The measures of the interior angles of Δ ABC
Proof :
$\because \overrightarrow{BD} \cap \overrightarrow{AE} = \{C\}$ (given)
$\therefore m(\angle ACB) = m(\angle DCE) = 55°$ (V.O.A.)

∵ $\overline{AF} \parallel \overline{BC}$, $\overline{AB}$ is a transversal to them

∴ m (∠ FAB) = m (∠ B) = 65° (alternate angles)

∵ The sum of measures of the interior angles of the triangle = 180°

∴ m (∠ BAC) = 180° − (55° + 65°) = 60° (The req.)

5

Given :

$A \in \overline{DC}$, $\overrightarrow{DE} \parallel \overrightarrow{CB}$, m (∠ D) = 100° ,

m (∠ B) = 40°

R.T.F. : m (∠ BAD)

Proof :

∵ $\overrightarrow{DE} \parallel \overrightarrow{CB}$, $\overleftrightarrow{CD}$ is a transversal

∴ m (∠ D) + m (∠ C) = 180°

(two interior angles in the same side of the transversal)

∵ m (∠ D) = 100°

∴ m (∠ C) = 180° − 100° = 80°

∵ ∠ BAD is an exterior angle of Δ ABC

∴ m (∠ BAD) = m (∠ B) + m (∠ C)

= 40° + 80° = 120° (The req.)

6

∵ $\overrightarrow{BD} \parallel \overrightarrow{CA}$, $\overleftrightarrow{AB}$ is a transversal to them

∴ m (∠ A) = m (∠ ABD) = 75° (alternate angles)

In Δ ABC :

m (∠ ABC) = 180° − (75° + 45°) = 60° (The req.)

7

∵ ∠ CAD is an exterior angle of Δ ABC

∴ m (∠ DAC) = m (∠ ACB) + m (∠ B)

∴ 112° = m (∠ ACB) + 58°

∴ m (∠ ACB) = 54° ∵ $\overrightarrow{CE}$ bisects ∠ ACB

∴ m (∠ ECB) = $\frac{54^\circ}{2}$ = 27°

∵ ∠ AEC is an exterior angle of Δ BCE

∴ m (∠ AEC) = 58° + 27° = 85° (The req.)

8

∵ $\overrightarrow{DE} \parallel \overline{BC}$, $\overline{DB}$ is a transversal to them

∴ m (∠ B) + m (∠ D) = 180°

(two interior angles in the same side of the transversal)

∴ m (∠ B) = 180° − 100° = 80°

∴ In Δ ABC :

m (∠ BAC) = 180° − (80° + 30°) = 70° (The req.)

9

∵ $\overrightarrow{EF} \parallel \overrightarrow{CD}$, $\overline{BE}$ is a transversal

∴ m (∠ ABC) = m (∠ E) = 50° (alternate angles)

∴ In Δ ABC :

m (∠ BAC) = 180° − (50° + 30°) = 100° (First req.)

∵ ∠ ABD is an exterior angle of Δ ABC

∴ m (∠ ABD) = m (∠ BAC) + m (∠ C)

= 100° + 30° = 130° (Second req.)

10

∵ $\overrightarrow{FR} \parallel \overline{BC}$, $\overline{FB}$ is a transversal to them

∴ m (∠ B) + m (∠ F) = 180°

(two interior angles in the same sid of the transversal)

∴ m (∠ B) = 180° − 135° = 45°

∵ $\overrightarrow{DE} \parallel \overline{BC}$ and $\overline{DC}$ is a transversal to them

∴ m (∠ C) + m (∠ D) = 180°

(two interior angles in the same side of the transversal)

∴ m (∠ C) = 180° − 120° = 60°

∴ In Δ ABC :

m (∠ BAC) = 180° − (45° + 60°) = 75° (The req.)

11

∵ $\overrightarrow{EF} \parallel \overline{AB}$, $\overrightarrow{AE}$ is a transversal to them

∴ m (∠ A) + m (∠ E) = 180°

∴ m (∠ A) = 180° − 110° = 70°

Similarly : m (∠ B) + m (∠ D) = 180°

∴ m (∠ B) = 180° − 120° = 60°

In Δ ABG

m (∠ BGA) = 180° − (60° + 70°) = 50°

∵ $\overline{AE} \cap \overline{BD}$ = {G}

∴ m (∠ EGD) = m (∠ BGA) = 50° (The req.)

12

∵ $\overline{DE} \parallel \overline{BC}$, $\overline{DB}$ is a transversal to them.

∴ m (∠ ADE) = m (∠ B) = 60° (corresponding angles)

∴ In Δ ADE

m (∠ AED) = 180° − (80° + 60°) = 40°

m (∠ DEC) = 180° − 40° = 140° (The req.)

13

$\because \overleftrightarrow{DF} \parallel \overline{BC}$, $\overrightarrow{EC}$ is a transversal to them

$\therefore m(\angle C) = m(\angle FEC) = 70^\circ$ (alternate angles)

$\because \overleftrightarrow{DF} \parallel \overline{BC}$, $\overrightarrow{AB}$ is a transversal to them

$\therefore m(\angle B) = m(\angle ADE) = 60^\circ$ (corresponding angles)

$\therefore$ In ΔABC :

$m(\angle A) = 180^\circ - (70^\circ + 60^\circ) = 50^\circ$ (The req.)

14

In ΔABC :

$m(\angle C) = 180^\circ - (64^\circ + 52^\circ) = 64^\circ$

$\because m(\angle DEC) + m(\angle C) = 116^\circ + 64^\circ = 180^\circ$

(but they are interior angles in the same side of the transversal)

$\therefore \overline{DE} \parallel \overline{BC}$ (Q.E.D.)

15

$\because \overleftrightarrow{AY} \cap \overleftrightarrow{BE} = \{C\}$

$\therefore m(\angle XCF) = m(\angle ACB) = 42^\circ$ (V.O.A.)

Similarly : $m(\angle XFC) = m(\angle YFZ) = 53^\circ$

$\therefore$ In ΔXFC :

$m(\angle FXC) = 180^\circ - (42^\circ + 53^\circ) = 85^\circ$

$\because \overleftrightarrow{EB} \cap \overleftrightarrow{DF} = \{X\}$

$\therefore m(\angle DXE) = m(\angle FXC) = 85^\circ$ (V.O.A.) (First req.)

$\because X \in \overleftrightarrow{EC}$

$\therefore m(\angle DXC) = 180^\circ - 85^\circ = 95^\circ$ (Second req.)

$\because \overleftrightarrow{DF} \cap \overleftrightarrow{EC} = \{X\}$

$\therefore m(\angle EXF) = m(\angle DXC) = 95^\circ$ (V.O.A.) (Third req.)

16

$\because \overleftrightarrow{ED} \parallel \overleftrightarrow{BF}$, $\overleftrightarrow{AE}$ is a transversal to them

$\therefore m(\angle E) + m(\angle CAF) = 180^\circ$

(two interior angles in the same side of the transversal)

$m(\angle E) = 180^\circ - 110^\circ = 70^\circ$

$\therefore$ In ΔDCE :

$m(\angle D) = 180^\circ - (60^\circ + 70^\circ) = 50^\circ$

$\because \overleftrightarrow{ED} \parallel \overleftrightarrow{BA}$, $\overleftrightarrow{DB}$ is a transversal to them

$\therefore m(\angle B) = m(\angle D) = 50^\circ$ (alternate angles)

, $m(\angle ACB) = m(\angle DCE) = 60^\circ$ (V.O.A.)

$\therefore$ In ΔABC :

$m(\angle BAC) = 180^\circ - (50^\circ + 60^\circ) = 70^\circ$ (The req.)

17

$\because$ ABCD is a rectangle

$\therefore m(\angle BAD) = m(\angle ADC) = 90^\circ$

$m(\angle EAD) = 90^\circ - 30^\circ = 60^\circ$

$m(\angle EDA) = 90^\circ - 60^\circ = 30^\circ$

$\therefore$ In ΔAED :

$m(\angle AED) = 180^\circ - (60^\circ + 30^\circ) = 90^\circ$ (The req.)

18

In ΔABC :

$m(\angle ABC) + m(\angle ACB) = 180^\circ - 80^\circ = 100^\circ$

$\because \overrightarrow{BM}$ bisects $\angle ABC$, $\overrightarrow{CM}$ bisects $\angle ACB$

$\therefore m(\angle MBC) + m(\angle MCB) = \frac{1}{2} \times 100^\circ = 50^\circ$

$\therefore$ In ΔMBC :

$m(\angle BMC) = 180^\circ - 50^\circ = 130^\circ$

$\therefore m(\angle EMD) = m(\angle BMC) = 130^\circ$ (V.O.A.) (The req.)

19

$\because \angle EBD$ is an exterior angle of ΔABC

$\therefore m(\angle EBD) = m(\angle A) + m(\angle C) = 63^\circ + 56^\circ = 119^\circ$

$\because \angle EDX$ is an exterior angle of ΔEDB

$\therefore m(\angle EDX) = m(\angle DEB) + m(\angle EBD)$

$= 34^\circ + 119^\circ = 153^\circ$ (The req.)

20

$\because \angle ADC$ is an exterior angle of ΔABD

$\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$

$= 35^\circ + 25^\circ = 60^\circ$

$\because \angle XAC$ is an exterior angle of ΔADC

$\therefore m(\angle XAC) = m(\angle ADC) + m(\angle C)$

$= 60^\circ + 60^\circ = 120^\circ$ (The req.)

21

$\because A \in \overleftrightarrow{BF}$

$\therefore m(\angle BAC) + m(\angle CAF) = 180^\circ$

$\therefore m(\angle BAC) = 180^\circ - 110^\circ = 70^\circ$

Similarly : $m(\angle EAB) = 180^\circ - 130^\circ = 50^\circ$

$\because \overrightarrow{AE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore m(\angle B) = m(\angle EAB) = 50^\circ$ (alternate angles)

$\because \angle ACD$ is an exterior angle of ΔABC

$\therefore m(\angle ACD) = m(\angle BAC) + m(\angle B)$

$= 70^\circ + 50^\circ = 120^\circ$ (The req.)

22

$\because \overrightarrow{OK} \mathbin{/\mkern-3mu/} \overrightarrow{ED}$, $\overrightarrow{EO}$ is a transversal to them

$\therefore m(\angle DEO) + m(\angle O) = 180^\circ$

(Two interior angles in the same side of the transversal)

$\therefore m(\angle DEO) = 180^\circ - 120^\circ = 60^\circ$

, $\because \overrightarrow{ED}$ bisects $\angle AEO$

$\therefore m(\angle AED) = 60^\circ$

, $\because \overline{AB} \mathbin{/\mkern-3mu/} \overline{ED}$, $\overline{AE}$ is a transversal to them

$\therefore m(\angle BAE) = m(\angle AED) = 60^\circ$ (Alternate angles)

, $\because \overrightarrow{AC}$ bisects $\angle BAE$

$\therefore m(\angle CAE) = 30^\circ$

, $\because \angle AED$ is an exterior angle of ΔACE

$\therefore m(\angle C) = 60^\circ - 30^\circ = 30^\circ$ (The req.)

23

$\because \overline{AC} \mathbin{/\mkern-3mu/} \overline{DF}$, $\overleftrightarrow{FC}$ is a transversal to them.

$\therefore m(\angle ACB) = m(\angle F)$ (corresponding angles)

$\therefore$ In $\Delta\Delta$ ABC , DEF :

$\because m(\angle B) = m(\angle DEF) = 90^\circ$ (given)

, $m(\angle ACB) = m(\angle F)$ (by proof)

$\therefore m(\angle A) = m(\angle D)$ (Q.E.D.)

24

$\because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore m(\angle BAD) = m(\angle CAD)$,

$\because m(\angle B) = m(\angle C)$ $\quad \therefore m(\angle ADB) = m(\angle ADC)$

$\therefore \Delta\Delta$ ABD , ACD in them :

$$\begin{cases} m(\angle BAD) = m(\angle CAD) \\ m(\angle ADB) = m(\angle ADC) \\ \overline{AD} \text{ is a common side} \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACD$, then we deduce that : $AB = AC$

(Q.E.D.)

25 $\because \overline{AB} \mathbin{/\mkern-3mu/} \overline{ED}$, $\overline{AD}$ is a transversal

$\therefore m(\angle BAC) = m(\angle ADE)$ (alternate angles)

$\therefore$ In $\Delta\Delta$ ABC and DEA :

$\because m(\angle B) = m(\angle E)$, $m(\angle BAC) = m(\angle ADE)$

$\therefore m(\angle ACB) = m(\angle DAE)$

(and they are alternate angles).

$\therefore \overline{BC} \mathbin{/\mkern-3mu/} \overline{AE}$ (Q.E.D.)

26

In Δ XYZ and Δ XML

$\because m(\angle XML) = m(\angle Y)$, $\angle X$ is a common angle.

$\therefore m(\angle XLM) = m(\angle Z) = 52^\circ$ (The req.)

27 $\because \angle ACB$ is an exterior angle of ΔACD

$\therefore m(\angle ACB) > m(\angle D)$

$\because m(\angle ACB) = m(\angle B)$

$\therefore m(\angle B) > m(\angle D)$ (Q.E.D.)

28 $\because \angle DAC$ is an exterior angle of ΔABC

$\therefore m(\angle DAC) = m(\angle B) + m(\angle C)$

$\because m(\angle B) = m(\angle C)$

$\therefore \frac{1}{2} m(\angle DAC) = m(\angle B)$

$\therefore m(\angle DAE) = m(\angle B)$

(and they are corresponding angles).

$\therefore \overrightarrow{AE} \mathbin{/\mkern-3mu/} \overline{BC}$ (Q.E.D.)

29 $\because m(\angle 1) = m(\angle A)$, $m(\angle 2) = m(\angle C)$

$\therefore m(\angle 1) + m(\angle 2) = m(\angle A) + m(\angle C)$

$\therefore m(\angle ABC) = m(\angle A) + m(\angle C)$

$\therefore \angle ABC$ is a right angle. (Q.E.D.)

30 In Δ ABC :

$\because m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$

$\therefore 2\,m(\angle C) + 4\,m(\angle C) + m(\angle C) = 180^\circ$

$\therefore 7\,m(\angle C) = 180^\circ$ $\qquad \therefore m(\angle C) = \frac{180^\circ}{7}$

$\therefore m(\angle B) = 4 \times \frac{180^\circ}{7} = 102\,\frac{6}{7}^\circ$

$\therefore \angle B$ is an obtuse angle. (Q.E.D.)

31

In Δ ABC :

$\because m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$

$\therefore 4X + 2X + 2^\circ + 28^\circ = 180^\circ$

$\therefore 6X + 30^\circ = 180^\circ$

$\therefore 6X = 150^\circ$ $\qquad \therefore X = 25^\circ$

$\therefore m(\angle A) = 4 \times 25^\circ = 100^\circ$

, $m(\angle B) = 2 \times 25^\circ + 2 = 52^\circ$ (The req.)

32

Comparing the measures of the angles of the two triangles ABC and ADE

$\because m(\angle B) = m(\angle EDA) = 90^\circ$

$, m(\angle ACB) = m(\angle E)$

$\therefore m(\angle CAD) = m(\angle DAE)$ (1)

$\because \angle CDB$ is an exterior angle of ΔCDA

$\therefore m(\angle CDB) > m(\angle CAD)$

From (1) : $\therefore m(\angle CDB) > m(\angle DAE)$ (Q.E.D.)

Answers of Exercise 6

1

1. bisects the third side.
2. parallel to
3. half the length of the third side
4. E , $\overline{AC}$
5. $\overline{DE} \parallel \overline{BC}$
6. 3
7. 90°
8. 12
9. 17
10. 2.5
11. 3
12. 3 , 45°

2

Given : ABCD is a parallelogram , DC = 5 cm. , $F \in \overline{BC}$, $\overrightarrow{DF} \cap \overrightarrow{AB} = \{E\}$, DF = FE

R.T.F. : The length of $\overline{BE}$

Proof : $\because$ ABCD is a parallelogram.

$\therefore \overline{AD} \parallel \overline{BC}$ $\quad$ $\therefore \overline{BF} \parallel \overline{AD}$

, In ΔAED :

$\because$ F is the midpoint of $\overline{DE}$, $\overline{BF} \parallel \overline{AD}$

$\therefore$ B is the midpoint of $\overline{AE}$

$\therefore AB = BE$

, $\because AB = DC = 5$ cm. (properties of parallelogram)

$\therefore BE = 5$ cm. (The req.)

3

Given : ABC is a triangle in which CA = CB , E is the midpoint of $\overline{AB}$, $\overline{EF} \parallel \overline{AC}$, H and G are the two midpoints of $\overline{BD}$ and $\overline{CD}$ respectively.

R.T.P. : EF = GH

Proof : In ΔABC :

$\because$ E is the midpoint of $\overline{AB}$, $\overline{EF} \parallel \overline{AC}$

$\therefore$ F is the midpoint of $\overline{BC}$ $\quad$ $\therefore EF = \frac{1}{2} AC$

In ΔBDC :

$\because$ H is the midpoint of $\overline{BD}$, G is the midpoint of $\overline{CD}$

$\therefore GH = \frac{1}{2} BC$ $\quad$, $\because CA = BC$

$\therefore EF = GH$ (Q.E.D.)

4

In ΔABC :

$\because$ E is the midpoint of $\overline{AB}$, $\overline{EX} \parallel \overline{BC}$

$\therefore$ X is the midpoint of $\overline{AC}$

, In ΔACD :

$\because$ X is the midpoint of $\overline{AC}$, $\overline{XY} \parallel \overline{CD}$

$\therefore$ Y is the midpoint of $\overline{AD}$

$\therefore AY = \frac{6}{2} = 3$ cm. (The req.)

5

In ΔDBC :

$\because$ X is the midpoint of $\overline{DC}$, $\overline{XY} \parallel \overline{BC}$

$\therefore$ Y is the midpoint of $\overline{DB}$

, In ΔADB :

$\because$ Y is the midpoint of $\overline{DB}$, Z is the midpoint of $\overline{AB}$

$\therefore \overline{ZY} \parallel \overline{AD}$

, $\because \overline{DB}$ is a transversal to them.

$\therefore m(\angle ZYB) = m(\angle ADB) = 40^\circ$

(corresponding angles) (The req.)

6

1. The perimeter of the parallelogram ABCD = 2 (12 + 8) = 40 cm.
2. $\because$ ABCD is a parallelogram , M is the point of intersection of its diagonals

   $\therefore$ M is the midpoint of $\overline{BD}$

   In ΔABD :

   $\because$ M is the midpoint of $\overline{BD}$, $\overrightarrow{MO} \parallel \overline{AD}$

   $\therefore$ O is the midpoint of $\overline{AB}$

   $\therefore AO = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4$ cm. (The req.)

7

$\because$ ABCD is a parallelogram , M is the point of intersection of its diagonals

$\therefore$ M is the midpoint of $\overline{BD}$, $\because \overline{MX} \parallel \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AD}$ (First req.)

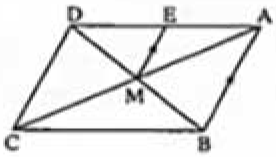
$\therefore MX = \frac{1}{2} AB$

$\therefore AB = 2 MX = 2 \times 5 = 10$ cm.

$\therefore DC = AB = 10$ cm. (Second req.)

8

∵ ABCD is a parallelogram.
, M is the intersection point of its diagonals.



∴ M is the midpoint of $\overline{BD}$

In Δ ABD :

∵ M is the midpoint of $\overline{BD}$, $\overline{ME}$ // $\overline{AB}$

∴ E is the midpoint of $\overline{AD}$

∴ ME = $\frac{1}{2}$ AB , ∵ AB = DC

∴ ME = $\frac{1}{2}$ DC (Q.E.D.)

9

∵ ABCD is a parallelogram.

∴ $\overline{AB}$ // $\overline{DC}$ ∴ $\overline{OC}$ // $\overline{AB}$

In Δ ABH : ∵ C is the midpoint of $\overline{BH}$, $\overline{CO}$ // $\overline{AB}$

∴ O is the midpoint of $\overline{AH}$

∴ AO = OH (Q.E.D.)

10

∵ ABCD is a trapezium. ∴ $\overline{AD}$ // $\overline{BC}$

, ∵ $\overline{AD}$ // $\overline{XY}$ ∴ $\overline{AD}$ // $\overline{XY}$ // $\overline{BC}$

, ∵ AX = XB ∴ DY = YC

i.e. : Y is the midpoint of $\overline{DC}$

In Δ HDC : ∵ Y is the midpoint of $\overline{DC}$

, $\overline{YZ}$ // $\overline{DH}$ ∴ Z is the midpoint of $\overline{CH}$

∴ CZ = ZH (Q.E.D.)

11

∵ D , E are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively.

∴ $\overline{DE}$ // $\overline{BC}$, ∵ $\overline{AX}$ // $\overline{BC}$

∴ $\overline{DE}$ // $\overline{AX}$

, ∵ Y ∈ $\overleftrightarrow{DE}$ ∴ $\overline{EY}$ // $\overline{AX}$

In Δ ACX :

∵ $\overline{EY}$ // $\overline{AX}$, E is the midpoint of $\overline{AC}$

∴ Y is the midpoint of $\overline{XC}$ (Q.E.D.)

12

In Δ ACD :

∵ Z is the midpoint of $\overline{AD}$, $\overline{ZY}$ // $\overline{CD}$

∴ Y is the midpoint of $\overline{AC}$

∴ AY = YC = 4 cm. (First req.)

, In Δ ABC :

∵ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$

∴ XY = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 6 = 3 cm.

∴ The perimeter of Δ AXY = 5 + 3 + 4 = 12 cm. (Second req.)

13

In Δ ABC :

∵ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$

∴ DE = $\frac{1}{2}$ AC = 3.5 cm.

, ∵ E is the midpoint of $\overline{BC}$, F is the midpoint of $\overline{AC}$

∴ EF = $\frac{1}{2}$ AB = 2.5 cm.

, ∵ F is the midpoint of $\overline{AC}$, D is the midpoint of $\overline{AB}$

∴ DF = $\frac{1}{2}$ BC = 4 cm.

∴ The perimeter of Δ DEF = 3.5 + 2.5 + 4 = 10 cm. (The req.)

14

In Δ XYZ :

∵ H is the midpoint of $\overline{XY}$, G is the midpoint of $\overline{XZ}$

∴ HG = $\frac{1}{2}$ YZ ∴ YZ = 2 HG (1)

∵ H is the midpoint of $\overline{XY}$

, O is the midpoint of $\overline{YZ}$

∴ HO = $\frac{1}{2}$ XZ ∴ XZ = 2 HO (2)

∵ O is the midpoint of $\overline{YZ}$, G is the midpoint of $\overline{XZ}$

∴ OG = $\frac{1}{2}$ XY ∴ XY = 2 OG (3)

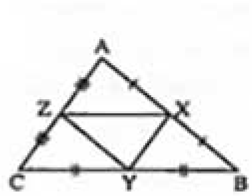
∵ The perimeter of Δ XYZ = XY + YZ + XZ

∴ From (1) , (2) , (3) : The perimeter of Δ XYZ

= 2 OG + 2 HG + 2 HO = 2 (OG + HG + HO)

= 2 the perimeter of Δ HOG = 2 × 18 = 36 cm. (The req.)

15



In Δ ABC :

∵ X is the midpoint of $\overline{AB}$

, Z is the midpoint of $\overline{AC}$

∴ XZ = $\frac{1}{2}$ BC (1)

∵ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$ ∴ XY = $\frac{1}{2}$ AC (2)

, ∵ Y is the midpoint of $\overline{BC}$, Z is the midpoint of $\overline{AC}$

∴ $YZ = \frac{1}{2} AB$ (3)

By adding (1) , (2) , (3) :

∴ $XZ + XY + YZ = \frac{1}{2} (BC + AC + AB)$

∴ The perimeter of Δ XYZ

$= \frac{1}{2}$ the perimeter of Δ ABC (Q.E.D.)

16

In Δ ABC :

∵ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$

∴ $DF = \frac{1}{2} BC = 6$ cm.

Similarly : ∵ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{BC}$

∴ $DE = \frac{1}{2} AC = 5$ cm.

∴ The perimeter of the figure DECF

$= 6 + 5 + 6 + 5 = 22$ cm. (The req.)

17

In Δ DBC :

∵ H is the midpoint of $\overline{BD}$

, O is the midpoint of $\overline{DC}$

∴ $\overline{HO} // \overline{BC}$

, ∵ $\overline{AD} // \overline{BC}$ ∴ $\overline{HO} // \overline{AD}$ (1)

, $HO = \frac{1}{2} BC$

, ∵ $AD = \frac{1}{2} BC$ ∴ HO = AD (2)

From (1) , (2) :

∴ AHOD is a parallelogram. (Q.E.D.)

18

In Δ ABC :

∵ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

∴ $\overline{DE} // \overline{BC}$, ∵ $F \in \overline{CB}$

∴ $\overline{DE} // \overline{BF}$ (1)

, ∵ $DE = \frac{1}{2} BC$, $BF = \frac{1}{2} BC$

∴ DE = BF (2)

From (1) and (2) :

∴ BEDF is a parallelogram. (Q.E.D.)

19

In Δ MBC :

∵ H is the midpoint of $\overline{MB}$, O is the midpoint of $\overline{MC}$

∴ $\overline{HO} // \overline{BC}$

, ∵ $\overline{BC} // \overline{AD}$

∴ $\overline{HO} // \overline{AD}$ (1)

, $HO = \frac{1}{2} BC$

, ∵ $AD = \frac{1}{2} BC$ ∴ HO = AD (2)

From (1) , (2) :

∴ AHOD is a parallelogram. (Q.E.D.)

20

In Δ ABC :

∵ D is the midpoint of $\overline{AB}$, E is the midpoint $\overline{AC}$

∴ $\overline{DE} // \overline{BC}$, $DE = \frac{1}{2} BC = 6$ cm.

, In Δ FDE :

∵ X is the midpoint of $\overline{DF}$, $\overline{XY} // \overline{DE}$

∴ Y is the midpoint of $\overline{EF}$

∴ $XY = \frac{1}{2} DE = 3$ cm. (The req.)

21

In Δ ABH :

∵ X is the midpoint of $\overline{AB}$, $\overline{XY} // \overline{BH}$

∴ Y is the midpoint of $\overline{AH}$

, ∵ ABCD is a parallelogram

∴ $\overline{AB} // \overline{CD}$

In Δ ABH : ∵ Y is the midpoint of $\overline{AH}$, $\overline{YC} // \overline{AB}$

∴ C is the midpoint of $\overline{BH}$ (Q.E.D.)

22

In Δ ABC :

∵ D is the midpoint of $\overline{AB}$, $\overline{DH} // \overline{BC}$

∴ H is the midpoint of $\overline{AC}$

∴ AH = HC

, ∵ HC = CO

∴ $AH = HC = CO = \frac{1}{3} AO$ (Q.E.D.1)

, In Δ DHO :

∵ C is the midpoint of $\overline{HO}$, $\overline{CX} // \overline{DH}$

∴ X is the midpoint of $\overline{DO}$

∴ OX = XD (Q.E.D.2)

23

In Δ BCO :

$\because$ D is the midpoint of $\overline{CB}$, $\overline{DZ} \parallel \overline{OC}$

$\therefore$ Z is the midpoint of $\overline{BO}$

$\therefore$ OZ = ZB (1)

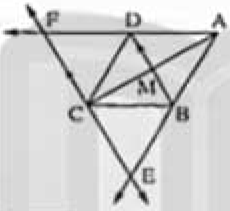
, In Δ AZD : $\because$ H is the midpoint of $\overline{AD}$, $\overline{HO} \parallel \overline{DZ}$

$\therefore$ O is the midpoint of $\overline{AZ}$

$\therefore$ AO = OZ (2)

From (1) , (2) : $\therefore$ AO = OZ = ZB (Q.E.D.)

24



$\because$ ABCD is a parallelogram

, M is the point of intersection of its diagonals

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore$ In Δ AEC :

$\because$ M is the midpoint of $\overline{AC}$, $\overline{MB} \parallel \overline{EC}$

$\therefore$ B is the midpoint of $\overline{AE}$

$\therefore$ AB = BE (Q.E.D.1)

, In Δ ACF :

$\because$ M is the midpoint of $\overline{AC}$, $\overline{MD} \parallel \overline{CF}$

$\therefore$ D is the midpoint of $\overline{AF}$

$\therefore$ AD = DF (Q.E.D.2)

25

$\because$ $\overline{AD} \parallel \overline{BC}$, $\overline{HX} \parallel \overline{BC}$

$\therefore$ $\overline{AD} \parallel \overline{BC} \parallel \overline{HX}$, $\because$ AH = HB

$\therefore$ DX = XC

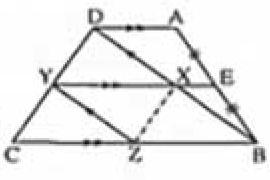
$\therefore$ X is the midpoint of $\overline{DC}$

, In Δ DBC :

$\because$ X is the midpoint of $\overline{DC}$, $\overline{XY} \parallel \overline{BD}$

$\therefore$ Y is the midpoint of $\overline{BC}$ (Q.E.D.)

26



Construction : Draw $\overline{XZ}$

Proof :

$\because$ $\overline{AD} \parallel \overline{EY} \parallel \overline{BC}$, AE = EB

$\therefore$ DX = XB , DY = YC

$\therefore$ In Δ DBC :

$\because$ Y is the midpoint of $\overline{DC}$, $\overline{YZ} \parallel \overline{BD}$

$\therefore$ Z is the midpoint of $\overline{BC}$

, $\because$ X is the midpoint of $\overline{BD}$

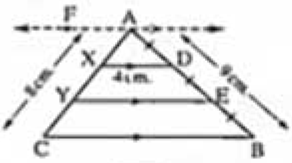
$\therefore$ $\overline{XZ} \parallel \overline{CD}$, $\because$ $\overline{XD} \parallel \overline{ZY}$

$\therefore$ XDYZ is a parallelogram.

$\therefore$ XD = YZ (Q.E.D.)

27

Construction : Draw $\overleftrightarrow{AF} \parallel \overleftrightarrow{DX}$



Proof :

$\because$ $\overleftrightarrow{AF} \parallel \overleftrightarrow{DX} \parallel \overleftrightarrow{EY} \parallel \overleftrightarrow{BC}$

, AD = DE = EB = 3 cm.

$\therefore$ AX = XY = YC = $\frac{8}{3}$

From Δ AEY : $\because$ D is the midpoint of $\overline{AE}$

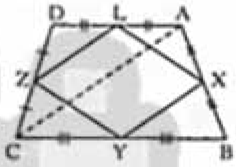
, X is the midpoint of $\overline{AY}$

$\therefore$ DX = $\frac{1}{2}$ EY $\therefore$ EY = 2 DX = 8 cm.

$\therefore$ The perimeter of the shape DEYX

$= 4 + 3 + 8 + \frac{8}{3} = 15 + 2\frac{2}{3} = 17\frac{2}{3}$ cm. (The req.)

28



Construction : Draw $\overline{AC}$

Proof : In Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore$ $\overline{XY} \parallel \overline{AC}$, XY = $\frac{1}{2}$ AC (1)

, In Δ ADC :

$\because$ L is the midpoint of $\overline{AD}$

, Z is the midpoint of $\overline{CD}$

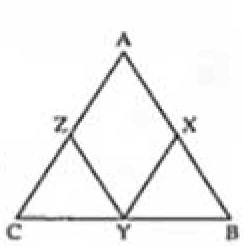
$\therefore$ $\overline{LZ} \parallel \overline{AC}$, LZ = $\frac{1}{2}$ AC (2)

From (1) and (2) :

$\therefore$ $\overline{XY} \parallel \overline{LZ}$, XY = LZ

$\therefore$ XYZL is a parallelogram. (Q.E.D.)

29



In Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore$ XY = $\frac{1}{2}$ AC = AZ (1)

, $\overline{XY} \parallel \overline{AC}$

$\therefore$ $\overline{XY} \parallel \overline{AZ}$ (2)

From (1) , (2) :

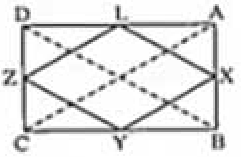
$\therefore$ AXYZ is a parallelogram

$\because$ AX = $\frac{1}{2}$ AB , AZ = $\frac{1}{2}$ AC

, $\because$ AC = AB $\therefore$ AZ = AX

$\therefore$ AXYZ is a rhombus (Q.E.D.)

30



Construction :
Draw $\overline{BD}$ and $\overline{AC}$

Proof : In Δ ABD

$\because$ X is the midpoint of $\overline{AB}$

, L is the midpoint of $\overline{AD}$

$\therefore \overline{XL} \parallel \overline{BD}$, XL $= \frac{1}{2}$ BD (1)

, In Δ DBC :

$\because$ Y is the midpoint of $\overline{BC}$, Z is the midpoint of $\overline{DC}$

$\therefore \overline{YZ} \parallel \overline{BD}$, YZ $= \frac{1}{2}$ BD (2)

From (1) , (2) : $\overline{XL} \parallel \overline{YZ}$, XL = YZ

$\therefore$ XYZL is a parallelogram , From Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore$ XY $= \frac{1}{2}$ AC (3)

From (1) , (3) :

, $\because$ The two diagonals of the rectangle are equal in length

$\therefore$ LX = XY

$\therefore$ XYZL is a rhombus (Q.E.D. 1)

, the perimeter of the rhombus

$= 4$ XL $= 4 \times \frac{1}{2}$ BD $= 2$ BD (Q.E.D. 2)

31

$\because$ ABCD is a parallelogram , its diagonals intersect at M

$\therefore$ M is the midpoint of each of $\overline{AC}$ and $\overline{DB}$

, $\because$ DBCH is a parallelogram , its diagonals intersect at L

$\therefore$ L is the midpoint of each of $\overline{DC}$ and $\overline{BH}$

In Δ DBC :

$\because$ M is the midpoint of $\overline{DB}$

, L is the midpoint of $\overline{DC}$

$\therefore \overline{ML} \parallel \overline{BC}$ (Q.E.D.1)

, ML $= \frac{1}{2}$ BC (1)

, $\because$ ABCD is a parallelogram $\therefore$ BC = AD

, $\because$ DBCH is a parallelogram $\therefore$ BC = DH

$\therefore$ AH = AD + DH = 2 BC $\therefore$ BC $= \frac{1}{2}$ AH

, From (1) :

$\therefore$ ML $= \frac{1}{2} \left(\frac{1}{2} \text{AH}\right) = \frac{1}{4}$ AH (Q.E.D.2)

32

In Δ ABC :

$\because$ F is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$

$\therefore$ FE $= \frac{1}{2}$ AC $\therefore X = \frac{1}{2}(3X - 6)$

$\therefore 2X = 3X - 6$ $\therefore X = 6$ cm.(First req.)

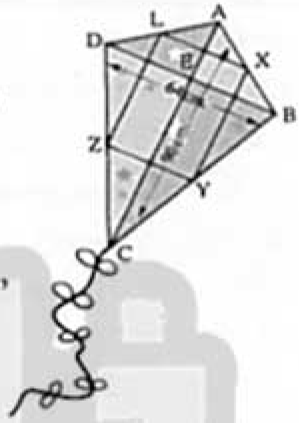
, AB $= 2 \times 6 + 1 = 13$ cm.

, $\because$ D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

$\therefore$ DE $= \frac{1}{2}$ AB $\therefore y = \frac{1}{2}$ AB

$\therefore y = 6.5$ cm. (Second req.)

33



In Δ ABD :

$\because$ X is the midpoint of $\overline{AB}$,

L is the midpoint of $\overline{AD}$

$\therefore$ XL $= \frac{1}{2}$ BD $= 32$ cm.

, In Δ ADC :

$\because$ L is the midpoint of $\overline{AD}$,

Z is the midpoint of $\overline{CD}$

$\therefore$ LZ $= \frac{1}{2}$ AC $= 45$ cm.

In Δ BCD :

$\because$ Y is the midpoint of $\overline{BC}$, Z is the midpoint of $\overline{CD}$

$\therefore$ YZ $= \frac{1}{2}$ BD $= 32$ cm.

In Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore$ XY $= \frac{1}{2}$ AC $= 45$ cm.

$\therefore$ The length of the stripe $= 32 + 45 + 32 + 45 = 154$ cm.

(The req.)

34

$\because \angle$ BAO is an exterior angle of Δ ABC

$\therefore$ m ($\angle$ BAO) = m ($\angle$ ABC) + m ($\angle$ ACB)

, $\because$ m ($\angle$ ABC) = m ($\angle$ ACB)

$\therefore$ m ($\angle$ BAO) = 2 m ($\angle$ ABC)

$\therefore \frac{1}{2}$ m ($\angle$ BAO) = m ($\angle$ ABC)

$\therefore$ m ($\angle$ HAB) = m ($\angle$ ABC) and they are alternate angles

$\therefore \overline{BC} \parallel \overline{AH}$

$\therefore$ In Δ AHD : $\because$ C is the midpoint of $\overline{AD}$, $\overline{CB} \parallel \overline{AH}$

$\therefore$ B is the midpoint of $\overline{DH}$

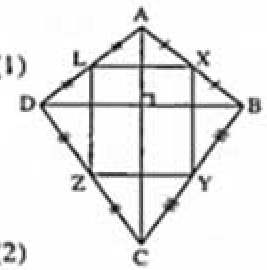
$\therefore$ DB = BH (Q.E.D.)

35

In ΔABC : $\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore \overline{XY} \parallel \overline{AC}$, $XY = \frac{1}{2} AC$ (1)



, From ΔADC :

$\because$ L is the midpoint of $\overline{AD}$

, Z is the midpoint of $\overline{DC}$

$\therefore \overline{LZ} \parallel \overline{AC}$, $LZ = \frac{1}{2} AC$ (2)

From (1) , (2) : $\therefore \overline{XY} \parallel \overline{LZ}$, $XY = LZ$

$\therefore$ XYZL is a parallelogram ,

, In ΔABD :

$\because$ X is the midpoint of $\overline{AB}$, L is the midpoint of $\overline{AD}$

$\therefore \overline{XL} \parallel \overline{BD}$, $XL = \frac{1}{2} BD$

, $\because \overline{AC}$, $\overline{BD}$ are perpendicular ,

$\overline{XY} \parallel \overline{AC}$, $\overline{XL} \parallel \overline{BD}$

$\therefore \overline{XY}$ and $\overline{XL}$ are perpendicular

$\therefore$ XYZL is a rectangle

$\therefore$ The area of the rectangle

$= XL \times XY = \frac{1}{2} BD \times \frac{1}{2} AC$

$= \frac{1}{4} AC \times BD$ (Q.E.D.)

Answers of Exercise 7

1

Fig. (1) : In ΔABC : $\because m(\angle A) = 90^\circ$

$\therefore (BC)^2 = (AB)^2 + (AC)^2 = 64 + 36 = 100$

$\therefore BC = \sqrt{100} = 10$ cm.

Fig. (2) : In ΔABC : $\because m(\angle B) = 90^\circ$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$= 144 + 25 = 169$

$\therefore AC = \sqrt{169} = 13$ cm.

Fig. (3) : In ΔABC : $\because m(\angle C) = 90^\circ$

$\therefore (AB)^2 = (AC)^2 + (BC)^2$

$= 256 + 144 = 400$

$\therefore AB = \sqrt{400} = 20$ cm.

Fig. (4) : In ΔABC : $\because m(\angle B) = 90^\circ$

$\therefore (BC)^2 = (AC)^2 - (AB)^2$

$= 225 - 81 = 144$

$\therefore BC = \sqrt{144} = 12$ cm.

Fig. (5) : In ΔABC : $\because m(\angle B) = 90^\circ$

$\therefore (BC)^2 = (AC)^2 - (AB)^2$

$= 625 - 225 = 400$

$\therefore BC = \sqrt{400} = 20$ cm.

Fig. (6) : In ΔABC : $\because m(\angle A) = 90^\circ$

$\therefore (AB)^2 = (BC)^2 - (AC)^2$

$= 625 - 576 = 49$

$\therefore AB = \sqrt{49} = 7$ cm.

2

$\because$ ABCD is a square.

$\therefore DC = 4$ cm. , $m(\angle DCB) = 90^\circ$

$\therefore$ In ΔDCE : $(DE)^2 = (DC)^2 + (CE)^2$

$= (4)^2 + (3)^2 = 16 + 9 = 25$

$\therefore DE = 5$ cm. (The req.)

3

In ΔADC : $\because m(\angle ADC) = 90^\circ$

$\therefore (AD)^2 = (AC)^2 - (DC)^2 = 400 - 256 = 144$

$\therefore AD = \sqrt{144} = 12$ cm. (First req.)

, In ΔABD : $\because m(\angle ADB) = 90^\circ$

$\therefore (AB)^2 = (AD)^2 + (BD)^2 = 144 + 81 = 225$

$\therefore AB = \sqrt{225} = 15$ cm. (Second req.)

, The area of $\Delta ABC = \frac{1}{2} BC \times AD = \frac{1}{2} \times 25 \times 12$

$= 150$ cm^2. (Third req.)

4

In ΔABD : $\because m(\angle ADB) = 90^\circ$

$\therefore (BD)^2 = (AB)^2 - (AD)^2 = 676 - 576 = 100$

$\therefore BD = \sqrt{100} = 10$ cm.

, In ΔADC : $\because m(\angle ADC) = 90^\circ$

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = 900 - 576 = 324$

$\therefore CD = \sqrt{324} = 18$ cm.

$\therefore BC = BD + DC = 10 + 18 = 28$ cm. (First req.)

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 28 \times 24$

$= 336$ cm^2. (Second req.)

5

In Δ XYZ : $\because$ m ($\angle$ Y) = 90°

$\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = 49 + 576 = 625$

$\therefore XZ = \sqrt{625} = 25$ cm. (First req.)

, In Δ XLZ : $\because$ m ($\angle$ L) = 90°

$\therefore (LZ)^2 = (XZ)^2 - (XL)^2 = 625 - 225 = 400$

$\therefore LZ = \sqrt{400} = 20$ cm. (Second req.)

6

In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 81 + 144 = 225$

$\therefore AC = \sqrt{225} = 15$ cm. (First req.)

, In Δ ACD : $\because$ m ($\angle$ ACD) = 90°

$\therefore (AD)^2 = (AC)^2 + (CD)^2 = 225 + 400 = 625$

$\therefore AD = \sqrt{625} = 25$ cm. (Second req.)

, The perimeter of the figure ABCD

$= 9 + 12 + 20 + 25 = 66$ cm. (Third req.)

, The area of the figure ABCD

= The area of Δ ABC + The area of Δ ACD

$= \frac{1}{2} \times 12 \times 9 + \frac{1}{2} \times 20 \times 15 = 54 + 150 = 204$ cm². (Fourth req.)

7

In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (BC)^2 = (AC)^2 - (AB)^2 = 100 - 64 = 36$

$\therefore BC = \sqrt{36} = 6$ cm. (First req.)

, In Δ ABD : $\because$ m ($\angle$ B) = 90°

$\therefore (BD)^2 = (AD)^2 - (AB)^2 = 289 - 64 = 225$

$\therefore BD = \sqrt{225} = 15$ cm. (Second req.)

, $\because$ CD = BD − BC

$\therefore$ CD = 15 − 6 = 9 cm. (Third req.)

8

In Δ ABD : $\because$ m ($\angle$ ABD) = 90°

$\therefore (BD)^2 = (AD)^2 - (AB)^2 = 225 - 81 = 144$

$\therefore BD = \sqrt{144} = 12$ cm.

, $\because \overline{AB} \parallel \overline{CD}$, $\overline{BD}$ is a transversal

$\therefore$ m ($\angle$ BDC) = m ($\angle$ ABD) = 90°

$\therefore$ In Δ BDC : m ($\angle$ BDC) = 90°

$\therefore (BC)^2 = (BD)^2 + (DC)^2 = 144 + 25 = 169$

$\therefore BC = \sqrt{169} = 13$ cm. (The req.)

9

$\because \overline{AD} \parallel \overline{EC}$, m ($\angle$ AEB) = m ($\angle$ C) = 90°

and they are corresponding angles

$\therefore \overline{AE} \parallel \overline{DC}$ $\therefore$ The figure AECD is a parallelogram

, $\because$ m ($\angle$ C) = 90°

$\therefore$ The figure AECD is a rectangle

$\therefore$ EC = AD = 9 cm. $\therefore$ BE = 17 − 9 = 8 cm.

$\therefore$ In Δ AEB : m ($\angle$ AEB) = 90°

$\therefore (AE)^2 = (AB)^2 - (BE)^2 = 289 - 64 = 225$

$\therefore AE = \sqrt{225} = 15$ cm.

$\therefore$ DC = AE = 15 cm. (First req.)

The area of the trapezium ABCD

= The area of Δ ABE + The area of the rectangle AECD

$= \frac{1}{2} \times 8 \times 15 + 9 \times 15 = 195$ cm². (Second req.)

10

In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 36 + 64 = 100$

$\therefore AC = \sqrt{100} = 10$ cm.

, $\because \overline{AB} \parallel \overline{DE}$, $\overline{BD}$ is a transversal

$\therefore$ m ($\angle$ D) = m ($\angle$ B) (alternate angles)

In $\Delta\Delta$ ABC , EDC

$$\left\{\begin{array}{l} m(\angle D) = m(\angle B) \\ m(\angle ACB) = m(\angle ECD) \quad \text{(V.O.A.)} \\ BC = DC \end{array}\right.$$

$\therefore \Delta ABC \equiv \Delta EDC$

$\therefore$ CE = CA = 10 cm. (The req.)

11

In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 16 + 9 = 25$

$\therefore AC = \sqrt{25} = 5$ cm.

, $\because$ The area of Δ ABC $= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times AC \times BD$

$\therefore \frac{1}{2} \times 3 \times 4 = \frac{1}{2} \times 5 \times BD$ $\therefore 6 = \frac{5}{2} \times BD$

$\therefore BD = 6 \div \frac{5}{2} = 2.4$ cm. (The req.)

12 1 The sum of areas of the squares on the sides of the right angle.

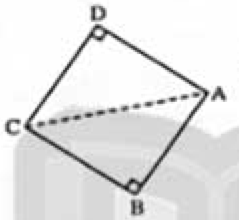
2 15   3 15   4 144   5 10

6 13   7 3   8 169   9 4

13 1 (d)   2 (b)   3 (c)   4 (c)

14

Constr. : Draw $\overline{AC}$



Proof : In Δ ABC :

$\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2$ (1)

In Δ ADC : $\because$ m ($\angle$ D) = 90°

$\therefore (AC)^2 = (AD)^2 + (DC)^2$ (2)

From (1) , (2) : $\therefore (AB)^2 + (BC)^2 = (AD)^2 + (DC)^2$

(Q.E.D.)

15

In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 400 - 144 = 256$

$\therefore AB = \sqrt{256} = 16$ cm.

, $\because$ BD = 9 cm. $\therefore$ AD = 16 – 9 = 7 cm. (First req.)

, $\because$ AE = 2 BC = 2 × 12 = 24 cm.

, $\because \overline{EA} \parallel \overline{BC}$, $\overline{AB}$ is a transversal

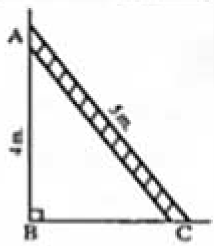
$\therefore$ m ($\angle$ EAB) = m ($\angle$ B) = 90° (alternate angles)

$\therefore$ In Δ EAD : $\because$ m ($\angle$ EAD) = 90°

$\therefore (ED)^2 = (EA)^2 + (AD)^2 = 576 + 49 = 625$

$\therefore ED = \sqrt{625} = 25$ cm. (Second req.)

16



In Δ ABC : $\because$ m ($\angle$ B) = 90°

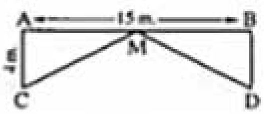
$\therefore (BC)^2 = (AC)^2 - (AB)^2$

$= 25 - 16 = 9$

$\therefore BC = \sqrt{9} = 3$ m.

$\therefore$ The distance = 3 m. (The req.)

17



$\because$ M is the midpoint of $\overline{AB}$

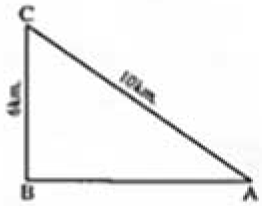
$\therefore AM = \frac{15}{2} = 7.5$ m.

In Δ ACM : $\because$ m ($\angle$ A) = 90°

$\therefore (MC)^2 = (AC)^2 + (AM)^2 = 16 + 56.25 = 72.25$

$\therefore MC = \sqrt{72.25} = 8.5$ m. (The req.)

18



In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AB)^2 = (AC)^2 - (BC)^2$

$= 100 - 36 = 64$

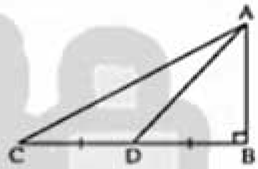
$\therefore AB = \sqrt{64} = 8$ km.

$\therefore$ If Mina takes the two roads (way to school and way to airport) , the distance will be = 6 + 8 = 14 km.

$\therefore$ If he takes the main road , the distance will be = 10 km.

$\therefore$ The distance he saves = 14 – 10 = 4 km. (The req.)

19



In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$= (AB)^2 + (2BD)^2$

$= (AB)^2 + 4\,(BD)^2$ (1)

, In Δ ABD : $\because$ m ($\angle$ B) = 90°

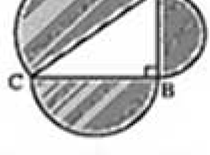
$\therefore (AD)^2 = (AB)^2 + (BD)^2$ (2)

By subtracting (2) from (1) :

$\therefore (AC)^2 - (AD)^2 = 3\,(BD)^2$ (Q.E.D.)

20

Let $r_1 = \frac{1}{2}AC$, $r_2 = \frac{1}{2}AB$, $r_3 = \frac{1}{2}BC$



In Δ ABC : $\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \left(\frac{1}{2}AC\right)^2 = \left(\frac{1}{2}AB\right)^2 + \left(\frac{1}{2}BC\right)^2$

$\therefore r_1^2 = r_2^2 + r_3^2$ multiplying by $\frac{1}{2}\pi$

$\therefore \frac{1}{2}\pi r_1^2 = \frac{1}{2}\pi r_2^2 + \frac{1}{2}\pi r_3^2$

$\therefore$ The area of the semicircle drawn on the hypotenuse equals the sum of areas of the two semicircles drawn on the two sides of the right angle. (Q.E.D.)

Answers of exams on the second part of unit three

Model 1

1 1 (b) 2 (d) 3 (a)
4 (c) 5 (c) 6 (b)

2 1 the sum of measures of its non adjacent interior angles.
2 acute 3 parallel
4 the sum of areas of squares on the sides of the right angle.
5 bisects the third side.

3 [a] m (∠ A) = 75° , m (∠ B) = 55° , m (∠ C) = 50°
[b] The perimeter of Δ DEF = $11\frac{1}{2}$ cm.

4 [a] XZ = 25 cm. , LZ = 20 cm.
[b] m (∠ A) = 40° , m (∠ B) = 60° , m (∠ C) = 80°

5 [a] Prove by yourself
[b] 1 BC = 28 cm.
2 The area of Δ ABC = 336 cm^2.

Model 2

1
1 (c) 2 (d) 3 (d) 4 (b) 5 (a) 6 (c)

2
1 obtuse-angled 2 $(AC)^2 - (BC)^2$
3 17 4 70° 5 5

3 [a] YZ = 4 cm.
[b] AB = 20 cm. , AD = 12 cm.

4 [a] m (∠ C) = 40°
, m (∠ ABC) = 45° , m (∠ BAC) = 95°
, m (∠ ABD) = 135°
[b] Prove by yourself

5 [a] 1 The perimeter of ABCD = 32 cm.
2 The area of ABCD = 36 cm^2.
[b] 1 m (∠ A) = 67°
2 m (∠ BCE) = 120°

Answers of Exercise 8

1 1 translation 2 rotation 3 reflection

2 1 rotation 2 rotation 3 translation.

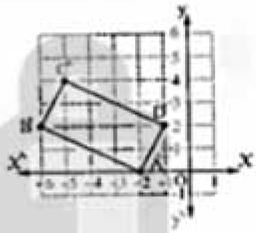
3 1 reflection 2 rotation 3 translation.

4 1 translation 2 rotation 3 reflection

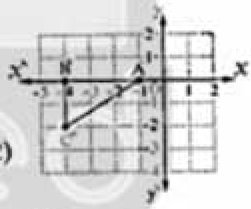
5 1 translation 2 rotation 3 reflection

6 1 translation 2 reflection 3 rotation
4 rotation 5 reflection

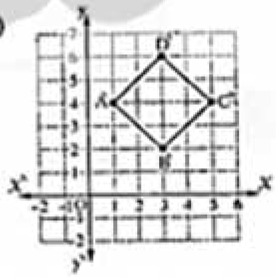
7
1 $(x , y) \longrightarrow (-x , y)$
∴ A (2 , 0) ⟶ À (−2 , 0)
, B (6 , 2) ⟶ B̀ (−6 , 2)
, C (5 , 4) ⟶ C̀ (−5 , 4)
, D (1 , 2) ⟶ D̀ (−1 , 2)
(The type is reflection)



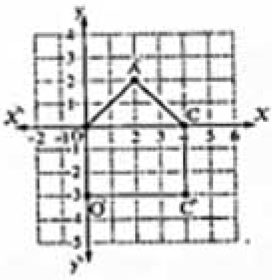
2 $(x , y) \longrightarrow (-x , -y)$
∴ A (1 , 0) ⟶ À (−1 , 0)
, B (4 , 0) ⟶ B̀ (−4 , 0)
, C (4 , 2) ⟶ C̀ (−4 , −2)
(The type is rotation)



3 $(x , y) \longrightarrow (x + 2 , y + 3)$
∴ A (−1 , 1) ⟶ À (1 , 4)
, B (1 , −1) ⟶ B̀ (3 , 2)
, C (3 , 1) ⟶ C̀ (5 , 4)
, D (1 , 3) ⟶ D̀ (3 , 6)
(The type is translation)



4 $(x , y) \longrightarrow (x , y - 3)$
∴ A (2 , 5) ⟶ À (2 , 2)
, B (4 , 3) ⟶ C (4 , 0)
, C (4 , 0) ⟶ C̀ (4 , −3)
, O (0 , 0) ⟶ Ò (0 , −3)
, E (0 , 3) ⟶ O (0 , 0)
(The type is translation)



5 $(x , y) \longrightarrow (y , -x)$

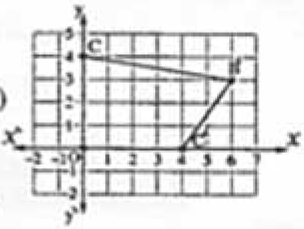
$\therefore A(-4 , 0) \longrightarrow C(0 , 4)$

, $B(-3 , 6) \longrightarrow \grave{B}(6 , 3)$

, $C(0 , 4) \longrightarrow \grave{C}(4 , 0)$

, $O(0 , 0) \longrightarrow O(0 , 0)$

(The type is rotation)



6 $(x , y) \longrightarrow (-x , y)$

$\therefore A(4 , 1) \longrightarrow \grave{A}(-4 , 1)$

, $B(3 , 2) \longrightarrow \grave{B}(-3 , 2)$

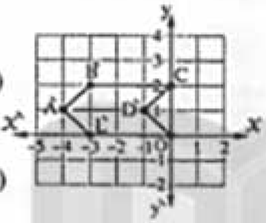
, $C(0 , 2) \longrightarrow C(0 , 2)$

, $D(1 , 1) \longrightarrow \grave{D}(-1 , 1)$

, $O(0 , 0) \longrightarrow O(0 , 0)$

, $L(3 , 0) \longrightarrow \grave{L}(-3 , 0)$

(The type is reflection)



8

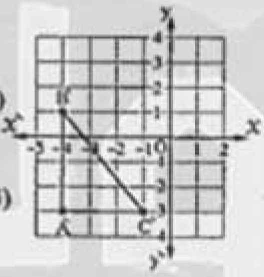
1 $(x , y) \longrightarrow (-x , y)$

$\therefore A(4 , -3) \longrightarrow \grave{A}(-4 , -3)$

, $B(4 , 1) \longrightarrow \grave{B}(-4 , 1)$

, $C(1 , -3) \longrightarrow \grave{C}(-1 , -3)$

(The type is reflection)



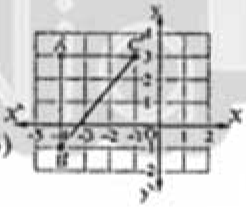
2 $(x , y) \longrightarrow (-x , -y)$

$\therefore A(4 , -3) \longrightarrow \grave{A}(-4 , 3)$

, $B(4 , 1) \longrightarrow \grave{B}(-4 , -1)$

, $C(1 , -3) \longrightarrow \grave{C}(-1 , 3)$

(The type is rotation)



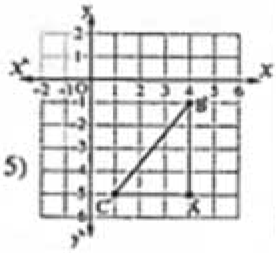
3 $(x , y) \longrightarrow (x , y - 2)$

$\therefore A(4 , -3) \longrightarrow \grave{A}(4 , -5)$

, $B(4 , 1) \longrightarrow \grave{B}(4 , -1)$

, $C(1 , -3) \longrightarrow \grave{C}(1 , -5)$

(The type is translation)



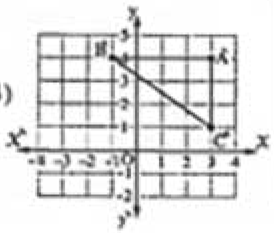
4 $(x , y) \longrightarrow (-y , x)$

$\therefore A(4 , -3) \longrightarrow \grave{A}(3 , 4)$

, $B(4 , 1) \longrightarrow \grave{B}(-1 , 4)$

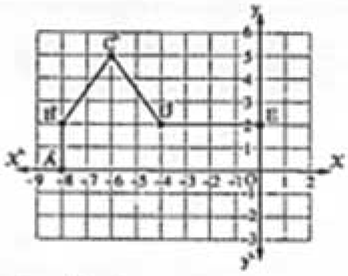
, $C(1 , -3) \longrightarrow \grave{C}(3 , 1)$

(The type is rotation)



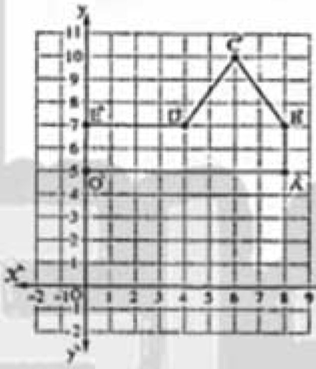
9

1 $(x , y) \longrightarrow (-x , y)$



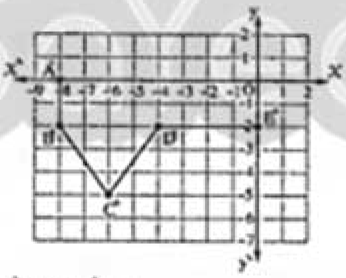
The type is reflection.

2 $(x , y) \longrightarrow (x , y + 5)$



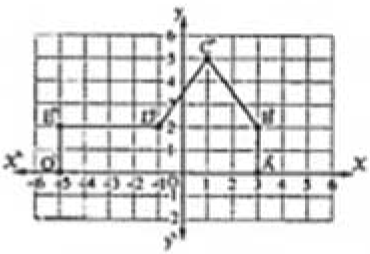
The type is translation.

3 $(x , y) \longrightarrow (-x , -y)$



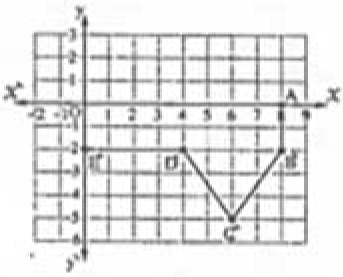
The type is rotation.

4 $(x , y) \longrightarrow (x - 5 , y)$



The type is translation.

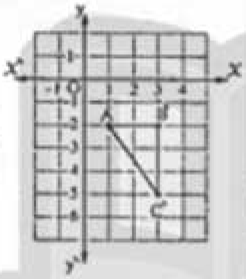
5 $(x , y) \longrightarrow (x , -y)$



The type is reflection.

10

1 $(x , y) \longrightarrow (x , -y)$

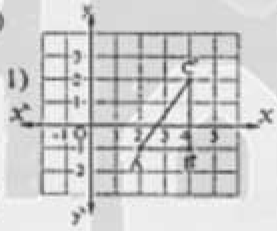


$\therefore A(1 , 2) \longrightarrow A'(1 , -2)$

$, B(3 , 2) \longrightarrow B'(3 , -2)$

$, C(3 , 5) \longrightarrow C'(3 , -5)$

2 $(x , y) \longrightarrow (x + 1 , y - 3)$

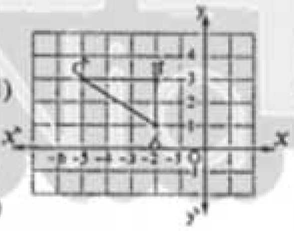


$\therefore A(1 , 2) \longrightarrow A'(2 , -1)$

$, B(3 , 2) \longrightarrow B'(4 , -1)$

$, C(3 , 5) \longrightarrow C'(4 , 2)$

3 $(x , y) \longrightarrow (-y , x)$



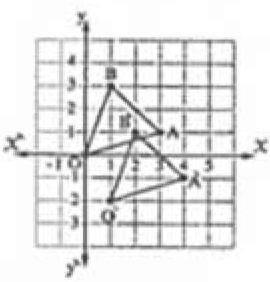
$\therefore A(1 , 2) \longrightarrow A'(-2 , 1)$

$, B(3 , 2) \longrightarrow B'(-2 , 3)$

$, C(3 , 5) \longrightarrow C'(-5 , 3)$

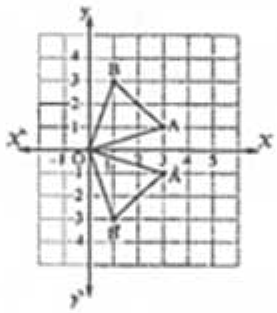
11

1 $(x , y) \longrightarrow (x + 1 , y - 2)$



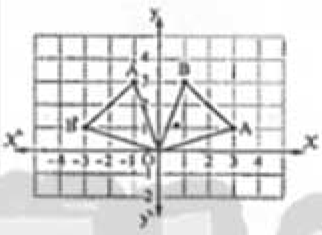
The type is translation.

2 $(x , y) \longrightarrow (x , -y)$



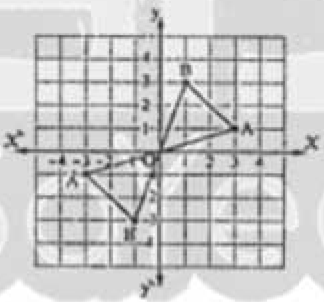
The type is reflection.

3 $(x , y) \longrightarrow (-y , x)$



The type is rotation.

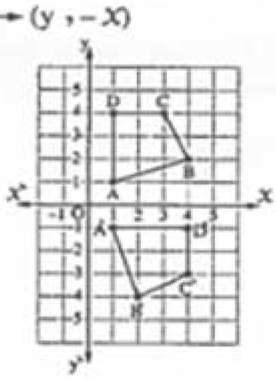
4 $(x , y) \longrightarrow (-x , -y)$



The type is rotation.

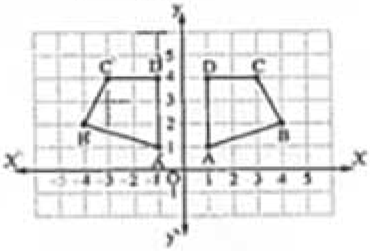
12

1 $(x , y) \longrightarrow (y , -x)$



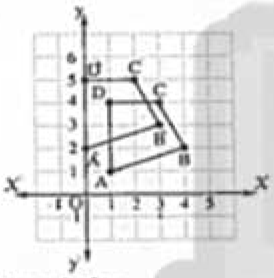
The type is rotation.

2 $(X, y) \longrightarrow (-X, y)$



The type is reflection.

3 $(X, y) \longrightarrow (X-1, y+1)$



The type is translation.

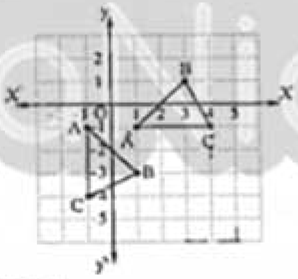
13

$\because (X, y) \longrightarrow (-y, X)$

$\therefore A(X, y) \longrightarrow \acute{A}(1, -1)$ $\therefore A(-1, -1)$

$, B(X, y) \longrightarrow \acute{B}(3, 1)$ $\therefore B(1, -3)$

$, C(X, y) \longrightarrow \acute{C}(4, -1)$ $\therefore C(-1, -4)$



The type is rotation.

Answers of Exercise 9

First : Problems on reflection in the plane :

1

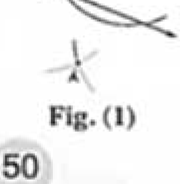


Fig. (1)

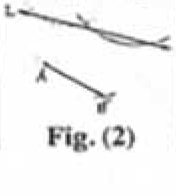


Fig. (2)

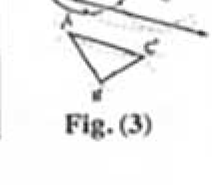
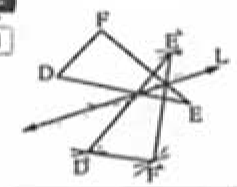


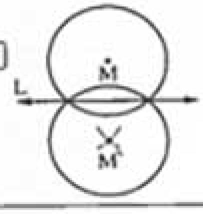
Fig. (3)

2

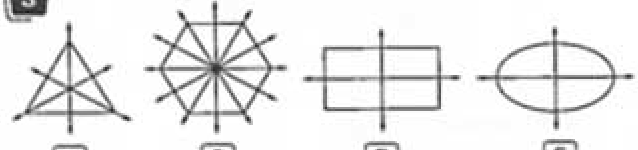
1



2



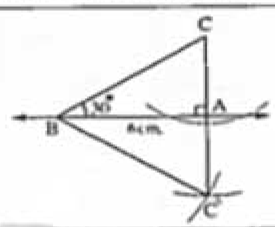
3



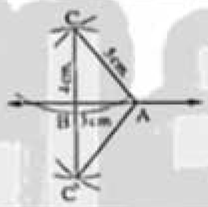
1 2 3 6

4 , 5 has no axis of symmetry.

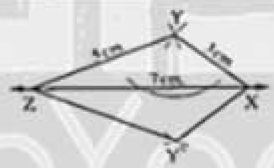
4



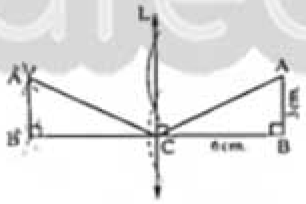
5



6

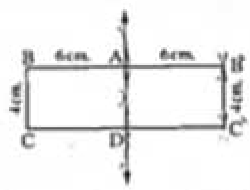


7

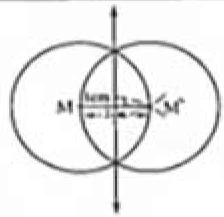


8

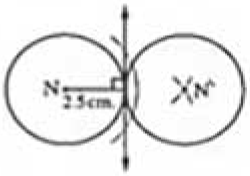
The resulting figure is a rectangle and its perimeter $= 2 \times (12 + 4) = 32$ cm.



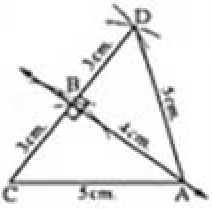
9



10



11



1 The perimeter of Δ ACD
= 16 cm.

2 The area of Δ ACD
$= \frac{1}{2} \times 6 \times 4 = 12$ cm^2.

12

1 The point D 2 $\overline{BM}$ 3 Δ DLM
4 Δ BYM 5 Δ AXM 6 Δ DMC
7 Δ BMA
8 The square DZML , the square ALMX
9 The square DCBA 10 $\overline{XZ}$ 11 $\overline{BD}$

13

1 $\overline{AE}$, $\overline{BF}$, $\overline{CD}$ 2 $\overline{AE}$ 3 $\overline{CF}$, $\overline{BD}$
4 Δ AMF , AMF , measures of the angles
5 Δ AMC
6 Δ AMC , Δ BMA , lengths of the line segments

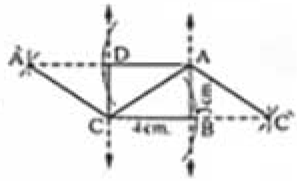
14

1 (a) the lengths of line segments.
(b) the measures of angles.
(c) parallelism (d) betweeness
2 the axis of symmetry of the figure.
3 (a) 3 (b) 1 (c) zero
(d) zero (e) 2 (f) 2
(g) 4 (h) zero
(i) 1 (j) an infinite number
4 20°

15

$\because$ the straight line L is the axis of symmetry of the figure ABCDE
$\therefore$ m ($\angle$ A) = m ($\angle$ E) = 130°
, m ($\angle$ D) = m ($\angle$ B) = 110°
from the pentagon ABCDE
$\therefore$ m ($\angle$ BCD) = 540° − (130° + 130° + 110° + 110°)
= 60° (The req.)

16



$\because$ A is the image of itself by reflection in $\overleftrightarrow{AB}$
, B is the image of itself by reflection in $\overleftrightarrow{AB}$
, $\grave{C}$ is the image of C by reflection in $\overleftrightarrow{AB}$
$\therefore$ Δ AB$\grave{C}$ is the image of Δ ABC by reflection in $\overleftrightarrow{AB}$
$\therefore$ m ($\angle$ $\grave{C}$AB) = m ($\angle$ CAB)
$\therefore$ m ($\angle$ $\grave{C}$AC) = 2 m ($\angle$ CAB) (Q.E.D.1) (1)
similarly we can prove that :
m (AC$\grave{A}$) = 2 m ($\angle$ ACD) (2)
$\because$ ABCD is a rectangle $\therefore$ $\overline{AB}$ // $\overline{CD}$
$\therefore$ m ($\angle$ BAC) = m ($\angle$ ACD) (alternate angles) (3)
from (1) , (2) , (3) :
$\therefore$ m ($\angle$ $\grave{C}$AC) = m ($\angle$ AC$\grave{A}$)
but they are alternate angles
$\therefore$ $\overleftrightarrow{A\grave{C}}$ // $\overleftrightarrow{\grave{A}C}$ (Q.E.D.2)

Second : Problems on reflection in the Cartesian plane :

1

1 A (2 , 4) $\xrightarrow[\text{X-axis}]{\text{its image in}}$ $\grave{A}$ (2 , −4)
B (4 , 0) $\xrightarrow[\text{X-axis}]{\text{its image in}}$ $\grave{B}$ (4 , 0)
C (0 , −2) $\xrightarrow[\text{X-axis}]{\text{its image in}}$ $\grave{C}$ (0 , 2)
D (−2 , 1) $\xrightarrow[\text{X-axis}]{\text{its image in}}$ $\grave{D}$ (−2 , −1)
E (−3 , −2) $\xrightarrow[\text{X-axis}]{\text{its image in}}$ $\grave{E}$ (−3 , 2)

2 A (2 , 4) $\xrightarrow[\text{y-axis}]{\text{its image in}}$ $\grave{A}$ (−2 , 4)
B (4 , 0) $\xrightarrow[\text{y-axis}]{\text{its image in}}$ $\grave{B}$ (−4 , 0)

$C(0\ ,-2) \xrightarrow[\text{y-axis}]{\text{its image in}} \grave{C}(0\ ,-2)$

$D(-2\ ,1) \xrightarrow[\text{y-axis}]{\text{its image in}} \grave{D}(2\ ,1)$

$E(-3\ ,-2) \xrightarrow[\text{y-axis}]{\text{its image in}} \grave{E}(3\ ,-2)$

2

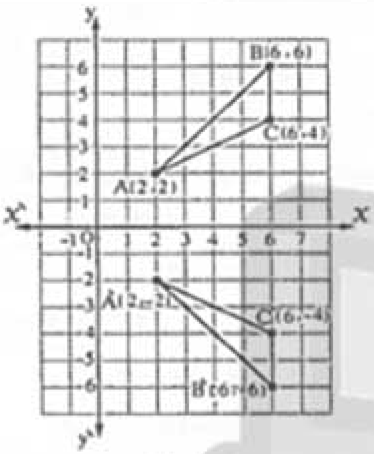


Fig. (1)

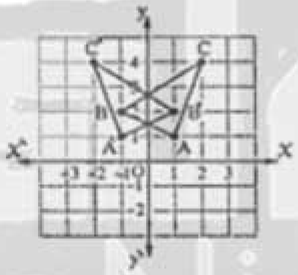


Fig. (2)

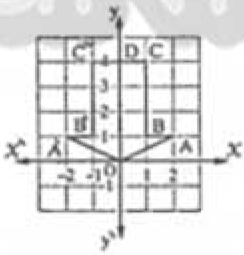


Fig. (3)

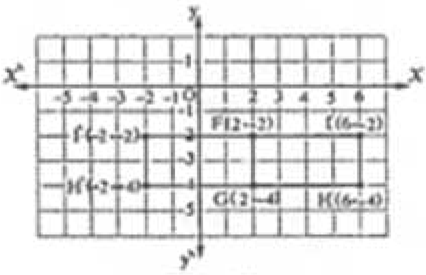
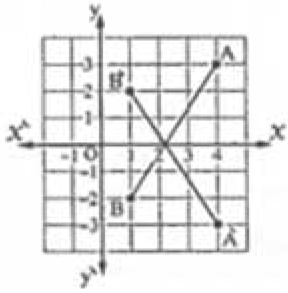


Fig. (4)

3

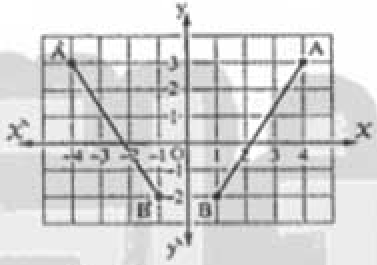
1. $A(4\ ,3) \xrightarrow[\text{X-axis}]{\text{its image in}} \grave{A}(4\ ,-3)$

   $B(1\ ,-2) \xrightarrow[\text{X-axis}]{\text{its image in}} \grave{B}(1\ ,2)$



2. $A(4\ ,3) \xrightarrow[\text{y-axis}]{\text{its image in}} \grave{A}(-4\ ,3)$

   $B(1\ ,-2) \xrightarrow[\text{y-axis}]{\text{its image in}} \grave{B}(-1\ ,-2)$

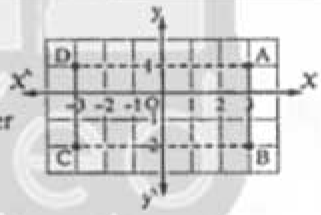


4

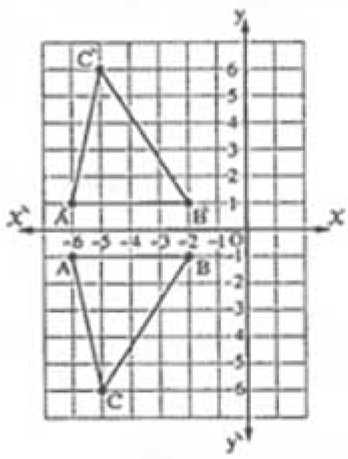
The figure ABCD is a rectangle and its perimeter

$= (6 + 3) \times 2$

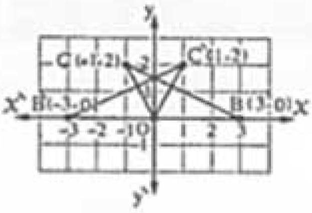
$= 18$ length units



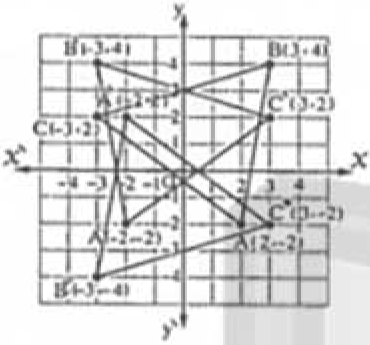
5



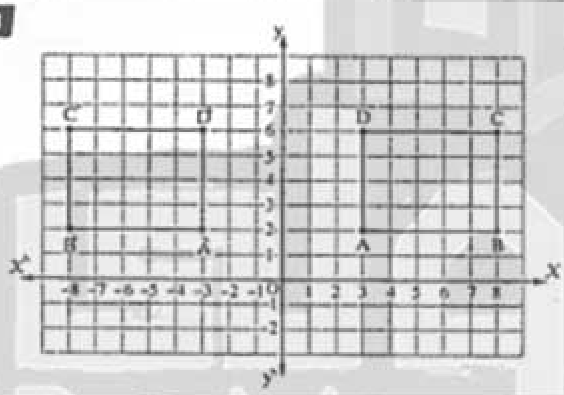
6



7



8



9

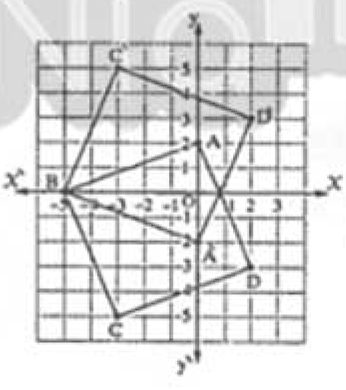
We notice that

$AB = A'B'$, $BC = B'C'$

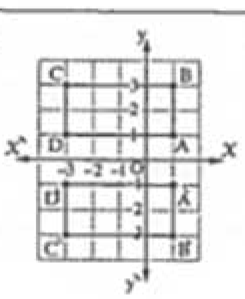
, $CD = C'D'$

, $AD = A'D'$

The area of the square ABCD = The area of the square $A'B'C'D'$



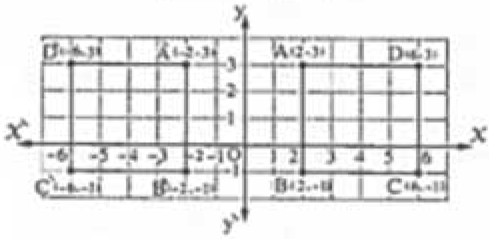
10 D = (−3 , 1)



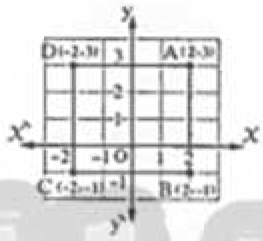
11

There are two cases to draw the square.

The first case



The second case



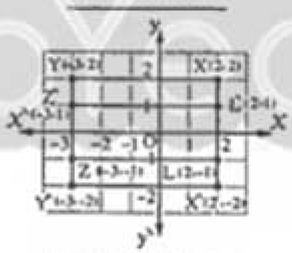
In the second case we notice that the image of the square ABCD by reflection in y-axis is DCBA

i.e. The same square.

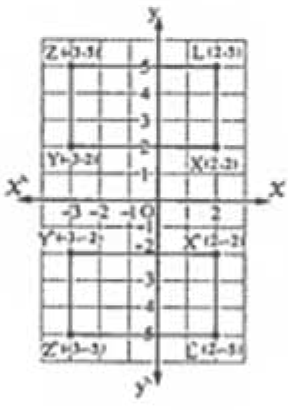
12

There are two cases to draw the rectangle

The first case



The second case



13

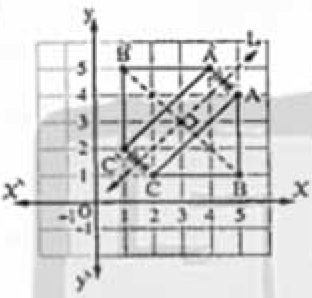
The point	Its image by reflection in the X-axis	Its image by reflection in the y-axis
$(3 , -2)$	$(3 , 2)$	$(-3 , -2)$
$(1 , -2)$	$(1 , 2)$	$(-1 , -2)$
$(2 , 4)$	$(2 , -4)$	$(-2 , 4)$
$(0 , 5)$	$(0 , -5)$	$(0 , 5)$
$(3 , 0)$	$(3 , 0)$	$(-3 , 0)$
$(0 , 0)$	$(0 , 0)$	$(0 , 0)$

14

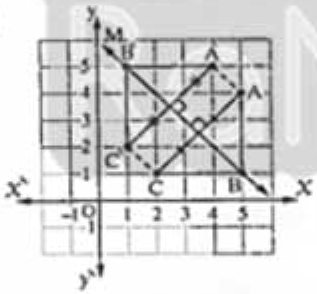
1. $(1 , -3)$
2. $(2 , 5)$
3. X-axis
4. y-axis
5. y-axis
6. X-axis
7. $(-2 , -1)$
8. $(-2 , 3)$
9. A $(2 , 3)$

15

1



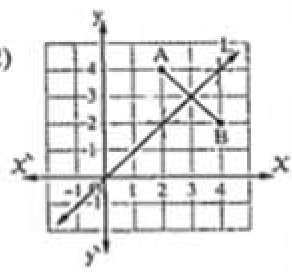
2



16

By taking A $(2 , 4)$, B $(4 , 2)$, then we draw $\overline{AB}$, then the axis of symmetry of $\overline{AB}$ is the straight line L

"try to solve this question by another way"



Answers of Exercise 10

First : Problems on reflection in the plane :

1

1. (c)
2. (b)
3. (d)
4. (c)

2

1. the point C
2. the point Z
3. $\overline{CY}$
4. $\overline{MX}$
5. $\overline{DM}$
6. $\overline{BX}$
7. Δ CYM
8. Δ DZM
9. Δ CMD
10. the square CZMY

3

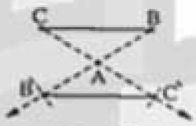


Fig. (1)

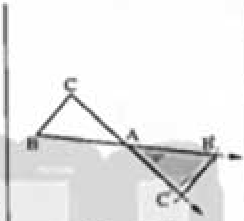


Fig. (2)

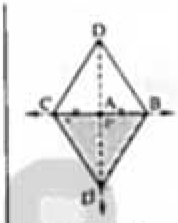
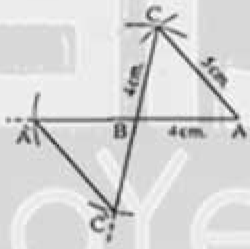


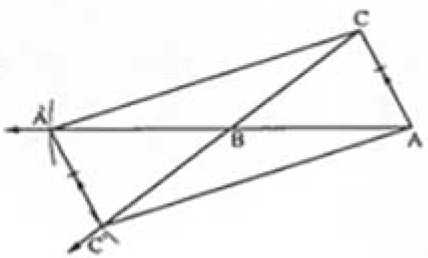
Fig. (3)

4



5

Fig. (1)



$\because$ $\acute{A}$ is the image of A by reflection in the point B

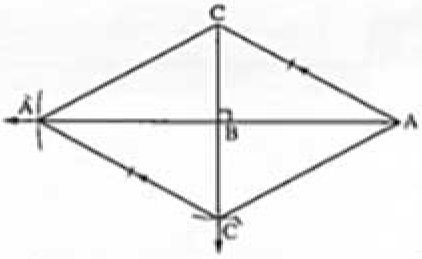
$\acute{C}$ is the image of C by reflection in the point B

$\therefore$ $\overline{\acute{A}\acute{C}}$ is the image of $\overline{AC}$ by reflection in the point B

$\therefore$ $\overline{AC} \parallel \overline{\acute{A}\acute{C}}$, $AC = \acute{A}\acute{C}$

$\therefore$ the quadrilateral $A\acute{C}\acute{A}C$ is a parallelogram.

Fig. (2)

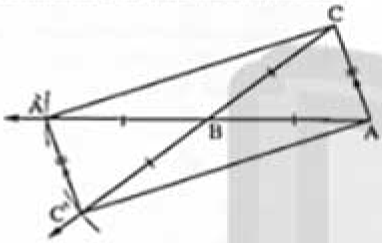


As we said previously the quadrilateral $AC'A'C$ is a parallelogram.

$\because \overline{AA'} \perp \overline{CC'}$

$\therefore$ the quadrilateral $AC'A'C$ is a rhombus.

Fig. (3)



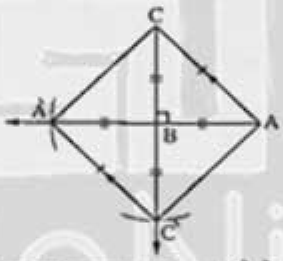
As we mentioned before the quadrilateral $AC'A'C$ is a parallelogram.

$\because AB = A'B$, $CB = C'B$, $AB = BC$

$\therefore AA' = CC'$

$\therefore$ the quadrilateral $AC'A'C$ is a rectangle.

Fig. (4)



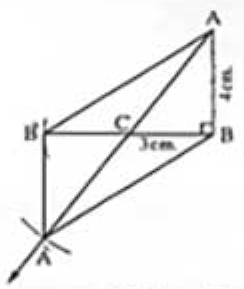
As we mentioned before $AC'A'C$ is a parallelogram.

$\because AB = BA'$, $CB = BC'$, $AB = BC$

$\therefore AA' = CC'$, $\overline{AA'} \perp \overline{CC'}$

$\therefore$ the quadrilateral $AC'A'C$ is a square.

6



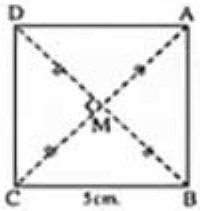
$\because A'$ is the image of A by reflection in C

, B' is the image of B by reflection in C

$\therefore AB = A'B'$, $\overline{AB} \parallel \overline{A'B'}$

$\therefore$ the figure $ABA'B'$ is a parallelogram. (Q.E.D)

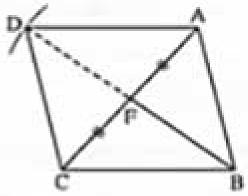
7



The image of the square ABCD by reflection in the point M is the square CDAB

We notice that we got the same square.

8

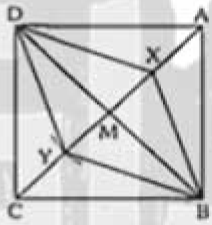


The figure ABCD is a parallelogram.

1 A right-angled triangle at B.

2 An isosceles triangle (AB = BC)

9



$\because$ ABCD is a square. $\therefore AM = MC$ (1)

$\because$ Y is the image of X by reflection in M

$\therefore XM = YM$ (2)

Subtracting (2) from (1) :

$\therefore AX = CY$

$\Delta\Delta$ DAX , BCY in them :

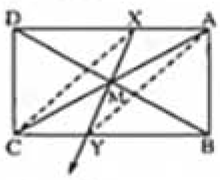
$\begin{cases} AD = CB \text{ (properties of square)} \\ m(\angle DAX) = m(\angle BCY) \text{ (alternate angles)} \\ AX = CY \end{cases}$

$\therefore \Delta DAX \equiv \Delta BCY$ (Q.E.D. 1)

$\because BM = MD$, $XM = YM$

$\therefore$ the figure DXBY is a parallelogram. (Q.E.D. 2)

10



$\because$ ABCD is a rectangle

$\therefore \overline{AD} \parallel \overline{BC}$

$\therefore$ m (∠ XAM) = m (∠ YCM) (alternate angles)

ΔΔ AMX , CMY in them :

$\begin{cases} \text{m (∠ XAM) = m (∠ YCM)} \\ \text{m (∠ AMX) = m (∠ CMY)} \quad \text{(V.O.A.)} \\ \text{AM = CM} \end{cases}$

$\therefore$ Δ AMX ≡ Δ CMY then we deduce that XM = MY

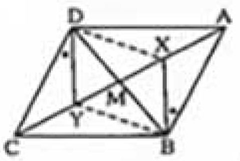
$\because$ Y ∈ $\overline{XM}$

$\therefore$ Y is the image of X by reflection in the point M (Q.E.D. 1)

$\therefore$ AM = CM , MX = MY

$\therefore$ the figure AXCY is a parallelogram. (Q.E.D. 2)

11



Δ Δ ABX , CDY in them

$\begin{cases} \text{AB = CD} \\ \text{m (∠ BAX) = m (∠ DCY)} \quad \text{(alternate angles)} \\ \text{m (∠ ABX) = m (∠ CDY)} \end{cases}$

$\therefore$ Δ ABX ≡ Δ CDY then we deduce that AX = CY

$\because$ AM = CM $\therefore$ AM – AX = CM – CY

$\therefore$ XM = YM

$\therefore$ Y is the image of X by reflection in M

$\because$ AM = CM

$\therefore$ C is the image of A by reflection in M

$\because$ BM = DM

$\therefore$ D is the image of B by reflection in M

$\therefore$ Δ ABX is the image of Δ CDY by reflection in M (Q.E.D. 1)

$\because$ XM = YM , BM = DM

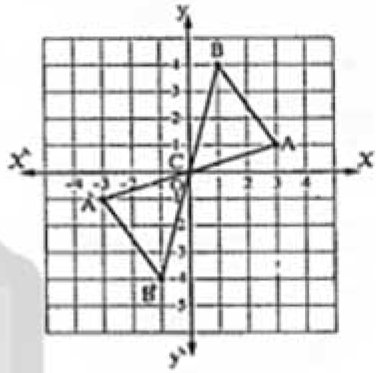
$\therefore$ the figure XBYD is a parallelogram. (Q.E.D. 2)

Second : Problems on reflection in the Cartesian plane :

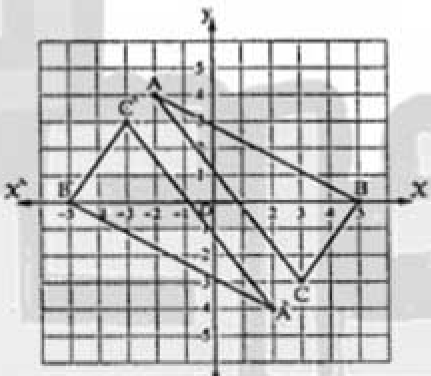
1

1 (c) 2 (c) 3 (c) 4 (b)

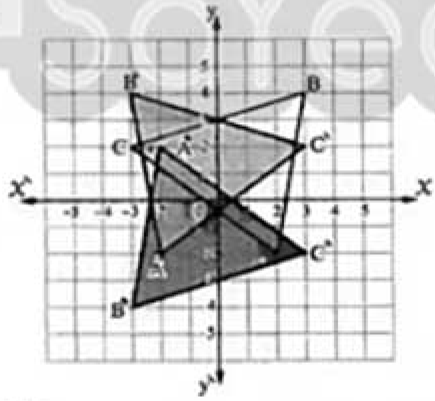
2



3

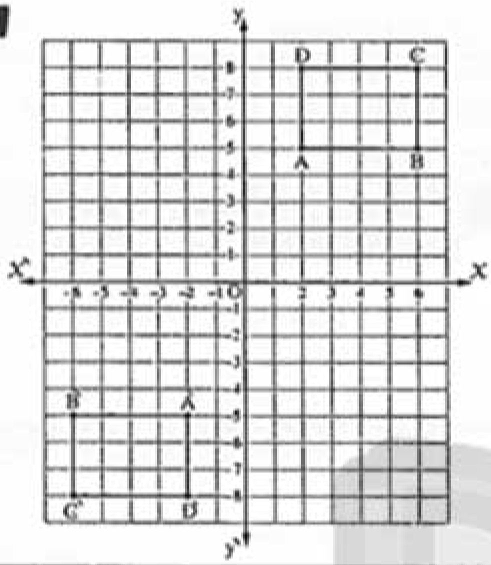


4



- Δ A`B`C` is the image of Δ ABC by reflection in the origin point.
- We deduce that the reflection in the origin point is equivalent to reflection in y-axis followed by reflection in X-axis.

5



6

$\because \overline{CD}$ is the image of $\overline{BA}$ by reflection in M

$\therefore$ BA = CD $\qquad \therefore 2x + 5 = x + 9 \quad \therefore x = 4$

$\therefore$ the length of $\overline{CD} = 4 + 9 = 13$ cm. (First req.)

$\because \overline{CD}$ is the image of $\overline{BA}$ by reflection in M

$\therefore$ BM = MC , AM = MD

$\therefore$ ACDB is a parallelogram.

$\therefore$ m ($\angle$ BAD) = m ($\angle$ CDA) (alternate angles)

$\therefore 2y = 60°$ $\qquad \therefore y = 30°$ (Second req.)

Answers of Exercise 11

First : Problems on translation in the plane :

1

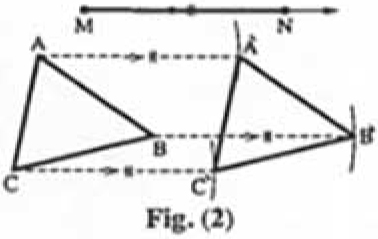
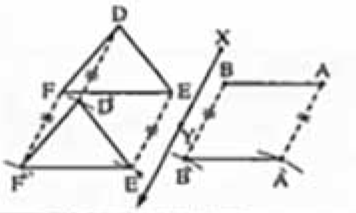


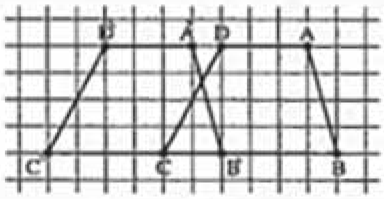
Fig. (1)

Fig. (2)

2

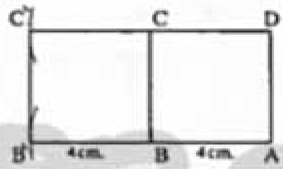


3



4

5



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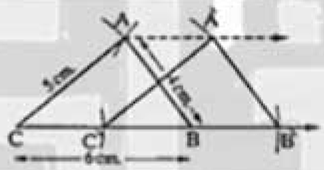
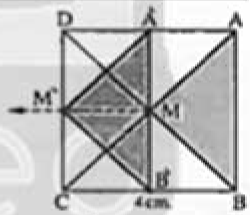
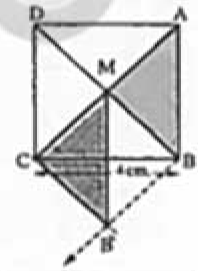


7

1. Δ M'A'B' is the image of Δ MAB by the translation of distance 2 cm. in the direction of $\overrightarrow{AD}$



2. Δ MCB' is the image of Δ AMB by the translation of distance AM in the direction of $\overrightarrow{AM}$



8 1. Δ DBE $\qquad$ 2. BE , $\overline{BE}$

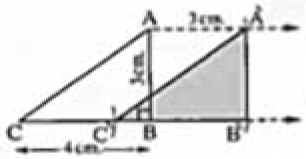
9

1. M $\qquad$ 2. $\overline{BM}$
3. Δ ABM $\qquad$ 4. AM in the direction $\overrightarrow{AM}$

10

1. $\overrightarrow{MN}$ $\qquad$ 2. The square FBLG $\qquad$ 3. 4 cm. , $\overline{CY}$

11



$\Delta A'B'C'$ is the image of ΔABC by the translation of distance 3 cm. in the direction of $\overrightarrow{CB}$

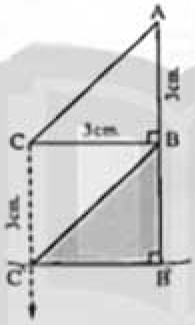
$\because A'$ is the image of A , C' is the image of C

$\therefore \overline{A'C'}$ is the image of $\overline{AC}$ by the translation of distance 3 cm. in the direction of $\overrightarrow{CB}$

$\therefore \overline{A'C'} \parallel \overline{AC}$, $A'C' = AC$

$\therefore AA'C'C$ is a parallelogram. (Q.E.D.)

12



$\Delta BB'C'$ is the image of ΔABC by the translation of distance 3 cm. in the direction of $\overrightarrow{AB}$

$\therefore B'$ is the image of B , C' is the image of C by the translation of distance 3 cm. in the direction of $\overrightarrow{AB}$

$\therefore \overline{B'C'} \parallel \overline{BC}$, $B'C' = BC$

$\therefore BB'C'C$ is a parallelogram.

$\because m(ABC) = 90°$ $\therefore m(\angle BB'C') = 90°$

"the translation reserves the measures of angles"

$\therefore BB'C'C$ is a rectangle. $\because AB = BC = 3$ cm.

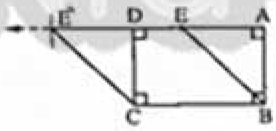
$\therefore$ the length of the image of $\overline{AB}$ = the length of the image of $\overline{BC} = 3$ cm.

"Translation reserves the lengths of line segments"

$\therefore BB' = B'C'$

$\therefore BB'C'C$ is a square. (Q.E.D.)

13



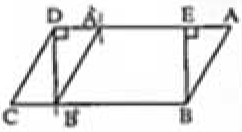
$\Delta DCE'$ is the image of ΔABE by the translation of distance AD in the direction of $\overrightarrow{AD}$

$\therefore \overline{CE'}$ is the image of $\overline{BE}$ by translation of distance AD in the direction of $\overrightarrow{AD}$

$\therefore \overline{CE'} \parallel \overline{BE}$, $CE' = BE$

$\therefore$ The figure $BCE'E$ is a parallelogram. (Q.E.D.)

14



$\Delta A'B'D$ is the image of ΔABE by the translation of distance ED in the direction of $\overrightarrow{AD}$

$\therefore \overline{B'D}$ is the image of $\overline{BE}$ by translation of distance ED in the direction of $\overrightarrow{AD}$

$\therefore \overline{B'D} \parallel \overline{BE}$, $B'D = BE$

$\therefore EBB'D$ is a parallelogram.

$\because m(\angle A'DB') = m(\angle AEB) = 90°$

"The translation reserves the measures of angles"

$\therefore$ The figure $EBB'D$ is a rectangle. (Q.E.D.)

Second : Problems on translation in the Cartesian plane :

1

1 (4 , 6) 2 (6 , 0) 3 (− 1 , − 1)

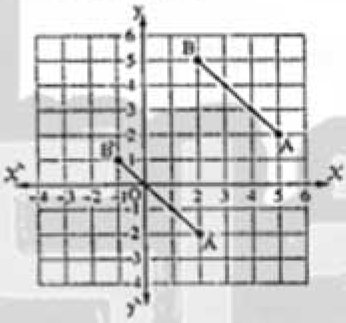
4 (− 4 , − 5) 5 (3 , 1)

2 1 (b) 2 (a) 3 (b) 4 (d) 5 (c) 6 (a)

3

1 $A(5 , 2) \longrightarrow A'(2 , -2)$

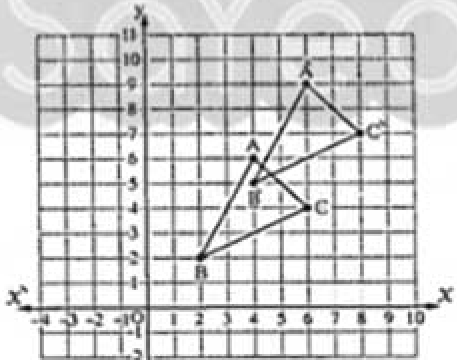
$B(2 , 5) \longrightarrow B'(-1 , 1)$



2 $A(4 , 6) \longrightarrow A'(6 , 9)$

$B(2 , 2) \longrightarrow B'(4 , 5)$

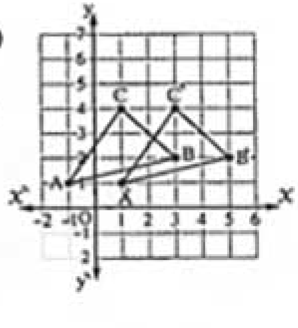
$C(6 , 4) \longrightarrow C'(8 , 7)$



3 $A(-1 , 1) \longrightarrow A'(1 , 1)$

$B(3 , 2) \longrightarrow B'(5 , 2)$

$C(1 , 4) \longrightarrow C'(3 , 4)$

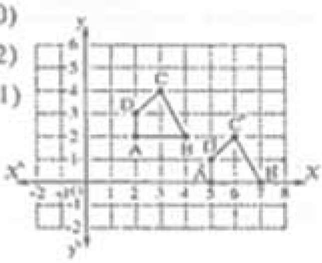


4 $A(2, 2) \longrightarrow \acute{A}(5, 0)$

$B(4, 2) \longrightarrow \acute{B}(7, 0)$

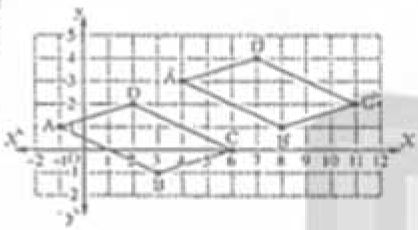
$C(3, 4) \longrightarrow \acute{C}(6, 2)$

$D(2, 3) \longrightarrow \acute{D}(5, 1)$



4

1



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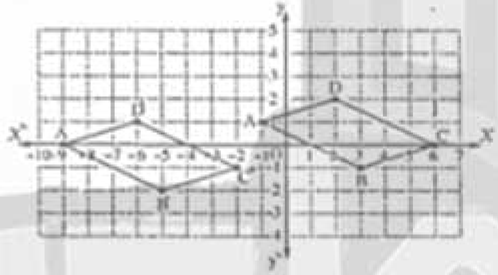

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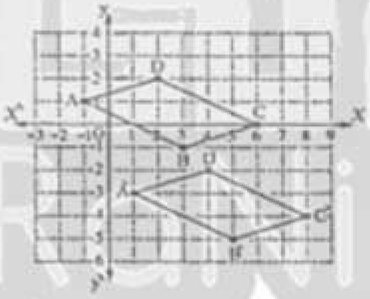

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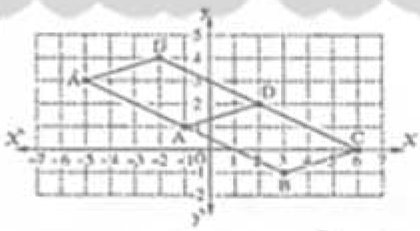


5

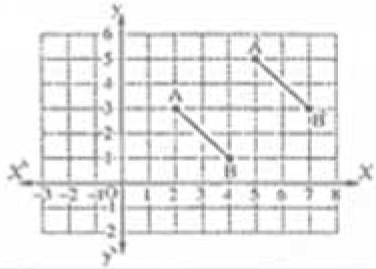
$\because$ Translation $(x, y) \longrightarrow (x+2, y-3)$ followed by translation $(x, y) \longrightarrow (x-3, y+1)$ is equivalent to $(x, y) \longrightarrow (x+2-3, y-3+1)$ and it is equivalent to $(x-1, y-2)$

1 The image of $A(4, -2)$ is $\acute{A}(3, -4)$

2 The image of $B(-1, 3)$ is $\acute{B}(-2, 1)$

3 The image of $C(0, 2)$ is $\acute{C}(-1, 0)$

6



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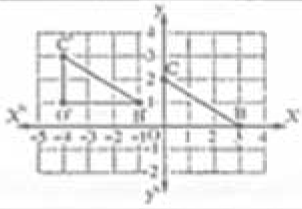

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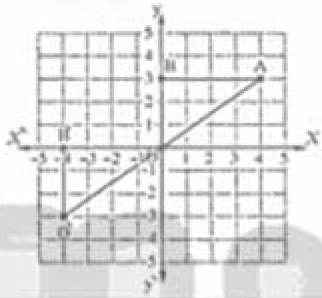


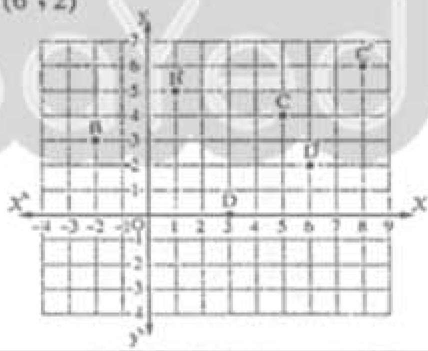
9

$(x, y) \longrightarrow (x+3, y+2)$

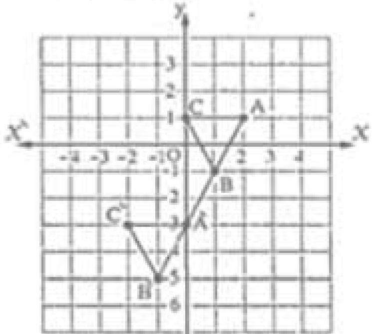
1 The image of $B(-2, 3)$ by translation LM is $\acute{B}(1, 5)$

2 The image of $C(5, 4)$ by translation LM is $\acute{C}(8, 6)$

3 The image of $D(3, 0)$ by translation LM is $\acute{D}(6, 2)$

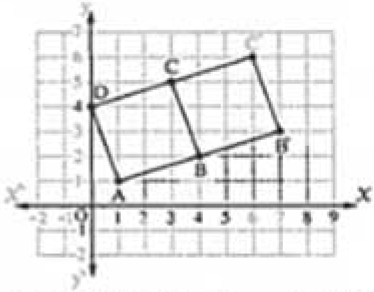


10



11

1



2 The mapping rule is $(x , y) \longrightarrow (x + 3 , y + 1)$

12 $(0 , 0)$

13

$\because$ $\acute{A}(2 , 2)$ is the image of the point $A(1 , 1)$

$\therefore$ The mapping rule of this translation is $(x , y) \longrightarrow (x + 1 , y + 1)$

$\therefore$ The image of $O(0 , 0)$ is $\acute{O}(1 , 1)$

, The image of $B(-1 , 3)$ is $\acute{B}(0 , 4)$

, The image of $C(-3 , 5)$ is $\acute{C}(-2 , 6)$

14 $(x , y) \longrightarrow (x + 4 , y - 3)$

15

$(x , y) \longrightarrow (x + 2 , y - 1)$

1 The image of $C(1 , -1)$ is $\acute{C}(3 , -2)$

2 Let $D(x , y)$ $\quad \therefore x + 2 = 2$

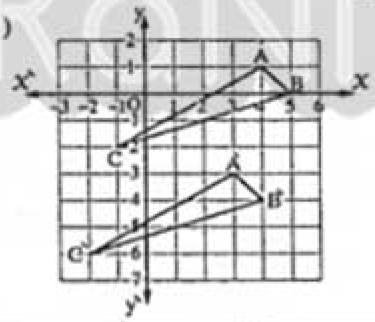
$\therefore x = 0 , y - 1 = 1 \quad \therefore y = 2 \quad \therefore D(0 , 2)$

16

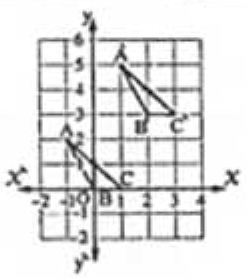
Let $A(x , y)$ $\quad \therefore x - 1 = 3$

$\therefore x = 4 \quad , y - 4 = -3 \quad \therefore y = 1$

$\therefore A(4 , 1)$



17



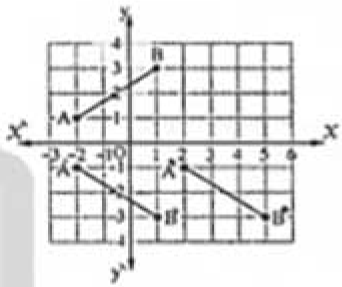
18

$A(-2 , 1) \xrightarrow[\text{in } x\text{-axis}]{\text{by reflection}} \acute{A}(-2 , -1)$

by translation $\acute{\acute{A}}(2 , -1)$

$B(1 , 3) \xrightarrow[\text{in } x\text{-axis}]{\text{by reflection}} \acute{B}(1 , -3)$

by translation $\acute{\acute{B}}(5 , -3)$



19

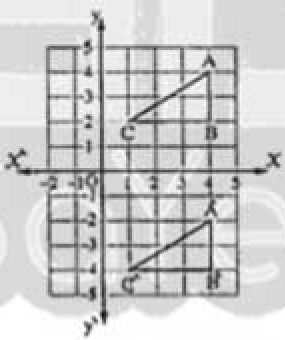
Fig. (1) Translation $(x , y) \longrightarrow (x - 5 , y - 3)$

Fig. (2) Reflection and the reflection axis is x-axis

Fig. (3) Reflection and the reflection axis is y-axis

Fig. (4) Translation $(x , y) \longrightarrow (x - 5 , y + 3)$

20



21

$\because$ $A(2 , 1)$ is the image of B by reflection in x-axis followed by reflection in y-axis

$\therefore B(-2 , -1)$

$\therefore$ The mapping rule of translation that makes the point $A(2 , 1)$ the image of the point $B(-2 , -1)$ is $(x , y) \longrightarrow (x + 4 , y + 2)$

Answers of Exercise 12

First : Problems on rotation in the plane :

1

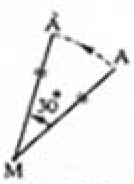


Fig. (1)

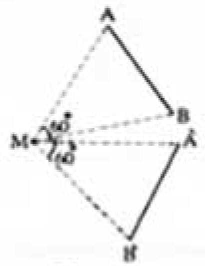


Fig. (2)

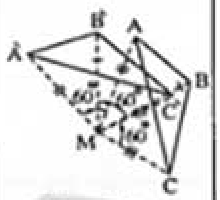
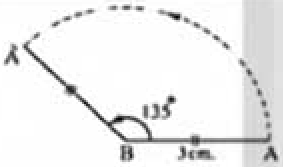


Fig. (3)

2



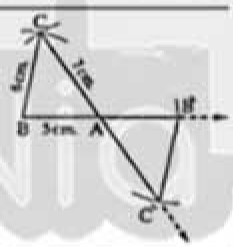
3

$\Delta A\acute{B}B$ is the image of ΔABC by the rotation R (A , 60°)

4

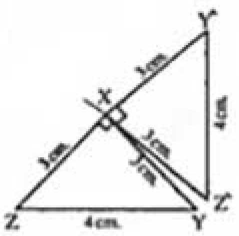
1 $\Delta A\acute{B}\acute{C}$ is the image of ΔABC by rotation R (A , 180°)

2 The image of ΔABC by rotation R (A , 360°) is itself.

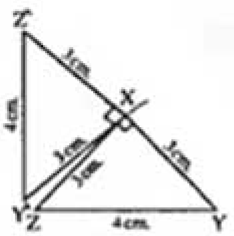


5

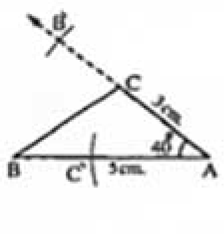
1



2

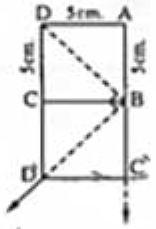


6

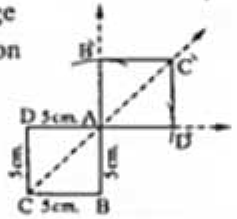


7

1 The square $CB\acute{C}\acute{D}$ is the image of the square ABCD by rotation R (B , 90°)

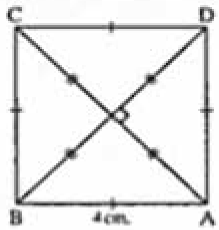


2 The square $A\acute{B}\acute{C}\acute{D}$ is the image of the square ABCD by rotation R (A , 180°)



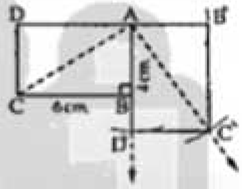
8

According to the shown figure , the square DABC is the image of the square ABCD by rotation about its centre with an angle of measure 90°

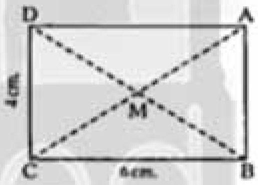


9

1 The rectangle $A\acute{B}\acute{C}\acute{D}$ is the image of the rectangle ABCD by the rotation R (A , 90°)



2 The rectangle CDAB is the image of the rectangle ABCD by the rotation R (M , 180°)



10

1 (d)	2 (b)	3 (c)
4 (c)	5 (c)	6 (a)

11

1 D , A , $\overline{DA}$, $\overline{DA}$

2 $\overline{BC}$, $\overline{BC}$, $\overline{BC}$

3 C , D , $\overline{CD}$

4 $\overline{CD}$

12

1 The point C	2 $\overline{DC}$	3 $\overline{EF}$
4 Δ MDE	5 M , 120°	6 Δ FMA

13 1 (b) 2 (d) 3 (c) 4 (a)

14

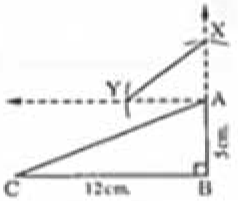
1 Δ DLO

2 Δ DLO

15

From the figure $XY \simeq 6.4$ cm.

(by measure)



16

$\because \overline{XZ} \parallel \overline{Y'Z'}$, $\overline{XY'}$ is a transversal to them.

$\therefore m(\angle ZXY') = m(\angle Y') = 30°$ (alternate angles)

$\therefore m(\angle YXY') = m(\angle YXZ) + m(\angle ZXY')$

$= 90° + 30° = 120°$

$\therefore$ The measure of the angle of rotation $= 120°$

(First req.)

$\because Z'$ is the image of Z and Y' is the image of Y by the same rotation and X is the image of itself.

$\therefore \Delta Y'Z'X$ is the image of ΔYZX by rotation R (X , 120°)

$\therefore Z'X = ZX = 5$ cm. (Second req.)

Second : Problems on rotation in the Cartesian plane :

1

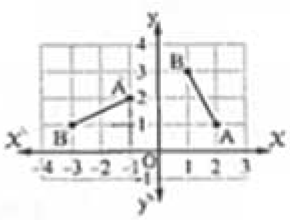
1. (3 , 2) , (− 2 , 3)
2. (0 , − 1) , (− 1 , 0)
3. − 90°
4. (4 , 1)
5. (− 5 , 2)
6. (7 , − 3)
7. (1 , 1)
8. (y , X)
9. ± 360°
10. zero

2

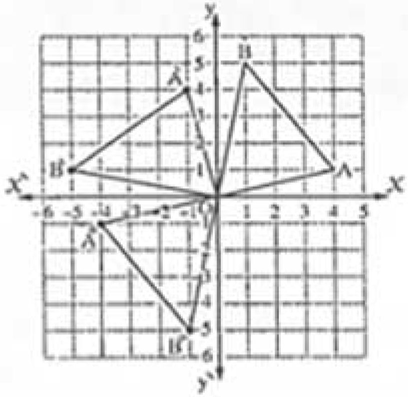
(2 , 1) , (− 2 , − 1) , (− 2 , 1) , (− 1 , 3) ,

(1 , 2) , (− 1 , − 2) , (− 2 , 1) , (2 , − 1)

3

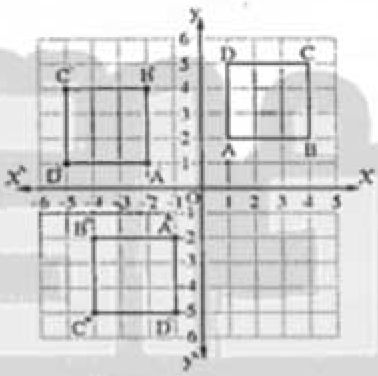


4



1. $\Delta A'B'O$ is the image of ΔABO by rotation about O with an angle of measure 90°
2. $\Delta A''B''O$ is the image of ΔABO by rotation about O with an angle of measure 180°

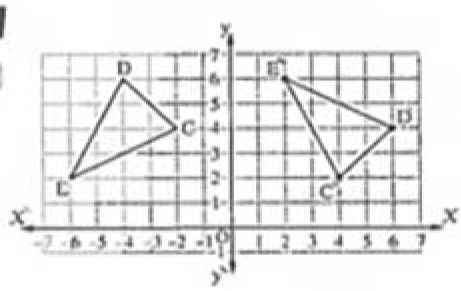
5



1. The square $A'B'C'D'$ is the image of ABCD by rotation about O with an angle of measure 90°
2. The square $A''B''C''D''$ is the image of the square ABCD by rotation about O with an angle of measure 180°

6

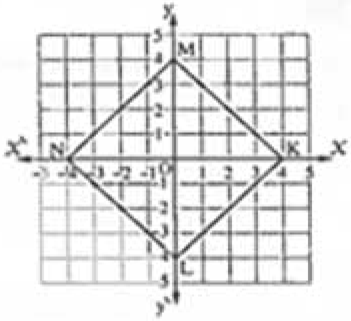
1.



$\Delta D'C'E'$ is the image of ΔDCE by rotation about O with an angle of measure − 90°

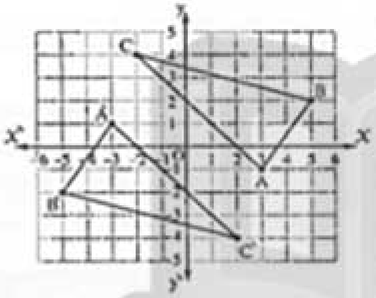
2

The square MNLK is the image of the square KMNL by rotation about O with an angle of measure 90°

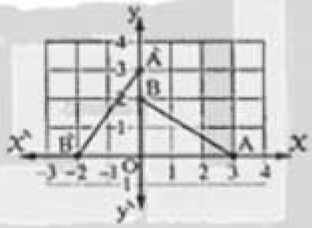


7

$\Delta \grave{A}\grave{B}\grave{C}$ is the image of ΔABC by rotation about the origin point with an angle of measure 180°

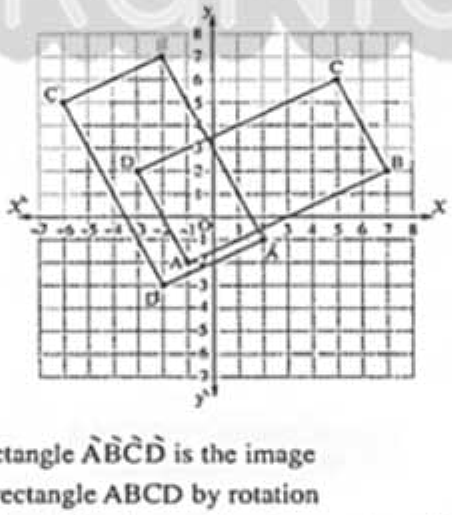


8



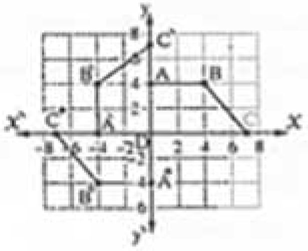
$\therefore \Delta \grave{A}O\grave{B}$ is the image of ΔAOB by rotation about O with an angle of measure 90°

9



The rectangle $\grave{A}\grave{B}\grave{C}\grave{D}$ is the image of the rectangle ABCD by rotation about the origin where : $(x , y) \longrightarrow (-y , x)$

10

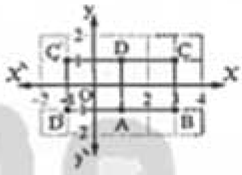


1. The figure $\grave{A}\grave{B}\grave{C}\grave{D}$ is the image of the figure ABCD by rotation about the origin with an angle of measure 90°
2. The figure $\grave{A}\grave{B}\grave{C}\grave{D}$ is the image of the figure ABCD by rotation about the origin with an angle of measure – 180°

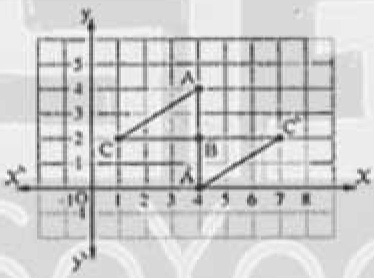
11 $C = (5 , 4)$, $\grave{C} = (-5 , -4)$

12

The square $AD\grave{C}\grave{D}$ is the image of the square ABCD by rotation about A with an angle of measure 90°

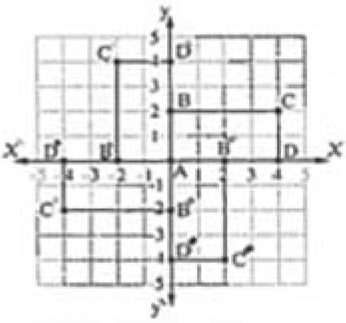


13



$\Delta \grave{A}B\grave{C}$ is the image of ΔABC by rotation about B with an angle of measure 180°

14 [a]



1. The rectangle $A\grave{B}\grave{C}\grave{D}$ is the image of the rectangle ABCD by rotation about the origin with an angle of measure 90°

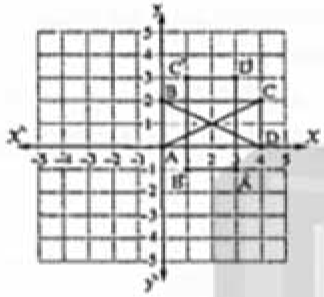
2 The rectangle $A''B''C''D''$ is the image of the rectangle ABCD by rotation about the origin with an angle of measure 180°

3 The rectangle $A'''B'''C'''D'''$ is the image of the rectangle ABCD by rotation about the origin with an angle of measure 270°

[b] (2 , 1)

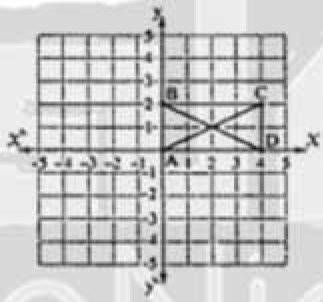
[c]

1



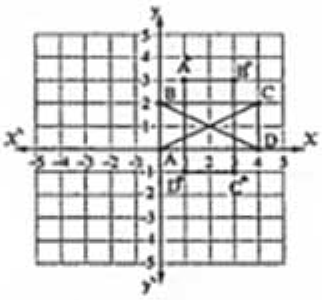
The rectangle $A'B'C'D'$ is the image of the rectangle ABCD by rotation about the centre of the rectangle with an angle of measure 90°

2



The rectangle CDAB is the image of the rectangle ABCD by rotation about the centre of the rectangle with an angle of measure 180°

3



The rectangle $A'B'C'D'$ is the image of the rectangle ABCD by rotation about the centre of the rectangle with an angle of measure 270°

15

Δ ABC is a right-angled triangle at A from Pythagoras $(AC)^2 = (BC)^2 - (AB)^2 = 25 - 9 = 16$

$\therefore$ AC = 4 cm.

, $\because$ $\Delta\, CA'B'$ is the image of Δ CAB by the given rotation

$\therefore \Delta\, CA'B' \equiv \Delta\, CAB$

i.e. $A'C = AC = 4$ cm. , then $AA' = 8$ cm.

$A'B' = AB = 3$ cm.

$\therefore$ the area of $\Delta\, AA'B'$

$= \frac{1}{2} AA' \times A'B' = \frac{1}{2} \times 8 \times 3 = 12 \text{ cm}^2$ (The req.)

Answers of exams on the third part of unit three

Model 1

1

1 (c)	2 (c)	3 (d)
4 (b)	5 (b)	6 (d)

2 1 2 , zero

2 lengths of line segments , measures of angles , parallelism. (There are other solutions)

3 (– 7 , – 3) 4 (5 , 2) 5 (2 , – 4)

3 [a] Draw by yourself

[b] 1 Δ AMB 2 Δ DMC

3 Δ MDE 4 Δ EMD

4 [a] Draw by yourself

[b] Draw by yourself

5 [a] Draw by yourself

[b] Draw by yourself

Model 2

1

1 (c)	2 (d)	3 (a)
4 (b)	5 (a)	6 (c)

2

1 Lengths of line segments , measures of angles , parallelism (There are other solutions)

2 1 3 360°

4 The magnitude of translation , the direction of translation

5 y-axis

3 [a] Draw by yourself

[b] 1 Δ CMD 2 $\overline{BC}$

4 [a] 1 The translation is : (0 , 2)

2 (3 , 3)

[b] Draw by yourself

5 [a] (0 , 4)

[b] Draw by yourself

Answers of accumulative basic skills

1

1 (b)	2 (c)	3 (d)	4 (d)
5 (b)	6 (c)	7 (d)	8 (a)
9 (c)	10 (b)	11 (d)	12 (b)

2

1 125	2 2	3 an acute	4 60°
5 115.5	6 50	7 6	8 360°
9 18	10 37.5 %	11 16	12 28

Guide Answers

of The Notebook

- Quizzes.
- Final Examinations.

Answers of the quizzes on Algebra and Statistics

Quiz 1

1

1 d — 2 c — 3 a

2

[a] $-\frac{2}{9}$ [b] $\frac{81}{16}$

Quiz 2

1

1 b — 2 c — 3 c

2

[a] 1 $\frac{27}{125}$ 2 x^4 3 $\frac{64}{729}$

[b] $8\frac{1}{2}$

Quiz 3

1

1 c — 2 c — 3 d

2

[a] 1 1 2 $\frac{7}{9}$ 3 $\frac{y^4}{x^6}$

[b] $\frac{8}{49}$

Quiz 4

1

1 b — 2 b — 3 c

2

[a] 1 5.832×10^7 2 1.31×10^{-3}

3 3×10^8

[b] 1 $\frac{1}{3}$ 2 1

Quiz 5

1

1 c — 2 b — 3 a

2

[a] 1 -3 2 22 3 3

[b] 49

Quiz 6

1

1 b — 2 b — 3 c

2

[a] 1 37 cm. 2 $-\frac{1}{9}$

[b] 8

Quiz 7

1

1 a — 2 d — 3 c

2

[a] 1 $\{3\}$ 2 5 , 10

[b] zero

Quiz 8

1

1 b — 2 c — 3 b

2

[a] 1 $\{x : x \in \mathbb{Q} , x > 2\}$

2 $\{\frac{2}{5}\}$

[b] 3

Quiz 9

1

1 c — 2 b — 3 d

2

[a] $\{\frac{5}{2}\}$

[b] $S = \{1 , 2 , 3 , 4 , 5 , 6 , 7 , 8\}$

1 $\frac{1}{2}$ 2 $\frac{1}{4}$

Answers of school book models on Algebra and Statistics

Model 1

1

1 -2 2 2 3 $\frac{7}{12}$ 4 3.5×10^{-3} 5 1

2

1 (b) 2 (a) 3 (a) 4 (a) 5 (a) 6 (a)

3 [a] $1 \times \frac{4}{25} \times \sqrt{\frac{25}{4}} = 1 \times \frac{4}{25} \times \frac{5}{2} = \frac{2}{5}$

[b] $3\,ab + 8\,a \div (4\,b)$

$= 3 \times 4 \times (-2) + 8 \times 4 \div (4 \times -2)$

$= -24 + 32 \div (-8)$

$= -24 - 4 = -28$

4

[a] $\because 3x + 1 = 25$ $\therefore 3x = 24$

$\therefore x = 8$ $\therefore$ The S.S. $= \{8\}$

[b] $\frac{8 \times 8^{-3}}{8^{-4}} = 8^{1-3+4} = 8^2 = 64$

5

[a] 1 The probability of change it before travelled 50 thousand km. $= \frac{80}{800} = \frac{1}{10}$

2 The probability of change it after travelled more than 100 thousand km. $= \frac{600}{800} = \frac{3}{4}$

[b] $\because 2x + 5 < 16$ $\therefore 2x < 11$

$\therefore x < \frac{11}{2}$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x < \frac{11}{2}\}$

Model 2

1

1 1 2 $\frac{4}{7}$ 3 zero

4 $13 , 21$ 5 510 students

2

1 (a) 2 (d) 3 (d) 4 (b) 5 (c) 6 (d)

3 [a] $\because 5x - 2x = 30$ $\therefore 3x = 30$ $\therefore x = 10$

$\therefore$ The two numbers are : $20 , 50$

[b] $\frac{5^{-4} \times 5^{7}}{5^{3}} = 5^{-4+7-3} = 5^0 = 1$

4

[a] 1 $\because 3x + 7 = 13$ $\therefore 3x = 6$

$\therefore x = 2$ $\therefore$ The S.S. $= \{2\}$

2 $\because 2x + 15 < 19$ $\therefore 2x < 4$

$\therefore x < 2$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x < 2\}$

[b] $\frac{1}{9} + \frac{8}{9} - 1 = 1 - 1 =$ zero

5

[a] 1 The probability of getting a prime even number $= \frac{1}{6}$

2 The probability of getting an odd number less than $4 = \frac{2}{6} = \frac{1}{3}$

[b] $\left(\frac{y}{x^2}\right)^{-2} = \left[\frac{\frac{-3}{4}}{\left(\frac{-1}{2}\right)^2}\right]^{-2} = \left(\frac{\frac{-3}{4}}{\frac{1}{4}}\right)^{-2} = (-3)^{-2}$

$= \frac{1}{(-3)^2} = \frac{1}{9}$

Model examination for the merge students

1 1 a 2 b 3 a

4 b 5 b

2 1 4 2 $\frac{1}{2}$ 3 zero

4 $\frac{2}{5}$ 5 42

3 1 $12 \times 2^2 \div 24 + 3^2 = 12 \times 4 \div 24 + 9$

$= 48 \div 24 + 9 = 2 + 9 = 11$

2 $\frac{8 + 20 - 4}{8 - 4} = \frac{28 - 4}{4} = \frac{24}{4} = 6$

4 1 ✔ 2 ✘ 3 ✔

4 ✘ 5 ✘

5 1 $\{8 , 6 , 4 , 2\}$ 2 $\frac{1}{2}$

3 $\{8 , 7\}$ 4 1 5 $\frac{1}{8}$

Answers of schools examinations on Algebra and Statistics

1 Cairo

1 1 b 2 d 3 c 4 a 5 c 6 b

2 1 $\left|\frac{5xy}{6}\right|$ 2 $\{-13\}$ 3 $-\frac{4}{25}$ 4 1 5 -3

3

[a] $x^2 y^2 z^2 = \left(\frac{-3}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{-4}{3}\right)^2$
$= \frac{9}{4} \times \frac{1}{4} \times \frac{16}{9} = 1$

[b] $\because 5x + 8 = 13 - 2x$ $\therefore 5x + 2x = 13 - 8$
$\therefore 7x = 5$ $\therefore x = \frac{5}{7}$
$\therefore$ The S.S. $= \left\{\frac{5}{7}\right\}$

4

[a] $\because 3x - 1 \leq 5$
$\therefore 3x \leq 6$ $\therefore x \leq 2$
$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x \leq 2\}$

[b] $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0 = \frac{1}{9} + \frac{8}{9} - 1 = 0$

5

[a] 1 The probability that the ball is red $= \frac{5}{15} = \frac{1}{3}$
2 The probability that the ball is black $= \frac{0}{15} = 0$
3 The probability that the ball is white or blue
$= \frac{4+6}{15} = \frac{10}{15} = \frac{2}{3}$

[b] Let the two numbers be : x and $2x$
$\therefore x + 2x = 24$ $\therefore 3x = 24$ $\therefore x = 8$
$\therefore$ The two numbers are : 8 , 16

2 Cairo

1 1 c 2 b 3 b 4 a 5 d 6 c

2 1 $\frac{1}{7}$ 2 2.5×10^{-4} 3 $\frac{4}{3}$ 4 1 5 $\frac{11}{36}$

3

[a] $\left(\frac{7^{-2} \times 7^5}{7^3}\right)^2 = \left(7^{-2+5-3}\right)^2 = (7^0)^2 = (1)^2 = 1$

[b] $\left(\frac{-3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{2}{5}\right)^0 = \frac{9}{4} \times \frac{8}{3} \times 1 = 6$

[c] $12 \times 2^2 \div 24 + 3^2 = 12 \times 4 \div 24 + 9$
$= 48 \div 24 + 9 = 2 + 9 = 11$

4

[a] 1 $\sqrt{100 - (-8)^2} = \sqrt{100 - 64} = \sqrt{36} = 6$
2 $(x^2 y^2)^{-3} = \left[\left(\frac{1}{2}\right)^2 \times \left(\frac{2}{3}\right)^2\right]^{-3}$
$= \left[\frac{1}{4} \times \frac{4}{9}\right]^{-3} = \left(\frac{1}{9}\right)^{-3} = (9)^3 = 729$

[b] $\because 3x + 6 > 3$ $\therefore 3x > -3$ $\therefore x > -1$
$\therefore$ The S.S. $= \{x : x \in \mathbb{Q} , x > -1\}$

5

[a] $\because 4x - 5 = 27$ $\therefore 4x = 32$
$\therefore x = 8$
$\therefore$ The S.S. $= \{8\}$

[b] 1 The probability of appearance of an even number $= \frac{3}{6} = \frac{1}{2}$
2 The probability of appearance of a number greater than $4 = \frac{2}{6} = \frac{1}{3}$

3 Cairo

1 1 b 2 d 3 c 4 a 5 c 6 a

2 1 -6 2 $\frac{3}{7}$ 3 $\frac{1}{6}$ 4 10 5 32

3

[a] $\left(\frac{5^5 \times 5^{-2}}{5^4}\right)^{-2} = \left(5^{5-2-4}\right)^{-2} = (5^{-1})^{-2} = 5^2 = 25$

[b] $\because -2 < 4x + 1 < 6$ $\therefore -3 < 4x < 5$
$\therefore \frac{-3}{4} < x < \frac{5}{4}$
$\therefore$ The S.S. $= \left\{x : x \in \mathbb{Q} , \frac{-3}{4} < x < \frac{5}{4}\right\}$

4

[a] $\left(\frac{-2}{7}\right)^{-2} \times \sqrt{\frac{16}{49}} \times \left(\frac{-1}{7}\right)^{0} = \frac{49}{4} \times \frac{4}{7} \times 1 = 7$

[b] Let the length of the rectangle be : x cm.

∴ The width $= (x - 5)$ cm.

∴ $2(x + x - 5) = 42$

∴ $2x - 5 = 21$ ∴ $2x = 26$ ∴ $x = 13$

∴ The length = 13 cm.

, the width = 8 cm.

∴ The area of rectangle $= 13 \times 8 = 104$ cm^2.

5

[a] $\frac{(4x^3y^2)^2}{(2xy^2)^3} = \frac{16x^6y^4}{8x^3y^6} = \frac{2x^3}{y^2}$

At $x = \frac{1}{2}$, $y = 3$

The result $= 2 \times \left(\frac{1}{2}\right)^3 \div 3^2 = 2 \times \frac{1}{8} \div 9$

$= \frac{1}{4} \times \frac{1}{9} = \frac{1}{36}$

[b] ∵ The probability of its loss $= 1 - (0.2 + 0.7) = 0.1$

∴ The number of loss matches $= 0.1 \times 30$

= 3 matches

4 Giza

1 1 d 2 b 3 a 4 b 5 c 6 c

2 1 6 2 $\frac{4}{9}$ 3 5 4 16 5 1

3

[a] $\left(\frac{-5}{7}\right)^0 \times \left(\frac{-3}{2}\right)^2 \times \sqrt{\frac{16}{9}} = 1 \times \frac{9}{4} \times \frac{4}{3} = 3$

[b] $(4.4 \times 10^3) \times (3 \times 10^5) = (4.4 \times 3) \times (10^3 \times 10^5)$

$= 13.2 \times 10^8 = 1.32 \times 10^9$

4

[a] ∵ $3x + 1 = 25$ ∴ $3x = 24$

∴ $x = 8$ ∴ The S.S. $= \{8\}$

[b] $\frac{3^{-4} \times 3^7}{3^3} = 3^{-4+7-3} = 3^0 = 1$

5

[a] ∵ $2x + 5 < 9$ ∴ $2x < 4$

∴ $x < 2$

∴ The S.S. $= \{x : x \in \mathbb{Q} , x < 2\}$

[b] $S = \{1 , 2 , 3 , 4 , 5 , 6\}$

1 The probability of getting a number greater than $6 = \frac{0}{6} = 0$

2 The probability of getting a number satisfies the inequality $1 < x < 6$ is $\frac{4}{6} = \frac{2}{3}$

3 The probability of getting a number divisible by $3 = \frac{2}{6} = \frac{1}{3}$

5 Giza

1 1 a 2 c 3 b 4 b 5 d 6 c

2 1 zero 2 $\frac{-1}{9}$ 3 5 4 $\frac{25}{4}$ 5 $\frac{5}{6}$

3

[a] 1 $\frac{9^{-2} \times 9^5}{9^3} = 9^{-2+5-3} = 9^0 = 1$

2 $\left(\frac{-3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0 = \frac{9}{4} \times \frac{8}{3} \times 1 = 6$

[b] ∵ $4x + 1 = 21$ ∴ $4x = 20$

∴ $x = 5$ ∴ The S.S. $= \{5\}$

4

[a] $(x\,y\,z)^2 = \left(\frac{1}{2} \times \frac{2}{3} \times \frac{-3}{2}\right)^2 = \left(\frac{-1}{2}\right)^2 = \frac{1}{4}$

[b] ∵ $3x - 1 \le 2x + 3$ ∴ $3x - 2x \le 3 + 1$

∴ $x \le 4$

∴ The S.S. $= \{x : x \in \mathbb{Q} , x \le 4\}$

5

[a] Let the numbers be : x , $x + 1$, $x + 2$

∴ $x + x + 1 + x + 2 = 24$

∴ $3x + 3 = 24$ ∴ $3x = 21$

∴ $x = 7$

∴ The numbers are : 7 , 8 , 9

[b] 1 The probability of appearance of an even number $= \frac{3}{6} = \frac{1}{2}$

2 The probability of appearance of a number greater than $5 = \frac{1}{6}$

6 Alexandria

1 (1) 1 (2) $\frac{5}{7}$ (3) 5 (4) 0.2 (5) X

2 (1) a (2) c (3) a (4) d (5) a (6) b

3

[a] $\frac{X^3 \times X^{-2}}{X^{-5} \times X} = X^{3-2+5-1} = X^5$ at $X = -2$

$\therefore X^5 = (-2)^5 = -32$

[b] (1) $\because 3X - 5 > 1$ $\therefore 3X > 6$

$\therefore X > 2$

$\therefore$ The S.S. $= \{X : X \in \mathbb{Q}, X > 2\}$

(2) $\because 3X + 6 = 30 - 5X$

$\therefore 3X + 5X = 30 - 6$

$\therefore 8X = 24$ $\therefore X = 3$

$\therefore$ The S.S. $= \{3\}$

4

[a] (1) The probability of getting an even number $= \frac{3}{6} = \frac{1}{2}$

(2) The probability of getting a number is a factor of $6 = \frac{4}{6} = \frac{2}{3}$

[b] $\sqrt{6\frac{1}{4}} \times \left(\frac{2}{7}\right)^{zero} \times \left(\frac{-2}{5}\right)^2 = \sqrt{\frac{25}{4}} \times 1 \times \frac{4}{25}$

$= \frac{5}{2} \times 1 \times \frac{4}{25} = \frac{2}{5}$

5

[a] Let the width be X cm. and the length be $(X+3)$ cm.

$\therefore 2(X + 3 + X) = 26$

$\therefore 2X + 3 = 13$ $\therefore 2X = 10$

$\therefore X = 5$ $\therefore$ The length = 8 cm.

and the width = 5 cm.

$\therefore$ The area $= 8 \times 5 = 40$ cm^2.

[b] $X^3 y^2 = \left(\frac{1}{2}\right)^3 \times \left(\frac{4}{3}\right)^2 = \frac{1}{8} \times \frac{16}{9} = \frac{2}{9}$

7 Alexandria

1 (1) b (2) b (3) c (4) c (5) a (6) c

2 (1) 1 (2) -4 (3) $\frac{-4}{9}$ (4) 1 (5) 2^{11}

3

[a] (1) $\because 3X + 1 = 16$ $\therefore 3X = 15$

$\therefore X = 5$ $\therefore$ The S.S. $= \{5\}$

(2) $\because 7X - 1 < 13$ $\therefore 7X < 14$

$\therefore X < 2$

$\therefore$ The S.S. $= \{X : X \in \mathbb{Q}, X < 2\}$

[b] $5^2 + [3 \times 8 \div 2^2 - 2 \times 3] = 25 + [24 \div 4 - 6]$

$= 25 + [6 - 6]$

$= 25 + 0 = 25$

4

[a] $(X + y)^{-1} = \left(\frac{1}{3} + \frac{1}{6}\right)^{-1} = \left(\frac{6+3}{18}\right)^{-1}$

$= \left(\frac{9}{18}\right)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2$

[b] Let the number be : X

$\therefore X + 3X = 24$ $\therefore 4X = 24$

$\therefore X = 6$

$\therefore$ The number is 6

5

[a] $\frac{a^3 \times a^{-8}}{a^{-5}} = a^{3-8+5} = a^0 = 1$

[b] (1) The probability of getting the ball is blue $= \frac{6}{15} = \frac{2}{5}$

(2) The probability of getting the ball is white or red $\frac{4+5}{15} = \frac{9}{15} = \frac{3}{5}$

8 El-Kalyoubia

1 (1) c (2) c (3) b (4) c (5) b (6) b

2 (1) $\frac{-16}{81}$ (2) $\frac{1}{36}$ (3) 8 (4) -5 (5) $\frac{2}{3}$

3

[a] $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{a}{b}\right)^0 = \frac{1}{9} + \frac{8}{9} - 1 = 0$

[b] $\left(\frac{y}{X}\right)^{-2} = \left[\left(\frac{-3}{4}\right) \div \left(\frac{-1}{2}\right)\right]^{-2}$

$= \left[\frac{-3}{4} \times \frac{-2}{1}\right]^{-2} = \left[\frac{3}{2}\right]^{-2} = \left[\frac{2}{3}\right]^2 = \frac{4}{9}$

4

1 $\because 3x + 1 = 25$ $\therefore 3x = 24$ $\therefore x = 8$

$\therefore$ The S.S. $= \{8\}$

2 $\because 15 + 2x < 1$ $\therefore 2x < -14$ $\therefore x < -7$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q}, x < -7\}$

5

[a] $\left(2 \times \sqrt{36} - 2^4\right) \div 4 = (2 \times 6 - 16) \div 4$

$= (12 - 16) \div 4 = -4 \div 4 = -1$

[b] 1 The probability of getting a prime even number $= \frac{1}{6}$

2 The probability of getting an odd number less than $4 = \frac{2}{6} = \frac{1}{3}$

9 El-Sharkia

1 1 1 2 1 3 $a^2 |b|$ 4 2.3×10^{-4} 5 42

2 1 c 2 b 3 d 4 a 5 b 6 c

3

[a] $\because 2x + 1 = 9$ $\therefore 2x = 8$

$\therefore x = 4$

$\therefore$ The S.S. $= \{4\}$

[b] $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} + \left(\frac{3}{7}\right)^0 = \frac{1}{9} + \frac{8}{9} + 1 = 2$

4

[a] $\frac{3^6 \times 3^{-2}}{3^2} = 3^{6-2-2} = 3^2 = 9$

[b] $\frac{y}{x^2} = \left(\frac{3}{4}\right) \div \left(\frac{-1}{2}\right)^2 = \frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times \frac{4}{1} = 3$

5

[a] $\because 3x - 2 \le 7$ $\therefore 3x \le 9$ $\therefore x \le 3$

$\therefore$ The S.S. $= \{x : x \in \mathbb{Q}, x \le 3\}$

[b] 1 The probability of the ball is white $= \frac{3}{14}$

2 The probability of the ball is not red

$= \frac{5+3}{14} = \frac{8}{14} = \frac{4}{7}$

10 El-Monofia

1 1 c 2 d 3 c 4 d 5 c 6 d

2 1 13 2 $\varnothing$ 3 3 4 $\frac{2}{3}$ 5 zero

3

[a] 1 $\because 5x - 2 = 8$ $\therefore 5x = 10$ $\therefore x = 2$

$\therefore$ The S.S. $= \{2\}$

2 $\because 2x + 3 > 4$ $\therefore 2x > 1$ $\therefore x > \frac{1}{2}$

$\therefore$ The S.S. $= \left\{x : x \in \mathbb{Q}, x > \frac{1}{2}\right\}$

[b] $2\left[(5^2 + 1) - (4^2 - 1)\right] = 2\left[(25 + 1) - (16 - 1)\right]$

$= 2\left[26 - 15\right] = 2 \times 11 = 22$

4

[a] $\left(\frac{-2}{3}\right)^3 \times \sqrt{\frac{81}{64}} \times \left(\frac{1}{3}\right)^{zero} = \frac{-8}{27} \times \frac{9}{8} \times 1 = \frac{-1}{3}$

[b] 1 The probability of the drawn ball is yellow $= \frac{4}{12} = \frac{1}{3}$

2 The probability of the drawn ball is not green $= \frac{3+4}{12} = \frac{7}{12}$

5

[a] $\frac{7^5 \times 7^{-2}}{7^3} = 7^{5-2-3} = 7^0 = 1$

[b] $(4x^2 - y)^2 = \left[4\left(\frac{1}{2}\right)^2 - \left(\frac{1}{3}\right)\right]^2 = \left[4 \times \frac{1}{4} - \frac{1}{3}\right]^2$

$= \left[1 - \frac{1}{3}\right]^2 = \left[\frac{3-1}{3}\right]^2 = \left[\frac{2}{3}\right]^2 = \frac{4}{9}$

11 El-Dakahlia

1 1 b 2 c 3 b 4 d 5 c 6 d

2 1 3 2 $13|K|$ 3 144 4 Event 5 $\{5, 6\}$

3

[a] $(25ab + 5a) \div 5a = 5b + 1$ at $a = 2, b = -1$

$\therefore 5b + 1 = 5(-1) + 1 = -5 + 1 = -4$

[b] $\left(\frac{-49}{25}\right)^0 \times \left(\frac{-2}{7}\right)^2 \times \sqrt{12\frac{1}{4}} = 1 \times \frac{4}{49} \times \sqrt{\frac{49}{4}}$

$= 1 \times \frac{4}{49} \times \frac{7}{2} = \frac{2}{7}$

4

[a] ① $\because 2x+7<15 \quad \therefore 2x<8 \quad \therefore x<4$

$\therefore$ The S.S. $=\{x : x \in \mathbb{Q} , x<4\}$

② $\because 6x+6=6 \quad \therefore 6x=0 \quad \therefore x=0$

$\therefore$ The S.S. $=\{0\}$

[b] $\frac{(3)^{-6}\times(3)^{11}}{(3)^3}=3^{-6+11-3}=3^2=9$

5

[a] $\because 5x-2\geq 3 \quad \therefore 5x\geq 5 \quad \therefore x\geq 1$

$\therefore$ The S.S. $=\{1 , 2 , 3 , 4 , 5 , ...\}$

[b] ① The probability of getting a black ball $=\frac{0}{15}=0$

② The probability of getting a red ball $=\frac{5}{15}=\frac{1}{3}$

12 El-Ismailia

1 ① b ② c ③ a ④ d ⑤ b ⑥ a

2 ① 30 ② $15y^4$ ③ $\frac{5x^2}{y^2}$ ④ 11 ⑤ 4

3

[a] $\left(\frac{-1}{3}\right)^2+\sqrt{\frac{64}{81}}-\left(\frac{3}{7}\right)^0=\frac{1}{9}+\frac{8}{9}-1=0$

[b] $\frac{a^7\times a^5}{a^4\times a^6}=a^{7+5-4-6}=a^2$

at $a=-3 \quad \therefore a^2=(-3)^2=9$

4

[a] $\because 3x+4<25 \quad \therefore 3x<21 \quad \therefore x<7$

$\therefore$ The S.S. $=\{x : x\in\mathbb{Q} , x<7\}$

[b] $x^2y^2+z=\left(\frac{1}{2}\right)^2\times\left(\frac{-2}{3}\right)^2+\left(\frac{3}{4}\right)=\frac{1}{4}\times\frac{4}{9}\times\frac{4}{3}=\frac{4}{27}$

5

[a] ① The probability of getting an even number $=\frac{4}{8}=\frac{1}{2}$

② The probability of getting a prime number $=\frac{4}{8}=\frac{1}{2}$

③ The probability of getting a number more than 7 $=\frac{1}{8}$

[b] $\because 6x-8=22 \quad \therefore 6x=30 \quad \therefore x=5$

$\therefore$ The S.S. $=\{5\}$

13 Damietta

1 ① b ② a ③ b ④ c ⑤ c ⑥ b

2 ① 1 ② 5.3×10^{-5} ③ $\frac{5}{3}$ ④ $\frac{4}{9}$ ⑤ 13

3

[a] $\left(\frac{-5}{3}\right)^2\times\left(\frac{-4}{9}\right)^0\times\sqrt{3\frac{6}{25}}=\frac{25}{9}\times1\times\sqrt{\frac{81}{25}}$

$=\frac{25}{9}\times1\times\frac{9}{5}=5$

[b] ① $\because 3x+1>25 \quad \therefore 3x>24$

$\therefore x>8$

$\therefore$ The S.S. $=\{x : x\in\mathbb{Q} , x>8\}$

② $\because 5x+8=15-2x$

$\therefore 5x+2x=15-8$

$\therefore 7x=7 \quad \therefore x=1$

$\therefore$ The S.S. $=\{1\}$

4

[a] $\frac{x^7\times x^9}{x^6\times x^8}=x^{7+9-6-8}=x^2$

at $x=-3 \quad \therefore x^2=(-3)^2=9$

[b] Let the numbers be : $x , x+2 , x+4$

$\therefore x+x+2+x+4=60$

$\therefore 3x+6=60 \quad \therefore 3x=54$

$\therefore x=18$

$\therefore$ The numbers are : 18 , 20 , 22

5

[a] $\sqrt{x^2+y^2}=\sqrt{3^2+(-4)^2}=\sqrt{9+16}=\sqrt{25}=5$

[b] ① The probability of getting a blue ball $=\frac{6}{15}=\frac{2}{5}$

② The probability of getting a white or red ball $=\frac{4+5}{15}=\frac{9}{15}=\frac{3}{5}$

③ The probability of getting a green ball $=\frac{0}{15}=0$

14 El-Fayoum

1
1 d 2 a 3 b
4 b 5 c 6 b

2
1 25 2 5 3 $\frac{3}{2}$
4 $\frac{7}{16}$, $\frac{9}{32}$ 5 4.5×10^6

3
[a] 1 $\frac{10^{-3} \times 10^6}{10^2} = 10^{-3+6-2} = 10$

2 $\frac{\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{-5}}{\frac{1}{2}} = \left(\frac{1}{2}\right)^{2-5-1} = \left(\frac{1}{2}\right)^{-4} = (2)^4 = 16$

[b] 1 $\because 8 + 2x = 14$ $\therefore 2x = 6$
$\therefore x = 3$ $\therefore$ The S.S. $= \{3\}$

2 $\because 3x - 1 = -10$ $\therefore 3x = -9$
$\therefore x = -3$
$\therefore$ The S.S. $= \{-3\}$

4
[a] 1 $\because 2(x-3) = -x + 12$
$\therefore 2x - 6 = -x + 12$
$\therefore 2x + x = 12 + 6$ $\therefore 3x = 18$
$\therefore x = 6$ $\therefore$ The S.S. $= \{6\}$

2 $\because 5x - 1 = 29$ $\therefore 5x = 30$
$\therefore x = 6$ $\therefore$ The S.S. $= \{6\}$

[b] 1 $\frac{9-b}{a^3} = \frac{9-5}{2^3} = \frac{4}{8} = \frac{1}{2}$

2 $\frac{6^2}{a+1} = \frac{6^2}{2+1} = \frac{36}{3} = 12$

5
[a] 1 $\because 3x - 4 \geq -10$ $\therefore 3x \geq -6$ $\therefore x \geq -2$
$\therefore$ The S.S. $= \{-2, -1, 0, 1, 2, ...\}$

2 $\because x + 2 \geq 2$ $\therefore x \geq 2 - 2$ $\therefore x \geq 0$
$\therefore$ The S.S. $= \{0, 1, 2, 3, ...\}$

[b] 1 The probability of the drawn card carries an odd prime number $= \frac{5}{15} = \frac{1}{3}$

2 The probability of the drawn card carries a number less than or equal to $1 = \frac{1}{15}$

3 The probability of the drawn card carries a number more than $15 = \frac{0}{15} = 0$

4 The probability of the drawn card carries the number $15 = \frac{1}{15}$

15 Qena

1
1 b 2 a 3 b
4 b 5 c 6 b

2
1 $x + 5$ 2 $\frac{4}{9}$ 3 zero
4 2 5 39

3
[a] $\because \frac{3}{5}x + 4 < 28$ $\therefore \frac{3}{5}x < 24$
$\therefore 3x < 120$ $\therefore x < 40$
$\therefore$ The S.S. $= \{x : x \in \mathbb{Q}, x < 40\}$

[b] $a^3 b^3 = \left(\frac{-2}{3}\right)^3 \times \left(\frac{3}{4}\right)^3 = \frac{-8}{27} \times \frac{27}{64} = \frac{-1}{8}$

4
[a] $\left(\frac{4}{9}\right)^{-2} \times \left(\frac{4}{9}\right)^6 = \left(\frac{4}{9}\right)^{-2+6} = \left(\frac{4}{9}\right)^4 = \frac{256}{6561}$

[b] Let the numbers be : x , $x + 2$, $x + 4$
$\therefore x + x + 2 + x + 4 = 156$
$\therefore 3x + 6 = 156$ $\therefore 3x = 150$
$\therefore x = 50$
$\therefore$ The numbers are : 50 , 52 , 54

5
[a] $3^{x+y} = 3^x \times 3^y = 7 \times 5 = 35$

[b] 1 The probability of appearance of an even number $= \frac{3}{6} = \frac{1}{2}$

2 The probability of appearance of a number greater than $3 = \frac{3}{6} = \frac{1}{2}$

3 The probability of appearance of the number $5 = \frac{1}{6}$

Answers of the quizzes on Geometry and Measurement

Quiz 1

1

1 equal in measure 2 360° 3 parallel

2

[a] Prove by yourself

[b] $X = 37.5°$

Quiz 2

1

1 900° 2 360° 3 6

2

[a] Prove by yourself

[b] 1 120° 2 60°

Quiz 3

1

1 125° 2 trapezium 3 120°

2

[a] Prove by yourself

[b] $X = 125°$

Quiz 4

1

1 square 2 45° 3 concave

2

[a] m (∠ D) = 116°

[b] 12 sides , 1800°

Quiz 5

1

1 180° 2 120° 3 120°

2

[a] m (∠ ABC) = 55° , m (∠ ACB) = 40° , m (∠ BAC) = 85°

[b] 1 Prove by yourself 2 CE = 4 cm.

Quiz 6

1

1 bisects the third side

2 obtuse-angled triangle 3 half

2

[a] LM = 5 cm.

[b] m (∠ A) = 100°

Quiz 7

1

1 5 2 right

3 The sum of areas of the two squares drawn on the right sides

2

[a] LX = 15 cm.

[b] Prove by yourself

Quiz 8

1

1 (1 , – 2) 2 $(XY)^2 - (YZ)^2$ 3 80°

2

[a] 150°

[b] Draw by yourself

Quiz 9

1

1 (3 , 5) 2 acute 3 (– 2 , 3)

2

[a] Prove by yourself [b] Draw by yourself

Quiz 10

1

1 $(-2, 4)$ 2 CMB

3 axis of symmetry

2

[a] BD = 5 cm. , AC = 20 cm.

[b] Draw by yourself

Quiz 11

1

1 measures of angles , parallelism

2 direction , magnitude 3 $(-1, 7)$

2 Draw by yourself

Quiz 12

1

1 90° 2 $(2, -4)$ 3 $\overline{NL}$

2

[a] 1 Δ DBE 2 Δ EDF 3 Δ FDE

[b] Draw by yourself

Answers of school book models on Geometry and Measurement

Model 1

1 1 (c) 2 (a) 3 (d) 4 (b) 5 (b) 6 (b)

2

1 (2 , – 1) 2 80° 3 4

4 160° 5 180°

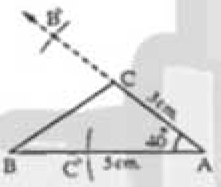
3

[a] $\because$ ∠ ACD is an exterior angle of △ ABC

$\therefore m(\angle ACD) = m(\angle A) + m(\angle B) = 25° + 25° = 50°$

(The req.)

[b]



4

[a] $\because M \in \overline{AC}$

$\therefore m(\angle BMC) = 180° - 70° = 110°$

$\therefore$ In △ BMC :

$m(\angle MCB) = 180° - (110° + 40°) = 30°$

$\therefore m(\angle MCB) = m(\angle MAD)$

and they are alternate angles

$\therefore \overline{AD} \parallel \overline{BC}$, $\because \overline{AB} \parallel \overline{DC}$

$\therefore$ ABCD is a parallelogram (Q.E.D.)

[b] The point (0 , 0)

5

[a] In △ ABD : $\because m(\angle ADB) = 90°$

$\therefore (BD)^2 = (AB)^2 - (AD)^2 = 676 - 576 = 100$

$\therefore BD = \sqrt{100} = 10$ cm.

In △ ADC : $\because m(\angle ADC) = 90°$

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = 900 - 576 = 324$

$\therefore CD = \sqrt{324} = 18$ cm.

$\therefore BC = 10 + 18 = 28$ cm. (First req.)

$\therefore$ The area of △ ABC $= \frac{1}{2}$ BC × AD

$= \frac{1}{2} \times 28 \times 24 = 336$ cm²

(Second req.)

[b] $\because$ ABCD is a squre

$\therefore \overline{AD} \parallel \overline{BC}$, $\because E \in \overline{BC}$

$\therefore \overline{AD} \parallel \overline{EC}$, $\because \overline{AC} \parallel \overline{DE}$

$\therefore$ ACED is a parallelogram (Q.E.D.)

Model 2

1

1 (a) 2 (c) 3 (b) 4 (c) 5 (c) 6 (d)

2

1 44 2 (5 , 5) 3 1728000

4 bisects the third side. 5 ZYC

3

[a] In △ XYZ : $\because m(\angle Y) = 90°$

$\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = 49 + 576 = 625$

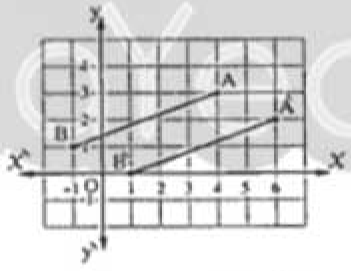
$\therefore XZ = \sqrt{625} = 25$ cm.

In △ LXZ : $\because m(\angle L) = 90°$

$\therefore (LZ)^2 = (XZ)^2 - (LX)^2 = 625 - 225 = 400$

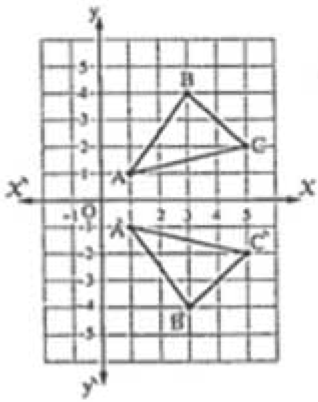
$\therefore LZ = \sqrt{400} = 20$ cm. (The req.)

[b]



4

[a]



[b] In ΔABC : $m(\angle ACB) = 180° - (90° + 30°) = 60°$

, $\because \overline{BD} \cap \overline{AO} = \{C\}$

$\therefore m(\angle ACB) = m(\angle OCD) = 60°$ (V.O.A)

$\therefore m(\angle E) = 360° - (60° + 120° + 90°) = 90°$

(The req.)

5

[a] $\because \overleftrightarrow{EO} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{EB}$ is a transversal

$\therefore m(\angle CBA) = m(\angle E) = 50°$ (alternate angles)

From ΔABC :

$m(\angle BAC) = 180° - (50° + 30°) = 100°$

$\because \angle ABD$ is an exterior angle of ΔABC

$\therefore m(\angle ABD) = 30° + 100° = 130°$ (The req.)

[b] $\because \overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, $\because AX = XB$

$\therefore DY = YC$

$\therefore Y$ is the midpoint of $\overline{CD}$

In ΔCDE : $\because \overline{ZY} \parallel \overline{DE}$

, Y is the midpoint of $\overline{CD}$

$\therefore Z$ is the midpoint of $\overline{CE}$

$\therefore CZ = ZE$ (Q.E.D.)

Model examination for the merge students

1
1 c 2 b 3 b
4 a 5 b

2
1 half 2 right 3 6
4 $(3 , -2)$ 5 $70°$

3
1 ✗ 2 ✗ 3 ✓
4 ✗ 5 ✓

4
1 $360°$ 2 $120°$ 3 $(4 , 0)$
4 $(-1 , -3)$ 5 $45°$

5
Fig (1) : $x = 8$
Fig (2) : $x = 90°$

Answers of schools examinations on Geometry and Measurement

1 Cairo

1

1 360° 2 equal in measure 3 90°
4 (6 , 1) 5 half

2

1 a 2 d 3 b 4 d 5 d 6 c

3

[a] $\because \overleftrightarrow{GH} \parallel \overline{AB}$, $\overrightarrow{AG}$ is a transversal to them

$\therefore m(\angle A) + m(\angle G) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle A) = 180° - 135° = 45°$

$\because \overleftrightarrow{DC} \parallel \overline{AB}$, $\overrightarrow{BD}$ is a transversal to them

$\therefore m(\angle B) + m(\angle D) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle B) = 180° - 145° = 35°$

$\therefore$ In Δ ABE :

$m(\angle BEA) = 180° - (45° + 35°) = 100°$

$\because \overline{AG} \cap \overline{BD} = \{E\}$

$\therefore m(\angle DEG) = m(\angle BEA) = 100°$ (The req.)

[b] In Δ ABC :

$\because$ E is the midpoint of $\overline{AB}$

, D is the midpoint of $\overline{AC}$

$\therefore DE = \frac{1}{2} BC$

, similarly : $ME = \frac{1}{2} AC$

$\therefore DE = MC$

, $ME = CD$

$\therefore$ DEMC is a parallelogram (Q.E.D.)

4

[a] In Δ ABC :

$\because m(\angle ACB) = 90°$

$\therefore (AC)^2 = (AB)^2 - (BC)^2 = (13)^2 - (12)^2 = 25$

$\therefore AC = \sqrt{25} = 5$ cm. (First req.)

, the perimeter of the figure ABCD

$= 13 + 12 + 4 + 3 = 32$ cm. (Second req.)

[b] In Δ ABC :

$\because$ D is the midpoint of $\overline{AB}$

, F is the midpoint of $\overline{BC}$

$\therefore AC = 2\ DF = 2 \times 4 = 8$ cm.

, $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore BC = 2\ DE = 2 \times 6 = 12$ cm.

, $\because$ F is the midpoint of $\overline{BC}$

, E is the midpoint of $\overline{AC}$

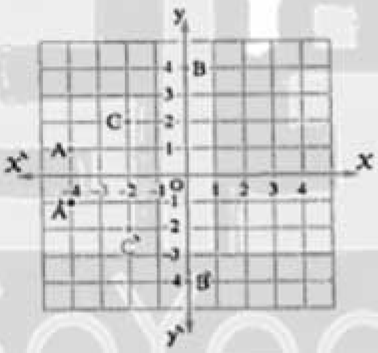
$\therefore AB = 2\ FE = 2 \times 3 = 6$ cm.

$\therefore$ The perimeter of $\Delta ABC = 8 + 12 + 6 = 26$ cm.

(The req.)

5

[a]



[b] In Δ ABC :

$m(\angle ACB) = 180° - (30° + 90°) = 60°$

$\therefore m(\angle DCF) = m(\angle ACB) = 60°$ (V.O.A.)

$\therefore$ From the quadrilateral CDEF :

$m(\angle E) = 360° - (120° + 60° + 90°) = 90°$

(The req.)

2 Cairo

1

1 b 2 d 3 a 4 a 5 d 6 c

2

1 a trapezium. 2 135°
3 (– 3 , 1) 4 9 5 130°

3

[a] In Δ ABC :

$\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore \overline{DE} \,/\!/\, \overline{BC}$, $\because F \in \overline{CB}$

$\therefore \overline{DE} \,/\!/\, \overline{BF}$ (1)

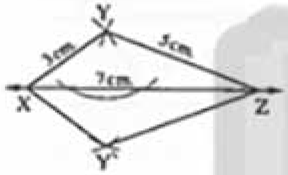
, $DE = \frac{1}{2}$ BC , $\because BF = \frac{1}{2}$ BC

$\therefore$ DE = BF (2)

From (1) and (2) :

$\therefore$ DFBE is a parallelogram. (Q.E.D.)

[b]



4

[a] In Δ ABC :

$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$= (9)^2 + (12)^2 = 81 + 144 = 225$

$\therefore AC = \sqrt{225} = 15$ cm. (First req.)

, In Δ ACD

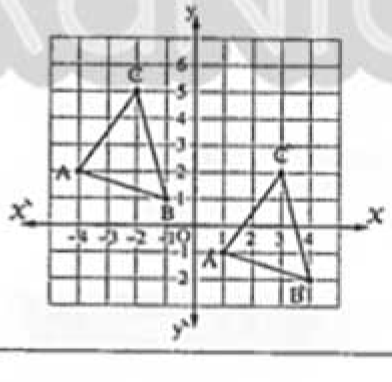
$\because m(\angle ACD) = 90°$

$\therefore (AD)^2 = (AC)^2 + (CD)^2$

$= (15)^2 + (20)^2 = 225 + 400 = 625$

$\therefore AD = \sqrt{625} = 25$ cm. (Second req.)

[b]



5

[a] In Δ COE :

$m(\angle COE) = 180° - (50° + 35°) = 95°$

$\therefore m(\angle BOD) = m(\angle COE) = 95°$ (V.O.A.)

from the quadrilateral ABOD

$\therefore m(\angle B) = 360° - (85° + 100° + 95°) = 80°$

(The req.)

[b] $\because \overleftrightarrow{AC} \cap \overleftrightarrow{DE} = \{B\}$

$\therefore m(\angle CBE) = m(\angle ABD) = 40°$ (V.O.A.)

, $\because \overrightarrow{BE}$ bisects $\angle$ CBF

$\therefore m(\angle EBF) = m(\angle CBE) = 40°$

$\therefore m(\angle ABF) = 180° - (40° + 40°) = 100°$

(The req.)

3 Cairo

1

1 c 2 b 3 c 4 a 5 b 6 a

2

1 the third side 2 (– 2 , – 4)

3 2 4 120° 5 216

3

[a] In Δ ABC :

$\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{BC}$

$\therefore DE = \frac{1}{2}$ AC $\therefore$ DE = 4 cm.

, $\because$ D is the midpoint of $\overline{AB}$

, F is the midpoint of $\overline{AC}$

$\therefore DF = \frac{1}{2}$ BC $\therefore$ DF = 3.5 cm.

, $\because$ F is the midpoint of $\overline{AC}$

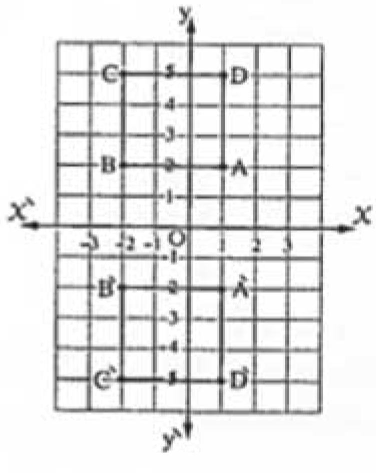
, E is the midpoint of $\overline{BC}$

$\therefore FE = \frac{1}{2}$ AB $\therefore$ FE = 2.5 cm.

$\therefore$ The perimeter of Δ DEF = 4 + 3.5 + 2.5 = 10 cm.

(The req.)

[b]



4

[a] In Δ ABC :

$\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (7)^2 + (24)^2 = 625$

$\therefore$ AC = $\sqrt{625}$ = 25 cm. (First req.)

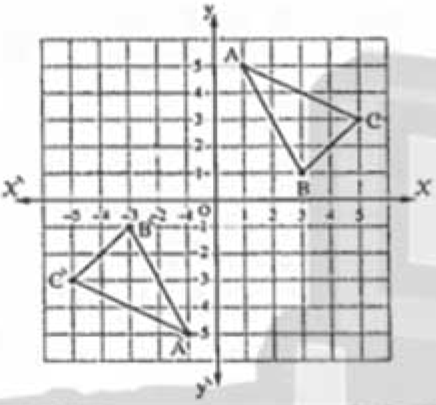
, In Δ ADC :

$\because$ m ($\angle$ D) = 90°

$\therefore (DC)^2 = (AC)^2 - (AD)^2 = (25)^2 - (15)^2 = 400$

$\therefore$ DC = $\sqrt{400}$ = 20 cm. (Second req.)

[b]



5

[a] In Δ ABC :

$\because$ m ($\angle$ ABC) = 180° − (50° + 70°) = 60°

$\therefore$ m ($\angle$ DBE) = m ($\angle$ ABC) = 60° (V.O.A.)

from the quadrilateral BDFE :

$\therefore$ m ($\angle$ E) = 360° − (60° + 130° + 90°) = 80°

(The req.)

[b] $\because$ ABCD is a rhombus

, $\overline{BD}$ is a diagonal

$\therefore$ m ($\angle$ B) = 2 × 62° = 124°

$\therefore$ m ($\angle$ A) = 180° − 124° = 56° (The req.)

4 Giza

1

1 c 2 c 3 a 4 b 5 a 6 b

2

1 180° 2 equal in measure.

3 half 4 (3 , −5) 5 (3 , 0)

3

[a] The perimeter of a regular hexagon

= 6 × 15 = 90 cm. (First req.)

the measure of each angle = $\frac{(6-2) \times 180°}{6}$

= 120°

(Second req.)

[b] $\because$ ABCD is a parallelogram , m ($\angle$ B) = 120°

$\therefore$ m ($\angle$ C) = 180° − 120° = 60°

, m ($\angle$ D) = m ($\angle$ B) = 120°

, AD = BC = 7 cm.

, DC = AB = 5 cm. (The req.)

4

[a] $\because \overline{DE} \parallel \overline{BC}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore$ m ($\angle$ B) = m ($\angle$ D) = 50° (alternate angles)

In Δ ABC :

$\therefore$ m ($\angle$ BAC) = 180° − (50° + 30°) = 100°

(The req.)

[b] $\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore$ XY = $\frac{1}{2}$ AC $\therefore$ XY = 3 cm.

, $\because$ X is the midpoint of $\overline{AB}$

, Z is the midpoint of $\overline{AC}$

$\therefore$ XZ = $\frac{1}{2}$ BC $\therefore$ XZ = 5 cm.

, $\because$ Y is the midpoint of $\overline{BC}$

, Z is the midpoint of $\overline{AC}$

$\therefore$ YZ = $\frac{1}{2}$ AB $\therefore$ YZ = 4 cm.

$\therefore$ The perimeter of Δ XYZ = 3 + 5 + 4 = 12 cm.

(The req.)

5

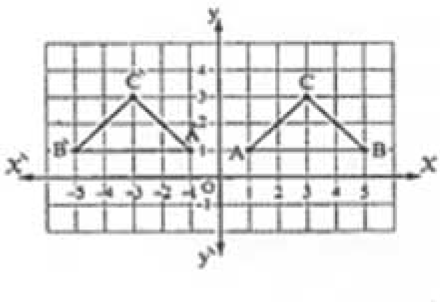
[a] In Δ ABC :

$\because$ m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (7)^2 + (24)^2 = 625$

$\therefore$ AC = $\sqrt{625}$ = 25 cm. (The req.)

[b]



5 Giza

1

1 (3 , 5) 2 bisects

3 perpendicular and bisect each other

4 360° 5 a right-angled triangle

2

1 d 2 d 3 a 4 d 5 a 6 c

3

[a] In ΔABC :

$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (4)^2 + (3)^2 = 25$

$AC = \sqrt{25} = 5$ cm. (First req.)

In ΔACD :

$\because m(\angle C) = 90°$

$\therefore (CD)^2 = (AD)^2 - (AC)^2 = (13)^2 - (5)^2 = 144$

$\therefore CD = \sqrt{144} = 12$ cm. (Second req.)

[b] $\because$ ABCD is a square

$\therefore BC = AD$

, $\because BC = CE$ $\therefore AD = CE$ (1)

$\because \overline{BC} \parallel \overline{AD}$, $E \in \overline{BC}$

$\therefore \overline{CE} \parallel \overline{AD}$ (2)

From (1) and (2) :

$\therefore AD = CE$ and $\overline{AD} \parallel \overline{CE}$

$\therefore$ ACED is a parallelogram (First req.)

$\because \overline{AC}$ is a diagonal in square ABCD

$\therefore m(\angle ACB) = 45°$

$\therefore m(\angle E) = m(\angle ACB) = 45°$

(corresponding angles) (Second req.)

4

[a] In ΔABC :

$\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore AC = 2\,XY = 2 \times 4 = 8$ cm.

, $\because$ X is the midpoint of $\overline{AB}$

, Z is the midpoint of $\overline{CA}$

$\therefore BC = 2\,XZ = 2 \times 5 = 10$ cm.

, $\because$ Z is the midpoint of $\overline{CA}$

, Y is the midpoint of $\overline{BC}$

$\therefore AB = 2\,YZ = 2 \times 6 = 12$ cm.

$\therefore$ The perimeter of $\Delta ABC = 8 + 10 + 12 = 30$ cm. (The req.)

[b] In parallelogram ABCD

$m(\angle BCD) = m(\angle A) = 60°$ (1)

, in parallelogram EBCF

$m(\angle EBC) = 180° - 130° = 50°$ (2)

In ΔXBC :

From (1) , (2) :

$m(\angle BXC) = 180° - (60° + 50°) = 70°$ (The req.)

5

[a] In ΔCEF :

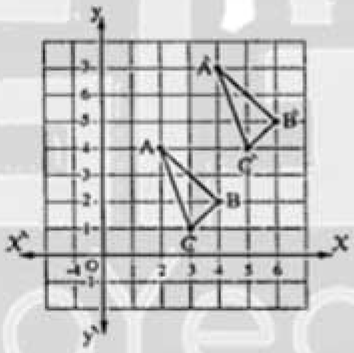
$m(\angle ECF) = m(\angle E) = m(\angle F) = 60°$

$\therefore m(\angle BCD) = m(\angle ECF) = 60°$ (V.O.A.)

from the quadrilateral ABCD

$\therefore m(\angle D) = 360° - (80° + 120° + 60°) = 100°$ (The req.)

[b]



6 Alexandria

1

1 a cute

2 half the length of the third side.

3 125 4 (2 , − 1) 5 120°

2

1 d 2 b 3 a 4 c 5 a 6 d

3

[a] $\because \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle ACD) = m(\angle A) = 50°$ (alternate angles)

, $\because \angle ACE$ is a right angle

$\therefore m(\angle ECD) = 90° - 50° = 40°$

$, \because m(\angle E) = m(\angle ECD) = 40°$

and they are alternate angles

$\therefore \overrightarrow{CD} \parallel \overrightarrow{EF}$

$, \because \overrightarrow{AB} \parallel \overrightarrow{CD}$ $\therefore \overrightarrow{AB} \parallel \overrightarrow{EF}$ (Q.E.D.)

[b] (0 , 0)

4

[a] In ΔABC :

$\because m(\angle B) = 90°$

$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 400 - 144 = 256$

$\therefore AB = \sqrt{256} = 16$ cm.

$, \because BD = 9$ cm.

$\therefore AD = 16 - 9 = 7$ cm. (First req.)

$, \because AE = 2\ BC = 2 \times 12 = 24$ cm.

$, \because \overline{AE} \parallel \overline{BC}$, $\overline{AB}$ is a transversal

$\therefore m(\angle EAB) = m(\angle B) = 90°$ (alternate angles)

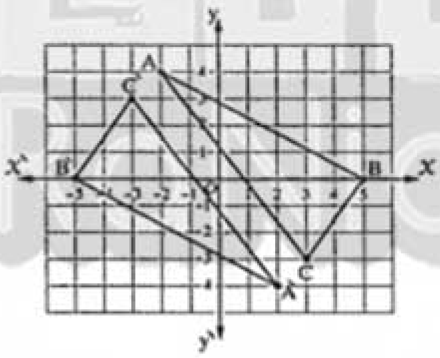
$\therefore$ In ΔEAD :

$\because m(\angle EAD) = 90°$

$\therefore (ED)^2 = (AE)^2 + (AD)^2 = (24)^2 + (7)^2 = 625$

$\therefore ED = \sqrt{625} = 25$ cm. (Second req.)

[b]



5

[a] $\because M \in \overline{AC}$

$\therefore m(\angle BMC) = 180° - 70° = 110°$

$\therefore$ In ΔBMC :

$m(\angle MCB) = 180° - (110° + 40°) = 30°$

$\therefore m(\angle MCB) = m(\angle MAD)$

and they are alternate angles.

$\therefore \overline{AD} \parallel \overline{BC}$

$, \because \overline{AB} \parallel \overline{DC}$

$\therefore ABCD$ is a parallelogram. (Q.E.D.)

[b] In ΔABC :

$\because D$ is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{BC}$

$\therefore DE = \frac{1}{2} AC$ $\therefore DE = 3.5$ cm.

$, \because D$ is the midpoint of $\overline{AB}$

, F is the midpoint of $\overline{AC}$

$\therefore DF = \frac{1}{2} BC$ $\therefore DF = 4$ cm.

$, \because E$ is the midpoint of $\overline{BC}$

, F is the midpoint of $\overline{AC}$

$\therefore EF = \frac{1}{2} AB$ $\therefore EF = 2.5$ cm.

$\therefore$ The perimeter of ΔDEF

$= 3.5 + 4 + 2.5 = 10$ cm. (The req.)

7 Alexandria

1

1 180° 2 (4 , 2)

3 bisects the third side.

4 (2 , − 1) 5 half the length of the third side.

6 parallelogram

2

1 c 2 d 3 b 4 d 5 c

3

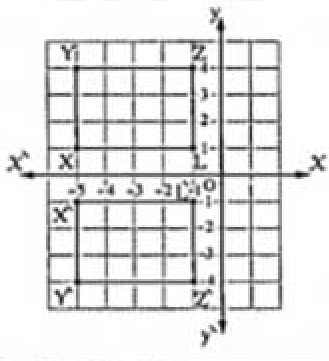
[a] In ΔABC :

$\because m(\angle A) = 90°$

$\therefore (AC)^2 = (CB)^2 - (AB)^2 = 100 - 36 = 64$

$\therefore AC = \sqrt{64} = 8$ cm. (The req.)

[b]



4

[a] In ΔABC :

$\because D$ is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{BC}$

$\therefore DE = \frac{1}{2} AC$ $\therefore DE = 4$

, $\because$ D is the midpoint of $\overline{AB}$

, F is the midpoint of $\overline{AC}$

$\therefore DF = \frac{1}{2} BC$ $\therefore DF = 5$

, $\because$ E is the midpoint of $\overline{BC}$

, F is the midpoint of $\overline{AC}$

$\therefore EF = \frac{1}{2} AB$ $\therefore EF = 3.5$ cm.

$\therefore$ The perimeter of Δ FDE

$= 4 + 5 + 3.5 = 12.5$ cm. (The req.)

[b] Let the measures of the interior angles of the quadrilateral be $2X, 2X, 3X, 5X$

$\because$ the sum of the measures of the interior angles of the quadrilateral $= 360°$

$\therefore 2X + 2X + 3X + 5X = 360°$

$\therefore 12X = 360°$ $\therefore X = \frac{360°}{12} = 30°$

$\therefore$ The measure of the biggest angle $= 5 \times 30° = 150°$

(The req.)

5

[a] $(3, -8)$

[b] In Δ ABC :

$\because \angle$ ACD is an exterior angle of Δ ABC

$\therefore m(\angle ACD) = m(\angle A) + m(\angle B)$

$= 30° + 30° = 60°$ (The req.)

8 El-Kalyoubia

1

1 b 2 b 3 d 4 c 5 d 6 b

2

1 2 2 8000 3 $(-3, -2)$

4 140° 5 120

3

[a] In Δ ABC :

$m(\angle ABC) = 180° - (70° + 50°) = 60°$

, $m(\angle DBF) = m(\angle ABC) = 60°$ (V.O.A.)

from the quadrilateral BDEF

$\therefore m(\angle F) = 360° - (60° + 140° + 90°) = 70°$

(The req.)

[b] $\because$ D is the midpoint of $\overline{AB}$

, F is the midpoint of $\overline{AC}$

$\therefore DF = \frac{1}{2} BC = 6$ cm.

, $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{BC}$

$\therefore DE = \frac{1}{2} AC = 5$ cm.

, $\because CE = \frac{1}{2} BC = 6$ cm. , $CF = \frac{1}{2} AC = 5$ cm.

$\therefore$ The perimeter of DECF

$= 6 + 5 + 6 + 5 = 22$ cm. (The req.)

4

[a] $\because$ ABCD is a square $\therefore \overline{AD} \parallel \overline{BC}$

, $\because E \in \overline{BC}$ $\therefore \overline{AD} \parallel \overline{CE}$

, $\because \overline{AC} \parallel \overline{ED}$

$\therefore$ ACED is a parallelogram. (First req.)

, $\because \overline{AC}$ is a diagonal in square ABCD

$\therefore m(\angle ACB) = 45°$

$\therefore m(\angle ACE) = 180° - 45° = 135°$ (Second req.)

[b] In Δ ADC :

$\because m(\angle ADC) = 90°$

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$

$\therefore CD = \sqrt{324} = 18$ cm.

In Δ ADB :

$m(\angle ADB) = 90°$

$\therefore (BD)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$

$\therefore BD = \sqrt{100} = 10$ cm.

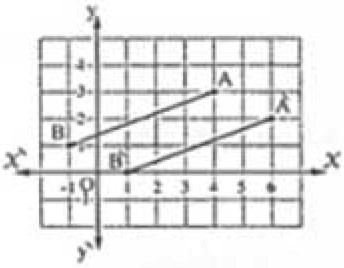
$\therefore BC = 18 + 10 = 28$ cm. (First req.)

$\therefore$ The area of Δ ABC

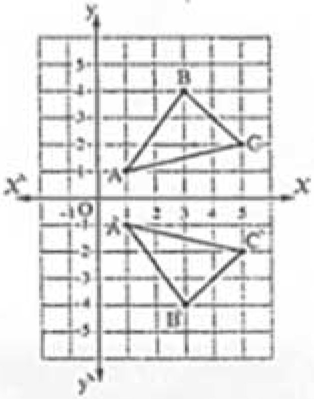
$= \frac{1}{2} \times 28 \times 24 = 336$ cm². (Second req.)

5

[a]



[b]



9 El-Gharbia

1

1 d 2 d 3 b 4 c 5 c 6 c

2

1 parallel to the third side.
2 120 3 $(-3, -2)$
4 equal in measure. 5 a right angle.

3

[a] $\because \overleftrightarrow{AC} \cap \overleftrightarrow{DE} = \{B\}$

$\therefore m(\angle CBE) = m(\angle ABD) = 40°$ (V.O.A.)

$\because \overrightarrow{BE}$ bisects $\angle CBF$

$\therefore m(\angle FBE) + m(\angle EBC) = 80°$

$\therefore m(\angle FBC) = 80°$

$\because B \in \overline{AC}$

$\therefore m(\angle ABF) = 180° - 80° = 100°$ (The req.)

[b] $\because \overleftrightarrow{DE} \parallel \overleftrightarrow{CB}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore m(\angle B) = m(\angle D) = 50°$ (alternate angles)

In ΔABC :

$m(\angle BAC) = 180° - (35° + 50°) = 95°$ (The req.)

4

[a] In ΔXYZ :

$\because$ D is the midpoint of $\overline{XY}$

, O is the midpoint of $\overline{YZ}$

$\therefore DO = \frac{1}{2} XZ = 3$ cm.

, $\because$ D is the midpoint of $\overline{XY}$

, E is the midpoint of $\overline{XZ}$

$\therefore DE = \frac{1}{2} YZ = 3.5$ cm.

, $\because$ O is the midpoint of $\overline{YZ}$

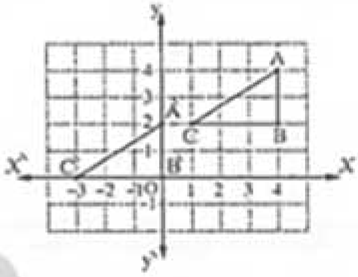
, E is the midpoint of $\overline{XZ}$

$\therefore OE = \frac{1}{2} XY = 2.5$ cm.

$\therefore$ The perimeter of ΔDOE

$= 3 + 3.5 + 2.5 = 9$ cm. (The req.)

[b]



5

[a] In ΔABC :

$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (12)^2 + (9)^2 = 225$

$\therefore AC = \sqrt{225} = 15$ cm.

In ΔACD :

$\because m(\angle ACD) = 90°$

$\therefore (AD)^2 = (AC)^2 + (CD)^2 = (15)^2 + (20)^2 = 625$

$\therefore AD = \sqrt{625} = 25$ cm. (The req.)

[b] $\because$ ABCD is a square $\therefore \overline{AD} \parallel \overline{BC}$

, $\because E \in \overline{BC}$ $\therefore \overline{AD} \parallel \overline{CE}$

, $\because \overline{AC} \parallel \overline{DE}$

$\therefore$ ACED is a parallelogram. (Q.E.D.)

10 El-Dakahlia

1

1 d 2 c 3 d 4 c 5 a 6 c

2

1 120° 2 35° 3 9 cm.
4 $(9, -6)$ 5 a right angle.

3

[a] In ΔADC :

$\because m(\angle ADC) = 90°$

$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$

$\therefore CD = \sqrt{324} = 18$ cm.

In ΔADB :

$\because m(\angle ADB) = 90°$

$\therefore (BD)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$

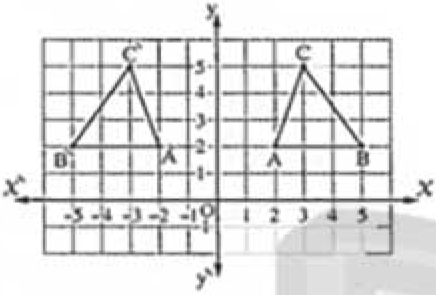
$\therefore BD = \sqrt{100} = 10$ cm.

$\therefore BC = 18 + 10 = 28$ cm. (First req.)

$\therefore$ The area of ΔABC

$= \frac{1}{2} \times 28 \times 24 = 336 \text{ cm}^2$. (Second req.)

[b]



4

[a] In ΔDEF :

$\because m(\angle DFE) = 60°$

$\therefore m(\angle CFG) = m(\angle DFE) = 60°$ (V.O.A.)

$\because$ the sum of measures of interior angles of the pentagon $= (5-2) \times 180° = 540°$

$\therefore m(\angle C) + m(\angle G)$

$= 540° - (100° + 90° + 60°) = 290°$

, $\because m(\angle C) = m(\angle G)$

$\therefore m(\angle C) = \frac{290°}{2} = 145°$ (The req.)

[b] In ΔABC :

$\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore \overline{DE} \parallel \overline{BC}$, $DE = \frac{1}{2} BC = 6$ cm.

, In ΔFDE :

$\because$ X is the midpoint of $\overline{DF}$

, $\overline{XY} \parallel \overline{DE}$

$\therefore$ Y is the midpoint of $\overline{EF}$

$\therefore XY = \frac{1}{2} DE = 3$ cm. (The req.)

5

[a] $\because$ ABCD is a square

$\therefore \overline{AD} \parallel \overline{BC}$

, $\because E \in \overrightarrow{BC}$ $\therefore \overline{AD} \parallel \overline{CE}$

, $\because \overline{AC} \parallel \overline{ED}$

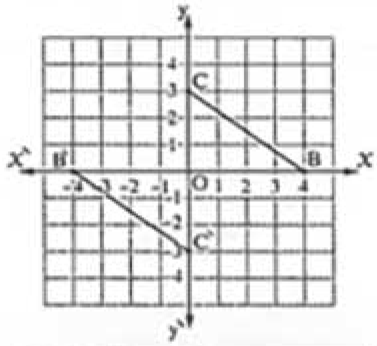
$\therefore$ ACED is a parallelogram. (First req.)

, $\because \overline{AC}$ is a diagonal in square ABCD

$\therefore m(\angle ACB) = 45°$

$\therefore m(\angle ACE) = 180° - 45° = 135°$ (Second req.)

[b]



11 Suez

1

1 b 2 b 3 d 4 b 5 d 6 c

2

1 140° 2 bisects the third side.

3 (1 , 2) 4 540° 5 $\overline{AC} \perp \overline{BD}$

3

[a] $m(\angle ABE) = 180° - 116° = 64°$

, $\because \overrightarrow{BD}$ bisects $\angle ABE$

$\therefore m(\angle ABD) = \frac{1}{2} m(\angle ABE) = 32°$ (The req.)

[b] $m(\angle DBE) = 360° - (110° + 35° + 140°) = 75°$ (The req.)

4

[a] $\because \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle ACD) = m(\angle A) = 50°$ (1) (alternate angles)

, $\because \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$, $\overleftrightarrow{CE}$ is a transversal

$\therefore m(\angle ECD) = m(\angle E) = 40°$ (2) (alternate angles)

From (1) , (2) :

$\therefore m(\angle ACE) = m(\angle ACD) + m(\angle ECD)$

$= 50° + 40° = 90°$ (The req.)

[b] In ΔABC :

$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (6)^2 + (8)^2$

$= 36 + 64 = 100$

$\therefore AC = \sqrt{100} = 10$ cm. (The req.)

5

[a] 1 (– 2 , 3) 2 (– 3 , – 2)

3 (– 3 , – 2) 4 3

[b] In Δ ABC :

$m(\angle ACB) = 180° - (60° + 90°) = 30°$

$\therefore m(\angle DCF) = m(\angle ACB) = 30°$ (V.O.A.)

$\therefore$ in the polygon CDEF :

$m(\angle E) = 360° - (30° + 120° + 90°) = 120°$

(The req.)

12 Port Said

1

1 b 2 c 3 b 4 b 5 b 6 d

2

1 $\frac{1}{2}$ 2 $(3, -2)$ 3 $360°$

4 $120°$ 5 6

3

[a] In Δ ABC :

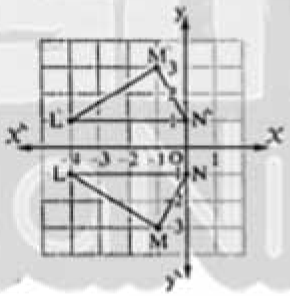
$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$= (3)^2 + (4)^2 = 9 + 16 = 25$

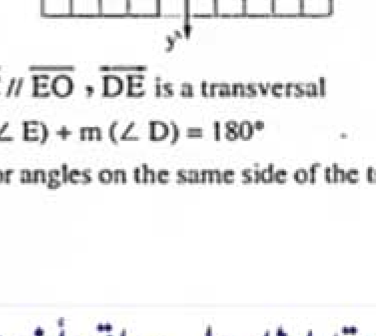
$\therefore AC = \sqrt{25} = 5$ cm. (The req.)

[b]



4

[a]



[b] $\because \overline{DC} \parallel \overline{EO}$, $\overleftrightarrow{DE}$ is a transversal

$\therefore m(\angle E) + m(\angle D) = 180°$

(interior angles on the same side of the transversal)

$\therefore m(\angle E) = 180° - 90° = 90°$

$\therefore m(\angle C) = 360° - (120° + 90° + 90°) = 60°$

(The req.)

5

[a] In Δ ABC :

$\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore CB = 2\, ED = 16$ cm. (The req.)

[b] $\because \angle ACD$ is an exterior angle of Δ ABC

$\therefore m(\angle ACD) = 25° + 25° = 50°$ (The req.)

13 Kafr El-Sheikh

1

1 c 2 b 3 b 4 c 5 d 6 b

2

1 $(1, 1)$ 2 a square 3 $(1, -2)$

4 half 5 28

3

[a] From the pentagon ABCDE

$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D)$

$+ m(\angle DEA) = 540°$

$\therefore m(\angle C) = 540° - (110° + 120° + 90° + 115°)$

$= 105°$ (The req.)

[b] $\because \overleftrightarrow{ED} \parallel \overline{BC}$, $\overleftrightarrow{EC}$ is a transversal

$\therefore m(\angle C) + m(\angle E) = 180°$

(interior angles on the same side of the transversal)

$\therefore m(\angle C) = 180° - 110° = 70°$

, $\because \overleftrightarrow{FH} \parallel \overline{BC}$, $\overleftrightarrow{FB}$ is a transversal

$\therefore m(\angle B) + m(\angle F) = 180°$

(interior angles on the same side of the transversal)

$\therefore m(\angle B) = 180° - 130° = 50°$

$\therefore$ In Δ ABC :

$m(\angle BAC) = 180° - (70° + 50°) = 60°$ (The req.)

4

[a] In Δ ABC :

$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (7)^2 + (24)^2 = 625$

$\therefore AC = \sqrt{625} = 25$ cm. (First req.)

In ΔACD :

$\because m(\angle D) = 90°$

$\therefore (DC)^2 = (AC)^2 - (AD)^2 = (25)^2 - (15)^2 = 400$

$\therefore DC = \sqrt{400} = 20$ cm. (Second req.)

[b] In ΔABC :

$\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$

$\therefore XY = \frac{1}{2} BC = 4$ cm.

, $\because$ X is the midpoint of $\overline{AB}$

, Z is the midpoint of $\overline{BC}$

$\therefore XZ = \frac{1}{2} AC = 4$ cm.

, $\because$ Y is the midpoint of $\overline{AC}$

, Z is the midpoint of $\overline{BC}$

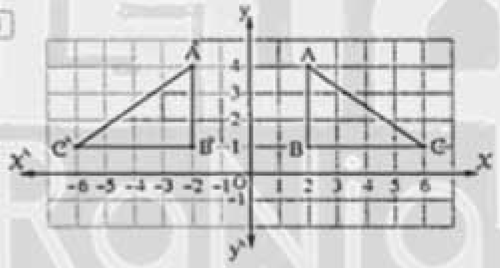
$\therefore YZ = \frac{1}{2} AB = 3$ cm.

$\therefore$ The perimeter of $\Delta XYZ = 4 + 4 + 3 = 11$ cm.

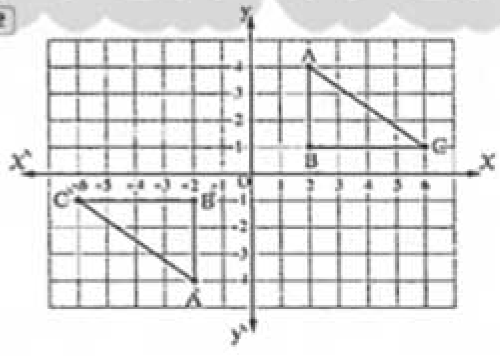
(The req.)

5

[a] 1



2



[b] $\because E \in \overleftrightarrow{CD}$

$\therefore m(\angle ADC) = 180° - 60° = 120°$

, $\because$ ABCD is a parallelogram

$\therefore m(\angle B) = m(\angle ADC) = 120°$ (The req.)

14 Beni Suef

1

1 c   2 a   3 c   4 c   5 c   6 d

2

1 equal in length.   2 bisects the third side.

3 the sum of areas of the squares on the sides of the right angle.

4 ΔZYC 5 equal in measure.

3

[a] $\because m(\angle EAC) + m(\angle BAC) + m(\angle EAB) = 360°$

$\therefore m(\angle BAC) = 360° - (130° + 90°) = 140°$

(First req.)

, $\because \overline{AB} \mathbin{/\mkern-5mu/} \overline{CD}$, $\overline{AC}$ is a transversal to them.

$\therefore m(\angle C) + m(\angle CAB) = 180°$

(Two interior angles in the same side of the transversal)

$\therefore m(\angle C) = 180° - 140° = 40°$ (Second req.)

[b] $\because$ ABCD is a rhombus , $\overline{AC}$ is a diagonal

$\therefore m(\angle BCD) = 2 \times 32° = 64°$

$\therefore m(\angle D) = 180° - 64° = 116°$ (The req.)

4

[a] In ΔABC :

$\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore BC = 2 DE = 2 \times 4 = 8$ cm.

, $BD = AD = 3$ cm.

, $CE = AE = 2$ cm.

$\therefore$ The perimeter of DBCE

$= 3 + 8 + 2 + 4 = 17$ cm. (The req.)

[b] In ΔADC :

$\because m(\angle ADC) = 90°$

$\therefore (AD)^2 = (AC)^2 - (CD)^2$

$= (20)^2 - (16)^2 = 144$

$\therefore AD = \sqrt{144} = 12$ cm. (First req.)

In ΔADB : $\because m(\angle ADB) = 90°$

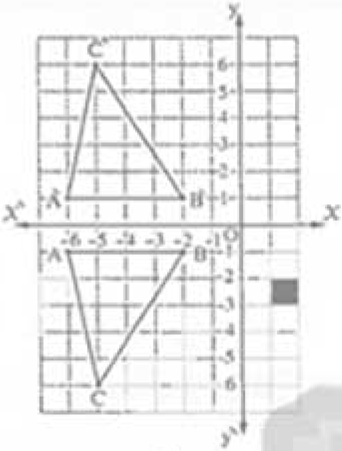
$\therefore (AB)^2 = (AD)^2 + (BD)^2$

$= (12)^2 + (9)^2 = 225$

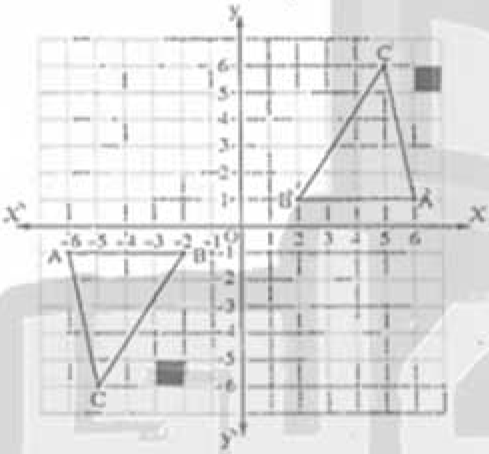
$\therefore AB = \sqrt{225} = 15$ cm. (Second req.)

5

1



2



15 Souhag

1

1 c   2 c   3 c   4 d   5 a   6 a

2

1 parallel to the third side , equal to half of its length.

2 15   3 $(-2, -1)$

4 Y   5 bisects the third side.

3

[a] In Δ ABC :

$\because$ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$

$\therefore XY = \frac{1}{2} BC = 4.5$ cm.

, $\because$ X is the midpoint of $\overline{AB}$

, Z is the midpoint of $\overline{BC}$

$\therefore XZ = \frac{1}{2} AC = 5$ cm.

, $\because$ Y is the midpoint of $\overline{AC}$

, Z is the midpoint of $\overline{BC}$

$\therefore YZ = \frac{1}{2} AB = 3.5$ cm.

$\therefore$ The perimeter of Δ XYZ $= 4.5 + 5 + 3.5$
$= 13$ cm. (The req.)

[b] $\because \overline{DE} \parallel \overline{CB}$, $\overline{BD}$ is a transversal

$\therefore m(\angle B) = m(\angle D) = 60°$ (alternate angles)

$\because \angle DAC$ is an exterior angle of Δ ABC

$\therefore m(\angle DAC) = 50° + 60° = 110°$ (The req.)

4

[a] $\because \overline{DA} \parallel \overline{CB}$, $\overline{AB}$ is a transversal

$\therefore m(\angle B) = m(\angle EAB) = 120°$
(alternate angles)

$\therefore m(\angle B) + m(\angle C) = 120° + 60° = 180°$

and they are interior angles on the same side of the transversal

$\therefore \overline{AB} \parallel \overline{CD}$

, $\because \overline{AD} \parallel \overline{BC}$

$\therefore$ ABCD is a parallelogram (Q.E.D)

[b] In Δ ABC :

$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (3)^2 + (4)^2 = 25$

$\therefore AC = \sqrt{25} = 5$ cm. (First req.)

In Δ ACD :

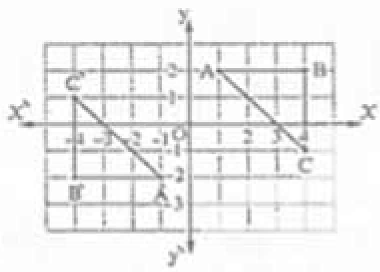
$\because m(\angle ACD) = 90°$

$\therefore (CD)^2 = (AD)^2 - (AC)^2 = (13)^2 - (5)^2 = 144$

$\therefore CD = \sqrt{144} = 12$ cm. (Second req.)

5

[a]



[b] 1 CBM   2 MCD

3 CDM   4 DEM